Problem 1: Let $X$ be a set with infinitely many members. We define a collection $\mathcal{T}$ of subsets of $X$ by saying that a set $\Omega \in \mathcal{T}$ if either $\Omega^c = X \setminus \Omega$ is finite, or if $\Omega$ is the empty set. Verify that $\mathcal{T}$ is a topology on $X$. This topology is called the “co-finite” topology on $X$. Describe the closed sets.

Problem 2: Let $X$ denote a finite set, and let $\mathcal{T}$ be a metrizable topology on $X$. Prove that $\mathcal{T}$ is the discrete topology on $X$.

Problem 3: Consider the set $X = \{a, b, c\}$, and the collection of subsets $\mathcal{T} = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. Is $\mathcal{T}$ a topology? Is $\mathcal{T}$ a metrizable topology?

Problem 4: Consider the integral equation

\[(*) \quad u(x) = \pi^2 \sin(x) + \frac{3}{2} \int_0^{\cos(x)} |x - y| u(y) dy.\]

Prove that $(*)$ has a unique solution in $C([0, 1])$. 