

Homework set 3 — CSE 383C / CS 383C / M 383E / ME 397, Fall 2026

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Due date: Friday October 9.

Hand in solutions to: 7.1, 7.5, 10.1, 10.2, 11.1 from the book. Also Problem 1 below.

Suggested problems (do not hand in): 7.3, 7.4, 8.3, 9.2, 10.4, 11.2 from the book.

Problem 1 (hand in solution): Let $\mathbf{A}$ be a matrix of size $m \times n$, with full column rank (so $m \geq n$, necessarily) with a reduced (“economy size”) QR factorization

$$\begin{array}{ccc} \mathbf{A} & = & \mathbf{Q} \quad \mathbf{R}. \\ m \times n & & m \times n \quad n \times n \end{array}$$

Throughout this problem, let $\kappa(\mathbf{X}) = \frac{\sigma_{\max}(\mathbf{X})}{\sigma_{\min}(\mathbf{X})}$ denote the standard 2-norm condition number. So far in class, we have defined this only for square matrices, but the same definition applies to a rectangular matrix with full column rank.

- (a) Show that $\mathbf{A}$ and $\mathbf{R}$ have the same singular values. Conclude that

$$\kappa(\mathbf{A}) = \kappa(\mathbf{R}).$$

- (b) Show that

$$\kappa(\mathbf{A}^* \mathbf{A}) = (\kappa(\mathbf{A}))^2.$$

- (c) Let ε be a number such that $0 < \varepsilon \leq 1$. Consider the matrix

$$\mathbf{A}_\varepsilon = \begin{bmatrix} 1 & 1 \\ \varepsilon & 0 \\ 0 & \varepsilon \end{bmatrix}.$$

Determine the singular values of $\mathbf{A}_\varepsilon$. (*Hint: Computing the eigenvalues of $\mathbf{A}_\varepsilon^* \mathbf{A}_\varepsilon$ may be helpful.*) Use them to compute

$$\kappa(\mathbf{A}_\varepsilon) \quad \text{and} \quad \kappa(\mathbf{A}_\varepsilon^* \mathbf{A}_\varepsilon).$$

How do these quantities behave as $\varepsilon \rightarrow 0$?

- (d) Briefly explain why these results give reason to be cautious about solving a least-squares problem by forming the normal equations when ε is small. (And more generally for ill-conditioned matrices.)