Homework set 4 — M341, Spring 2024

Hand in solutions: 5df and 16 from Section 2.3.

Suggested problems (do not hand in): 4, 5b, 6b, 12, and 18 from Section 2.3.

Problem 1: Let c be a real number, and consider the matrix

$$\mathsf{E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}.$$

(a) Let A be a matrix with three rows, and consider the matrix $\mathsf{B} = \mathsf{EA}$. The matrix B is the result of performing an elementary row operation on A. Which one?
(b) Specify a matrix F such that $\mathsf{EF} = \mathsf{I}$. (In other words, $\mathsf{F} = \mathsf{E}^{-1}$.) Observe that such a matrix F exists for *every* real number c, including $c = 0$.

Problem 2: Let c be a real number such that $c \neq 0$, and consider the matrix

$$\mathsf{E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & c \end{bmatrix}.$$

(a) Let A be a matrix with three rows, and consider the matrix $\mathsf{B} = \mathsf{EA}$. The matrix B is the result of performing an elementary row operation on A. Which one?
(b) Specify a matrix F such that $\mathsf{EF} = \mathsf{I}$. (In other words, $\mathsf{F} = \mathsf{E}^{-1}$.)
(c) Set $\mathsf{G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Prove that there cannot exist a matrix H such that $\mathsf{GH} = \mathsf{I}$.

Problem 3: Consider the matrix

$$\mathsf{E} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

(a) Let A be a matrix with three rows, and consider the matrix $\mathsf{B} = \mathsf{EA}$. The matrix B is the result of performing an elementary row operation on A. Which one?
(b) Specify a matrix F such that $\mathsf{EF} = \mathsf{I}$. (In other words, $\mathsf{F} = \mathsf{E}^{-1}$.)

Problem 4: Consider the two linear systems

$$\left\{ \begin{array}{rrrcr} x_1 & -x_2 & +x_3 & = & -1 \\ x_1 & -x_2 & +2x_3 & = & -2 \\ -x_1 & +2x_2 & -x_3 & = & 2 \end{array} \right. \qquad \text{and} \qquad \left\{ \begin{array}{rrrcr} y_1 & -y_2 & +y_3 & = & -1 \\ y_1 & -y_2 & +2y_3 & = & 1 \\ -y_1 & +2y_2 & -y_3 & = & 1 \end{array} \right.$$

Specify a matrix A, and column vectors B and C so that the two systems can be written as $\mathsf{AX} = \mathsf{B}$ and $\mathsf{AY} = \mathsf{C}$, respectively. Then form the extended matrix $\big[\mathsf{A}|\mathsf{B}\ \mathsf{C}\big]$, and transform it to its reduced row echelon form. Finally, specify the solutions X and Y.

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