Quiz:
Feel free to hand this in anonymously.

**Question 1:** Mark in each slot:
- “no” if the value does not exist
- $\pm \infty$ if the value is infinite
- “finite” if the value exists and is finite (give an exact value if you know it but don’t spend time on trying to figure it out).

<table>
<thead>
<tr>
<th>$x_n$</th>
<th>$\lim_{n\to\infty} x_n$</th>
<th>$\sup{x_n}_{n=1}^\infty$</th>
<th>$\limsup_{n\to\infty} x_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/n$</td>
<td>$\frac{1}{n}$</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$(-1)^n + \sin(n)/n$</td>
<td>$\text{skip}$</td>
<td>$\text{skip}$</td>
<td>$\text{skip}$</td>
</tr>
<tr>
<td>$\sum_{j=1}^n \frac{1}{j}$</td>
<td>$\text{skip}$</td>
<td>$\text{skip}$</td>
<td>$\text{skip}$</td>
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<tr>
<td>$\sum_{j=1}^n \frac{1}{j} - \log(n)$</td>
<td>$\text{skip}$</td>
<td>$\text{skip}$</td>
<td>$\text{skip}$</td>
</tr>
<tr>
<td>$\sum_{j=1}^n \frac{1}{j^2}$</td>
<td>$\text{skip}$</td>
<td>$\text{skip}$</td>
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</tr>
<tr>
<td>$\sum_{j=1}^n (-1)^j/j$</td>
<td>$\text{skip}$</td>
<td>$\text{skip}$</td>
<td>$\text{skip}$</td>
</tr>
<tr>
<td>$\sum_{j=1}^n (-1)^j/j^2$</td>
<td>$\text{skip}$</td>
<td>$\text{skip}$</td>
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</tr>
</tbody>
</table>

**Question 2:** Circle the sums that are absolutely convergent:

$$\sum_{j=1}^n \frac{1}{j}, \quad \sum_{j=1}^n \frac{1}{j^2}, \quad \sum_{j=1}^n (-1)^j/j, \quad \sum_{j=1}^n (-1)^j/j^2.$$

**Question 3:** Let $\alpha$ denote a real number, let $B = \{x \in \mathbb{R}^2 : |x| \leq 1\}$ and set

$$f(\alpha) = \int_B \frac{1}{|x|^{\alpha}} dA.$$

(a) For which values of $\alpha$ is $f(\alpha)$ finite?

(b) What is the answer if $B$ is the unit ball in $\mathbb{R}^n$ rather than $\mathbb{R}^2$?

**Question 4:** Let $f$ be a continuous function defined on the set $\Omega$. For each of the examples of sets given below, answer the following questions: Is $f$ necessarily bounded? Is $f$ necessarily uniformly continuous? (Give a counter examples if the answer is no.)

(a) $\Omega = \{x \in \mathbb{R}^2 : |x| \leq 2\}$.

(b) $\Omega = \{x \in \mathbb{R}^2 : 0 < |x| \leq 2\}$.

(c) $\Omega = \{x \in \mathbb{R}^2 : |x| \geq 2\}$.

(d) $\Omega = \bigcup_{n=1}^\infty [1/n, 1/n + 1/n^3]$. 
**Question 5:** Let \( \{ F_n \}_{n=1}^\infty \) be a sequence of closed sets in \( \mathbb{R}^2 \) and let \( \{ G_n \}_{n=1}^\infty \) be a sequence of open sets in \( \mathbb{R}^2 \). Which of the following four sets are necessarily open? Necessarily closed?

(a) \( \bigcup_{n=1}^\infty F_n \)
(b) \( \bigcap_{n=1}^\infty F_n \)
(c) \( \bigcup_{n=1}^\infty G_n \)
(d) \( \bigcap_{n=1}^\infty G_n \)

**Question 6:** The parallelogram law in \( \mathbb{R}^n \) says that for any \( x, y \in \mathbb{R}^n \)

\[ |x + y|^2 + |x - y|^2 = \]

**Question 7:** Let \( \Omega \) be a bounded set in \( \mathbb{Q} \) (the set of rational numbers). Does the set \( \Omega \) necessarily have a least upper bound in \( \mathbb{Q} \)? If no, give a counter example.

**Question 8:** Let \( \Omega \) be a closed set in \( \mathbb{R}^3 \) (not necessarily bounded) and let \( \{ x_n \}_{n=1}^\infty \) denote a Cauchy sequence in \( \Omega \). Does \( x_n \) necessarily have a limit value in \( \Omega \)? If no, give a counter example.

**Question 9:** Let \( A \) be an \( n \times n \) matrix of real numbers. Give a sufficient condition for there to exist a unitary matrix \( U \), and a diagonal matrix \( D \) such that \( A = U D U^T \).

**Question 10:** Let \( f \) be a continuous function on the interval \([-\pi, \pi]\) and define for \( n = \ldots, -2, -1, 0, 1, 2, \ldots \) the complex number \( a_n \) by

\[ a_n = \int_{-\pi}^{\pi} e^{inx} f(x) \, dx. \]

Give the right hand side of the following equality:

\[ \sum_{n=-\infty}^{\infty} |a_n|^2 = \]