Homework set 7 — M341, TuTh 9:30am - 10:45am section, Fall 2022

Hand in solutions to:
Section 3.3: 8.
Section 3.4: 1(a,d), 2(b), 3(b,d,f).

Problem 1 (hand in): Evaluate the determinants of the following matrices:

$$\mathsf{A} = \begin{bmatrix} 7 & 1 & 13 \\ 0 & 3 & -5 \\ 0 & 0 & 5 \end{bmatrix} \qquad \mathsf{B} = \begin{bmatrix} -4 & 3 & -2 & -2 \\ 0 & 2 & 3 & -9 \\ 4 & -3 & 7 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathsf{C} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{bmatrix} \qquad \mathsf{D} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Please motivate briefly how you arrived at the answer. (You are welcome to use a computer to verify your answers, of course. But please do not answer with just a number.)

Problem 2 (optional):

(a) Determine the characteristic polynomial of $\begin{bmatrix} 0 & 1 \\ -c_0 & -c_1 \end{bmatrix}$.

(b) Determine the characteristic polynomial of $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -c_0 & -c_1 & -c_2 \end{bmatrix}$.

(c) Determine the characteristic polynomial of the $n \times n$ matrix $\begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ -c_0 & -c_1 & -c_2 & -c_3 & \cdots & -c_{n-1} \end{bmatrix}$.

(d) Justify the claim that "an algorithm that computes all eigenvalues of a given $n \times n$ matrix can be used to find the roots of any polynomial of degree $n - 1$".

Optional problems: You are encouraged to work these! But do not hand in.
Section 3.2: 7, 9.
Section 3.4: 5(c,e), 11, 16.

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