Homework set 4 — M341, TuTh 9:30am - 10:45am section, Spring 2021

Hand in solutions to problems 4b, 5bdf, 6b, 9ab, 12, and 18 from Section 2.3.

Problem 1: Let c be a real number, and consider the matrix

$$\mathbf{E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}.$$

(a) Let $\mathbf{A}$ be a matrix with three rows, and consider the matrix $\mathbf{B} = \mathbf{EA}$. The matrix $\mathbf{B}$ is the result of performing an elementary row operation on $\mathbf{A}$. Which one?

(b) Specify a matrix $\mathbf{F}$ such that $\mathbf{EF} = \mathbf{I}$. (In other words, $\mathbf{F} = \mathbf{E}^{-1}$.) Observe that such a matrix $\mathbf{F}$ exists for *every* real number c, including $c = 0$.

Problem 2: Let c be a real number such that $c \neq 0$, and consider the matrix

$$\mathbf{E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & c \end{bmatrix}.$$

(a) Let $\mathbf{A}$ be a matrix with three rows, and consider the matrix $\mathbf{B} = \mathbf{EA}$. The matrix $\mathbf{B}$ is the result of performing an elementary row operation on $\mathbf{A}$. Which one?

(b) Specify a matrix $\mathbf{F}$ such that $\mathbf{EF} = \mathbf{I}$. (In other words, $\mathbf{F} = \mathbf{E}^{-1}$.)

(c) Set $\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Prove that there cannot exist a matrix $\mathbf{H}$ such that $\mathbf{GH} = \mathbf{I}$.

Problem 3: Consider the matrix

$$\mathbf{E} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

(a) Let $\mathbf{A}$ be a matrix with three rows, and consider the matrix $\mathbf{B} = \mathbf{EA}$. The matrix $\mathbf{B}$ is the result of performing an elementary row operation on $\mathbf{A}$. Which one?

(b) Specify a matrix $\mathbf{F}$ such that $\mathbf{EF} = \mathbf{I}$. (In other words, $\mathbf{F} = \mathbf{E}^{-1}$.)