Finite Element Simulation of Thermal Vortices

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## Simulation Targets

### Natural vortex development
- Thermal convection plume from heated surface
- “Vortex stretching” of far field vorticity
- Sensitivity to initial, boundary conditions

### Artificial vortex development
- Vortex pinning
  - Surface heating
  - Surface geometry
- Vorticity introduction
  - Surface vanes
- Power generation
  - Near-surface turbine

### Vortex stability and control
- Parameter sensitivities
- Turning vane design
- Turbine sizing
Modular Design

Simulation application factoring

Interfaces must expose complex functionality but still enable independent implementation development for:

- Numerics optimization: discretization (libMesh) vs. algebraic solver (PETSc, AMGCL, Trilinos)
- Code optimization: physics (GRINS) vs. assembly (FEMSystem)
- Code verification: module uses vs. unit tests
- Solution verification: physics (GRINS) vs. error estimation (libMesh)
- Uncertainty quantification: simulation vs. UQ (QUESO, DAKOTA)
- Interdisciplinary collaboration: physics vs. any code

The libMesh Finite Element Method (FEM) library provides:

- Application Programming Interfaces (APIs) for such designs
- Implementations of physics-independent FEM code
- Common interfaces to third-party tools
libMesh

- Distributed open source development
- Hybrid parallel execution
  - MPI + TBB well-tested
  - + MIC + GPU in development
- Serialized or distributed parallel meshes
- Object-oriented finite element, quadrature, time integration, geometry, etc. classes
- Assorted error indicators, adaptive mesh refinement/coarsening
- Automatic adjoint solves
  - Quantity of Interest error estimation
  - Goal-oriented refinement
  - Sensitivity calculations in high-dimensional parameter spaces
GRINS

General Reacting Incompressible Navier-Stokes

- Created for a sub-physics calibration during PSAAP I
- Evolved into more general FEM multiphysics framework
- Solvers, meshing, parallelism via FEMSystem framework in libMesh
- Designed for flexibility in adding new physics, solvers, initial/boundary conditions, etc.
- New simulation designs via input file
- Easy experimentation with new numerical methods in code
- Statistical forward and inverse problems
Simulation Formulation

Physics

• 3-D Incompressible Navier-Stokes
  ▶ Fully coupled implicit finite element discretization
• Heat convection/diffusion
• Boussinesq buoyancy approximation
• Neglecting viscous heating

\[
\begin{bmatrix}
  \frac{\partial (\rho u)}{\partial t} + \rho u \cdot \nabla u + \nabla p - \mu \nabla^2 u + \rho_0 \beta T (T - T_0) g \\
  \nabla \cdot u \\
  \frac{\partial (\rho c_p T)}{\partial t} + \rho c_p u \cdot \nabla T - k \nabla^2 u
\end{bmatrix} = 0
\]
Simulation Formulation

- $H^1$ Finite Element weak formulation

$$R^G \left( \begin{bmatrix} \mathbf{u} \\ p \\ T \end{bmatrix}, \begin{bmatrix} \mathbf{v} \\ q \\ S \end{bmatrix} \right) \equiv \left( \frac{\partial (\rho \mathbf{u})}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} + \rho_0 \beta_T (T - T_0) \mathbf{g}, \mathbf{v} \right)_\Omega$$

- Adjoint-based residual-consistent stabilization terms

$$R(\mathbf{U}, \mathbf{V}) = R^G(\mathbf{U}, \mathbf{V}) + (\tau R(\mathbf{U}), \delta R^*(\mathbf{V}))_{\Omega'}$$
Finite Element Formulation

Approximation Details

- Taylor-Hood triquadratic+trilinear elements in initial experiments
- Equal-order trilinear elements in current stabilized runs
- 25600 and 86400 element mesh experiments
- Using uniform meshes to test stabilization options
- Analytic stabilization terms
- Finite-differenced Jacobians
Simulation Numerics

**Solvers**

- Theta method implicit time integration
  - 0.1s time steps
  - Adaptive time stepping options
- Inexact Newton-Krylov, $10^{-10}$ relative tolerance
- Automatic line search (triggered on initial time step(s))
- GMRes linear solver
- Stampede supercomputer, 8 nodes, 16 cores per node
- Additive Schwarz parallel preconditioning, one level overlap
- LU solve on each subdomain
  - Beat ILUn, SOR, etc in benchmarks
  - Slight speed improvements by increasing subdomain count
Simulation Conditions

“Laboratory” vortex

- 20m × 20m room, 4m ceilings
- Adiabatic (optionally periodic) side wall boundary conditions
- Isothermal ceiling, 280K
- 4m × 4m floor “hot spot”, 300 − 370K
- Optional far field “breeze” (peak 0.1m/s) velocity on one wall
- Optional forcing at vortex base (turning vanes) simulated via small (peak up to 2m/s) Rankine vortex Dirichlet conditions
Current Results

“Laboratory” vortices

- Thermal plume formation as expected
- Vorticity convected upward
- Plume narrowing depending on “hot spot” parameters
Current Results

Vortex Stability

- Unforced initial vorticity dissipates
- Far field “breeze” vorticity can be overwhelmed by thermal plume
- Forcing at vortex base (i.e. turning vanes) is required

Simulation Fidelity

- Far field is indirectly important, may be overresolved
- Refinement needed to better resolve vortex, boundary layers
Simulation Requirements

- **Laboratory domain simulations**
  - Confirm simulation of observed phenomena
  - Angular momentum introduced through boundary conditions
  - Angular momentum introduced through turning vanes
  - Effect of turning vanes modeled through body forces

- **Array of atmospheric Solar Vortices (periodic BC)**
  - Assessment of vortex conditions for use by UTRC
  - Assess effects of array spacing & pattern
  - Dependence on external conditions (wind, slope, solar input)
Next Simulation Targets

Atmospheric Solar Vortex simulations

- Periodic side boundary conditions for spaced arrays
- Constant thermal flux boundary conditions at ground
- Neumann vs Robin boundary conditions at altitude?

Parameter investigations

- Cross wind
- Ground slope
- Array spacing, orientation
Work in Progress

Grid improvements

- Turning vanes mesh from Gridgen (Sigfried Haering, student)
  - Replaces boundary-condition-based forcing
  - Body forces still necessary to model turbine
- Vortex, boundary layer adaptive refinement via libMesh
- Far-field adaptive coarsening via libMesh

Stabilization testing

- MMS verification of adjoint-based stabilization physics
- Comparison to new SPGSM (Gravemeier & Wall 2011) stabilization physics option
Updated Future Simulation Plans

- Simulate effects of turbine using body forces
  - Evaluate disruption of vortex by turbine
  - Evaluate effects/utility of shrouding
  - Determine maximum power extraction

- Simulate detailed rotor configurations
  - Simulate in rotating reference frame with body force representation of turning vanes
  - Evaluate effectiveness of rotor designs

- Vane & Rotor Design
  - Need to develop vane & rotor design principles
  - Use CFD to help evaluate