Quiz for final exam

ⓘ This is a preview of the published version of the quiz

Started: May 15 at 1:10pm

Quiz Instructions

This exam is timed. You have 60 minutes from the time that you start. There are 12 questions total, so do not linger if you have difficulties with one question - you only have 5 minutes for each one.

Several of the questions ask you to identify which of a given number of statements are true or false. Be sure to write "T" or "F" in the given box (without the quotation marks). Do NOT write "t" or "f" or "true" or"false" or anything like that, just T or F.

Question 1

1 pts

Let *A* and *B* be 5 x 5 matrices.

Mark which of the following statements are necessarily true (write either T or F in the box provided):

(a) If rank(*A*)=5 and rank(*B*)=3, then rank(*AB*)=3. T ← If A is invertible, then AB has the same RREF as B.

(b) If rank(*A*)=3 and rank(*B*)=3, then rank(*AB*)=3. F

(c) If rank(*A*)=3, then dim(ker(*A*))=2. T $2 = 5 - 3$

Question 2

1 pts

Consider the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ t & 0 & 3 \end{bmatrix}$ where *t* is a real number.

You know that det(*A*)=3.

Specify the value of *t*.

$3 = \det(A) = 6 - 2t$

1.5

Question 3

1 pts

Let *A* be a 3x3 ***symmetric*** matrix. You know that *A* has three distinct eigenvalues, and that $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $v = \begin{bmatrix} 3 \\ t \\ -1 \end{bmatrix}$ are both eigenvectors of *A*. Specify *t*.

$U \cdot V = 0$ so $3 + 2t - 3 = 0 \Rightarrow t = 0$

0

Question 4

1 pts

Let *A* be a 3x3 matrix. You know that *A* has precisely two eigenvalues, $\lambda_1 = 1$ and $\lambda_2 = 2$. Let *p* denote the characteristic polynomial of *A*, so that $p(\lambda) = \det(\lambda I - A)$ as usual. Which of the following three options *could* be the characteristic polynomial of *A*:

(a) $p(\lambda) = \lambda^3 + \lambda$ $\Leftarrow$ $\lambda = 1$ is not a root

(b) $p(\lambda) = \lambda^3 - 3\lambda^2 + 3\lambda - 1$ $\Leftarrow$ $\lambda = 2$ is not a root

(c) $p(\lambda) = \lambda^3 - 3\lambda^2 + 2\lambda$ $\Leftarrow$ $\lambda = 0$ is a root

(d) $p(\lambda) = \lambda^3 - 4\lambda^2 + 5\lambda - 2$ $= (\lambda - 1)^2 (\lambda - 2)$

- (d) [selected]
- (a)
- (b)
- (c)

Question 5

1 pts

The matrix $\begin{bmatrix} -4 & -2 & -3 & 0 \\ -3 & -1 & -3 & -2 \\ 4 & 2 & 1 & -4 \end{bmatrix}$ is row equivalent to the matrix $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$.

$x_3 = 2$

$x_1 = 1 - x_3 = -1$

$x_2 = -x_1 - x_3 = 1 - 2 = -1$

Let $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ denote the solution to the linear system $\begin{bmatrix} -4 & -2 & -3 \\ -3 & -1 & -3 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ -4 \end{bmatrix}$.

Specify the value of x_2 .

-1

Question 6

1 pts

The matrices *A* and *B* are both of size 4x4. You know that rank(*A*)=3 and that det(*B*)=1.

Mark whether the following statements are necessarily true (write either T or F in the box provided):

(a) The equation *Bx=y* has a solution for every 4x1 vector *y*. T

(b) The equation *ABx=0* has infinitely many solutions. T $\det(AB) = \det(A)\det(B) = 0 \cdot 1 = 0$

(c) The equation *Bx=Ay* has a solution for every 4x1 vector *y*. T

(d) dim(ker(*A*+*B*)) = 1. F ← Not enough information

Question 7

1 pts

The matrix $A = \begin{bmatrix} -1 & -2 & -3 & -5 & -1 \\ 1 & 0 & 1 & 1 & -1 \\ -1 & 0 & -1 & -1 & 2 \\ -2 & -1 & -3 & -4 & 1 \end{bmatrix}$ has the RREF $B = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Mark whether the following statements are necessarily true (write either T or F in the box provided):

(a) The first, second, and fifth columns of *A* form a basis for the column space of *A*. T

(b) The first, second, and fifth columns of *B* form a basis for the column space of *A*. F

(c) The first, second, and third rows of *B* form a basis for the row space of *A*. T

(d) rank(*A*) = 3. T

Question 8

1 pts

Let $V = \mathcal{P}_4$ denote the vector space consisting of all polynomials of degree four or less.

Consider the subspace *W* consisting of all polynomials in *V* that are *even*. In other words, $W = \{p \in V : p(x) = p(-x) \text{ for every } x\}$.

Specify the dimension of *W*. W has a basis $\{1, x^2, x^4\}$

3

Question 9

1 pts

Let *A* be a 3x3 matrix that has the eigenvalues $\lambda_1 = 0$, $\lambda_2 = 1$, and $\lambda_3 = 3$.

Mark whether the following statements are necessarily true (write either T or F in the box provided):

(a) The matrix A^2 has 9 as an eigenvalue. T

(b) There exists a positive integer *n* such that A^n = 0. F A^n must have an eval 3^n.

(c) *A* is necessarily invertible, and A^{-1} has 1/3 as an eigenvalue. F A is <u>not</u> invertible.

(d) There is a basis $\{v_1, v_2, v_3\}$ for $\mathbb{R}^3$ consisting entirely of eigenvectors of *A*. T

Question 10

1 pts

Let *A* be a 3x3 matrix, and let *x*, *y*, and *b* be vectors such that *Ay*=0 and *Ax*=*b*.

Mark whether the following statements are necessarily true (write either T or F in the box provided):

(a) The vector *u* = *x*+*y* satisfies *Au*=*b*. T

(b) The vector *v* = *x* -2*y* satisfies *Av*=*b*. T

(c) The vector *w* = 2*x*+*y* satisfies *Aw*=2*b*. T

Question 11

1 pts

Let $V = \text{span}\{[1,-2,1],\ [-1,0,1]\}$ denote a plane through the origin in $\mathbb{R}^3$.

Let $x = [1,-1,3]$ denote a vector in $\mathbb{R}^3$.

You know that the orthogonal projection of *x* onto *V* is the vector $y = [0,-2,2]$.

Specify the ***square*** of the distance *d* between *V* and *x*, so that $d = \inf\{\|x-v\| : v \in V\}$.

NOTE: Be sure to specify d^2 , and not d itself. (To avoid having to evaluate a square root, of course.)

$d^2 = \|x-y\|^2 = \|[1,1,1]\| = 1^2+1^2+1^2 = 3$

3

Question 12

1 pts

Consider the matrices $A = \begin{bmatrix} 2 & 1 & t & 0 \\ 3 & -1 & -2 & 7 \\ -1 & 2 & 2 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$.

Specify the value of *t* for which *B* is the RREF of *A*.

Hint: *You can determine t through a one-line calculation.*

Think of the matrices as extended coefficient matrices of linear systems.

Then $x_1 = 2$ $x_2 = -3$ $x_3 = 1$

$2x_1 + x_2 + t x_3 = 0$

$2 \cdot 2 - 3 + t = 0 \Rightarrow t = -1$

-1

No new data to save. Last checked at 1:42pm

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