Quiz Instructions

This exam is timed. You have 50 minutes from the time that you start. There are 10 questions total, so do not linger if you have difficulties with one question - you only have 5 minutes for each one.

Several of the questions ask you to identify which of a given number of statements are true or false. Be sure to write "T" or "F" in the given box (without the quotation marks). Do NOT write "t" or "f" or "true" or "false" or anything like that, just T or F.

Question 1

Consider a linear system $AX=B$. You know that the extended coefficient matrix $[A|B]$ takes the form

$$[A|B] = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 2 & 0 & 6 \\ 1 & 0 & 1 & 1 \end{bmatrix},$$

The solution to the system is:

$X_1 = \fbox{}$

$X_2 = \fbox{}$

$X_3 = \fbox{}$

Question 2

1 pts
Let $A$ be a rectangular matrix of size $m \times n$. Is the following statement true or false:
You can always multiply the transpose of $A$ with the matrix $A$ itself to form a new matrix $B$ that is necessarily symmetric. (In other words, the matrix $B = A^T A$ necessarily exists, and $B^T = B$.)

- True
- False

**Question 3**

Consider a linear system $AX = B$. After some elementary row operations, you convert the extended coefficient matrix $[A|B]$ to a row equivalent system

$$\begin{bmatrix}
1 & 2 & 0 & 1 & 1 \\
0 & 0 & 1 & 2 & -1 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}.$$

The rank of $A$ is

The number of free variables in the system $AX = B$ is

The value of $X_3$ is

**Question 4**

Let $n$ be a positive integer, and let $x$ and $y$ be vectors in $\mathbb{R}^n$. Mark whether the following statements are true.

(Write T or F in the corresponding box.)
(a) If \( \|x\| = \|y\| = 1 \), then \( x \cdot y \geq -1 \).

(b) \((x + y) \cdot (x - y) = \|x\|^2 - \|y\|^2\).

(c) If \( x \cdot y = 0 \), then \( \|x + y\|^2 = \|x\|^2 + \|y\|^2 \).

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**Question 5**

1 pts

The matrix \( A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \) is invertible and has an inverse of the form \( A^{-1} = \begin{bmatrix} s & t \\ u & v \end{bmatrix} \). Specify the value of \( s \).

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**Question 6**

1 pts

The matrix \( \begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 1 \\ -2 & 1 & 3 \end{bmatrix} \) is row equivalent to the matrix \( \begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 1 \\ 0 & 3 & t \end{bmatrix} \). Specify the value of \( t \).
Question 7

Consider the vectors $x = [1,2,0,2]$ and $y = [3,0,-4,0]$. Let $\theta$ denote the angle between $x$ and $y$. Specify the value of $\cos \theta$.

\[
\text{value of } \cos \theta.
\]

Question 8

Consider a $2 \times 4$ matrix $A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$, where $A_1$ and $A_2$ are $1 \times 4$ matrices that form the rows of $A$. Specify whether the following statements are true or false. (Write T or F in the space provided.)

(a) If $A_1 = 2A_2$ and $A_1 \neq 0$ then the rank of $A$ is necessarily 1.

(b) The vector $A_1 - 2A_2$ is necessarily in the row space of $A$.

(c) If the $4 \times 1$ matrix $X$ satisfies $A_1X = 0$ and $A_2X = 0$, then $X = 0$.

Question 9

Let $n$ be a positive integer, and let $x$ and $y$ be vectors in $\mathbb{R}^n$. You know that $\|x\| = 3$ and $\|y\| = 2$. Specify the largest possible value of $\|x-y\|$.
Question 10

Below, you will find 4 matrices of size 3x3. In this question, you are asked to tell whether the matrix corresponds to an elementary row operation. In case the answer is yes, you are further asked to identify which one. Use the following answer key:

1: This matrix corresponds to a "Type 1" elementary row operation (scaling a row by a non-zero scalar number).

2: This matrix corresponds to a "Type 2" elementary row operation (adding a scalar multiple of one row to another).

3: This matrix corresponds to a "Type 3" elementary row operation (swapping two rows).

4: That's not an elementary row operation, crazy.

In other words, enter either 1, 2, 3, or 4, in each of the boxes below.

(a) \[
\begin{bmatrix}
1 & 0 & 2 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]
Answer is ______________________ .

(b) \[
\begin{bmatrix}
1 & 0 & 2 \\
0 & 1 & 0 \\
0 & 0 & -2 \\
\end{bmatrix}
\]
Answer is ______________________ .

(c) \[
\begin{bmatrix}
0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0 \\
\end{bmatrix}
\]
Answer is ______________________ .

(d) \[
\begin{bmatrix}
0.5 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]
Answer is ______________________ .