Homework set 1 — APPM5450

From the textbook: 7.1, 7.3, 7.4. Optional: 7.2, 7.5.

Problem 1: Suppose that $H$ is a Hilbert space, and that $(\psi_n)_{n=1}^{\infty}$ is an ON-set in $H$. Let $P$ denote the set of finite linear combinations of elements in $(\psi_n)_{n=1}^{\infty}$. Prove that:

$(\psi_n)_{n=1}^{\infty}$ is a basis for $H$ ⇔ $P$ is dense in $H$.

Problem 2: Suppose that $f, g \in C(\mathbb{T})$. Prove that:

(a) $f \ast g \in C(\mathbb{T})$.

(b) $f \ast g = g \ast f$. 