Quiz for section exam 2

ⓘ This is a preview of the published version of the quiz

Started: Apr 25 at 4:29pm

Quiz Instructions

This exam is timed. You have 50 minutes from the time that you start. There are 10 questions total, so do not linger if you have difficulties with one question - you only have 5 minutes for each one.

Several of the questions ask you to identify which of a given number of statements are true or false. Be sure to write "T" or "F" in the given box (without the quotation marks). Do NOT write "t" or "f" or "true" or"false" or anything like that, just T or F.

Question 1 — 1 pts

Let $A = \begin{bmatrix} -4 & 12 & 6 \\ -2 & 5 & 2 \\ 1 & 0 & 1 \end{bmatrix}$. The matrix A has an eigenvector $v = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$. Specify the corresponding eigenvalue.

-1

$$Av = \begin{bmatrix} -4 & 12 & 6 \\ -2 & 5 & 2 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -8+12-6 \\ -4+5-2 \\ 2+0-1 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix} = (-1)v$$

Question 2 — 1 pts

Let t be a real number, and let $A = \begin{bmatrix} 2 & t & 3 \\ -1 & 4 & -1 \\ -3 & 9 & -4 \end{bmatrix}$. For which value of t is the vector $v = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ an eigenvector of A?

$Av = \begin{bmatrix} 2 & t & 3 \\ -1 & 4 & -1 \\ -3 & 9 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5+t \\ 2 \\ 2 \end{bmatrix}$ These imply that $\lambda = 2$?

-3

Since $\lambda = 2$, we have $(5+t) = 2 \cdot 1 = 2$

So $t = -3$

Question 3

1 pts

The matrix A has the characteristic polynomial $p(\lambda) = \lambda^3 - \lambda$ How many distinct eigenvalues does A have?

$\lambda^3 - \lambda = 0 \iff \lambda(\lambda - 1)(\lambda + 1) = 0$

3

Question 4

1 pts

Let A be a matrix of size *n x n*.

Which of the following statements are necessarily true?

Write either "T" or "F" in the space provided.

(a) If λ is an eigenvalue of A, then λ^2 is an eigenvalue of A^2. T

If $Av = \lambda$, then $A^2 v = A(Av) = A\lambda v = \lambda^2 v$

(b) If *0* is an eigenvalue of *A*, then $\det(A) = 0$ T

If 0 is an eval, then $Ax = 0$ for some nonzero x.

(c) If A^{17}=0, then at least one eigenvalue of A is zero. T

(d) If A^{17}=0, then all eigenvalues of A are zero. T

If λ is an eval of A, then λ^{17} is an eval of A^{17}.

Since $A^{17} = 0$, we know $\lambda^{17} = 0$, so $\lambda = 0$

Question 5 — 1 pts

Let *A* be a 3 x 3 matrix that whose rank is 2.

Which of the following statements are true?

Write either "T" or "F" in the space provided.

If $\operatorname{rank}(A)=2$, then $\exists x \neq 0$ s.t. $Ax=0$.
Then $A^2x=0$ so $\operatorname{rank}(A^2) \leq 2$.

(a) It is possible that the rank of A^2 is 3. F

(b) It is possible that the rank of A^2 is 2. T ← $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is an example

(c) It is possible that the rank of A^2 is 1. T ← $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is an example

(d) It is possible that the rank of A^2 is 0. F ← This one is a bit tricky!

Let x & y be s.t. $\{Ax, Ay\}$ is a lin. indep set. (Possible since A has rank 2)
Then if $A^2=0$, this means $A(Ax)=0$ and $A(Ay)=0$.
But this implies $\operatorname{rank}(A) \leq 1$, since there are two lin. indep vectors z s.t. $Az=0$.
Contradiction!!

Question 6 — 1 pts

Let *A* be a 3 x 3 matrix, and let $\{u, v, w\}$ be a set of three linearly independent (nonzero) vectors in $\mathbb{R}^3$. You know that $Au = 3u$, that $Av = 3v$, and that $Aw = 7w$.

Which of the following statements are true?

Write either "T" or "F" in the space provided.

(a) The vector *u*+*v* is an eigenvector of *A*. T

(b) The vector *u*+*w* is an eigenvector of *A*. F

(c) The vector *u-v* is an eigenvector of *A* with eigenvalue 0. F

u-v is an evec, but it has evl 3, not 0.

Question 7

1 pts

Let *A* and *B* be matrices of the same size. Suppose further that there exists an invertible matrix *V* for which $A = VBV^{-1}$.

Which of the following statements are necessarily true?

Write either "T" or "F" in the space provided.

(a) $\det(A) = \det(B)$ T

$\det(A) = \det(V)\det(B)\frac{1}{\det(V)} = \det(B)$

(b) If λ is an eigenvalue of *A*, then λ is also an eigenvalue of *B*.

T

$p_A(\lambda) = \det(\lambda I - A) = \det(\lambda I - VBV^{-1}) =$
$= \det(V(\lambda I - B)V^{-1}) = \cancel{\det(V)}\det(\lambda I - B)\frac{1}{\cancel{\det(V)}} = p_B(\lambda)$

(c) If *v* is an eigenvector of *A*, then *v* is also an eigenvector of *B*.

F

Question 8

1 pts

Let $V = \mathbb{R}^3$ and $W = \mathbb{R}^4$. Further, let *A* be a matrix of size 4 x 3.

Which of the following statements are necessarily true?

Write either "T" or "F" in the space provided.

(a) If $\{u, v, w\}$ is a linearly independent set in *V*, then $\{Au, Av, Aw\}$ is a linearly independent set in *W*. F

$A = 0$ provides a counterexample.

(b) If $\{Au, Av, Aw\}$ is a linearly independent set in *W*, then $\{u, v, w\}$ is a linearly independent set in *V*. T

Suppose $c_1 u + c_2 v + c_3 w = 0$, then $c_1 Au + c_2 Av + c_3 Aw = 0$.
Since $\{Au, Av, Aw\}$ is lin indep, we know $c_1 = c_2 = c_3 = 0$.

(c) If $\{Au, Av, Aw\}$ is a linearly independent set in *W*, then $\{Au, Av, Aw\}$ is a basis for *W*. F

A basis for $\mathbb{R}^4$ must have precisely 4 elements.

Question 9

1 pts

Let *A* be a matrix of size *4 x 3*. You know that there exists a nonzero vector *x* such that *Ax=0*. You also know that there exist two vectors *y* and *z* such that $\{Ax, Ay\}$ is a linearly independent set in $\mathbb{R}^4$.

$Ax = 0 \Rightarrow \text{rank}(A) \leq 3 - 1 = 2$

Which of the following statements are necessarily true?

$\{Ax, Ay\}$ lin indep $\Rightarrow \text{rank}(A) \geq 2$

Write either "T" or "F" in the space provided.

(a) It is possible that the rank of *A* is 1. F

(b) It is possible that the rank of *A* is 2. T

(c) It is possible that the rank of *A* is 3. F

(d) It is possible that the rank of *A* is 4. F

Question 10

1 pts

Let $V = \mathcal{P}_2$ and $W = \mathcal{P}_3$. Further, let *M* be the subspace of *W* defined by

$M = \{x\,p(x) : p \in V.\}$In other words, a polynomial *q* belongs to *M* if and only if it is of the form *q(x)* = *xp(x)* for some polynomial *p* in *V*. What is the dimension of *M*?

A polynomial q is in M iff it takes the form $q(x) = ax + bx^2 + cx^3$.

3

So $\{x, x^2, x^3\}$ is a basis for M.

Quiz saved at 4:29pm

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