Quiz for Section Exam 2

ⓘ This is a preview of the published version of the quiz

Started: Apr 23 at 8:04am

Quiz Instructions

This exam is timed. You have 45 minutes from the time that you start. There are 14 questions total, so do not linger if you have difficulties with one question - you only have about 3 minutes for each one.

Several of the questions ask you to identify which of a given number of statements are true or false. Be sure to write "T" or "F" in the given box (without the quotation marks). Do NOT write "t" or "f" or "true" or"false" or anything like that, just T or F.

Question 1 — 1 pts

Let A and B be square invertible matrices of the same size.

Which of the following statements are necessarily true?

Write either "T" or "F" in the space provided.

(a) det(A + B) = det(A) + det(B): FALSE

(b) det(AB) = det(A)det(B): TRUE

(c) det(A^T) = det(A): TRUE

(d) det(A^{-1}) = det(A)$^{-1}$: TRUE

Question 2

1 pts

Let A be a square invertible matrix of size 3x3. What is det(2A)/det(A)?

RECALL: $\det(cA) = c^n \det(A)$

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Here: $c=2$, $n=3$

Question 3

1 pts

Let V = R^3 denote three dimensional Euclidean space, as usual. Let L denote the set consisting of all vectors x = [x_1, x_2, x_3] such that $x_1 - 2x_2 + 3x_3 = c$ for some real number c. For which values of c is L a subspace of V?

RECALL: If L is a hyperplane in $\mathbb{R}^n$, then:

- ○ For all c except for c=0.
- ● For c=0 only.
- ○ For all c.
- ○ For no values c.

L is a subspace $\Leftrightarrow$ L goes through the origin

Question 4

1 pts

The matrix A = $\begin{bmatrix} 1 & -1 & 4 & 0 \\ 0 & 1 & -1 & -1 \\ -1 & -1 & -2 & 1 \end{bmatrix}$ has the RREF B = $\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Which of the following statements are necessarily true?

Please fill in either "T" or "F" after each statement:

(a) The columns of A span R^3. TRUE

Since $Ax=b$ has a sol^n for every b.

(b) The columns of A are linearly independent. FALSE

IMPOSSIBLE! At most three vectors can be linearly indep in $\mathbb{R}^3$, and there are four columns.

(c) The rows of A span R^4. FALSE

IMPOSSIBLE! You need at least four vectors to span $\mathbb{R}^4$, and there are only three rows.

(d) The rows of A are linearly independent. TRUE

Recall that the nonzero rows of the RREF form a basis for the row space.

Question 5

1 pts

Let V denote the linear space of all continuous functions on the interval [-1,1]. This question asks you to identify whether certain subsets of V form subspaces.

Please write "T" or "F" in the blanks provided.

These sets do not contain the zero vector!

(a) The set of all functions f such that f(0)=0 is a subspace. TRUE

(b) The set of all functions f such that f(0)=1 is a subspace. FALSE

(c) The set of all functions f such that f(1)=0 is a subspace. TRUE

(d) The set of all functions f such that f(1)=1 is a subspace. FALSE

(e) The set of all functions f such that $\int_{-1}^{1} f(x)\,dx = 0$ is a subspace.

TRUE

Question 6

1 pts

Let V denote the linear space of all polynomials of degree at most 2.

Which of the following statements are necessarily true?

Please write "T" or "F" in the given box.

(a) The set $\{1,x,x^2\}$ is a basis for V. TRUE

(b) The set $\{1+x,x,x^2\}$ is a basis for V. TRUE

(c) The set $\{1+x^2,1-x^2,x^2\}$ is a basis for V. FALSE

$x \notin \text{span}\{1+x^2, 1-x^2, x^2\}$

(d) The set $\{1+x,2+2x,x^2\}$ is a basis for V. FALSE

$2+2x = 2(1+x)$

so the set is not linearly indep.

Question 7

1 pts

Let λ be an eigenvalue of a square matrix A. Which of the following statements are necessarily true?

Fill in each blank with either "T" or "F".

(a) The set V of all vectors x such at Ax=λx forms a vector space.

TRUE This is E_λ, the "eigenspace".

(b) The matrix A^T also has λ as an eigenvalue. TRUE $\det(\lambda I - A^T) = \det(\lambda I - A)$

(c) det(A - λI) = 0. TRUE Since $(A-\lambda I)x = 0$ for the evec x.

(d) The matrix A^2 has the eigenvalue λ^2. TRUE

If v is such that $Av = \lambda v$, then

$A^2 v = A(Av) = A(\lambda v) = \lambda A v = \lambda^2 v$

Question 8 — 1 pts

Let A be a matrix of size n x n such that det(A) ≠ 0.

Which of the following statements are necessarily true?

Please fill in each blank with either "T" or "F".

(a) The rows of A form a basis for R^n. TRUE

(b) The columns of A form a basis for R^n. TRUE

(c) The equation Ax=b has a solution for every vector b in R^n. TRUE

Question 9 1 pts

Let V be the linear space formed by all 3 x 3 matrices. This question asks you to identify whether certain given subsets of V form linear subspaces.

Fill in each blank with either "T" or "F" for true or false, respectively.

(a) The set of all upper triangular matrices is a subspace. TRUE

(b) The set of all diagonal matrices is a subspace. TRUE

(c) The set of every matrix A such that det(A)≠0 is a subspace. FALSE

Note that the zero matrix is not in this set.

(c) The set of every matrix A such that det(A)=0 is a subspace. FALSE

Counterexample: $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Then $\det(A) = 0$ and $\det(B) = 0$.

But $\det(A+B) = 1$

Question 10 1 pts

Let V denote the linear space formed by all 3 x 3 matrices. What is the dimension of V?

Recall: $\dim M_{m,n} = mn$

9

Question 11 1 pts

Let A be a square matrix for which we know that ||Ax|| = ||x|| for every vector x.

Which of the following statements are necessarily true?

Please write either "T" or "F" in the given boxes.

(a) A is invertible. TRUE

If $Ax=0$, then $\|x\| = \|Ax\| = 0$, so $\ker(A) = \{0\}$.

(b) det(A) = 1. FALSE

Counterexample: $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \det(A) = -1$

(c) If λ is an eigenvalue of A, then either λ=1 or λ=-1. TRUE

If λ is an eval with evec v, then

$\|v\| = \|Av\| = \|\lambda v\| = |\lambda| \|v\| \Rightarrow |\lambda| = 1$

Question 12

1 pts

For a 3 x 3 matrix A, we know that det(tI - A) = $t^3 - 2t^2 - 3t$. What are the eigenvalues of A?

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12pt Paragraph

$\lambda_1 = 0 \quad \lambda_2 = -1 \quad \lambda_3 = 3$ (in any order)

Since $t^3 - 2t^2 - 3t = t(t^2 - 2t - 3)$

and the roots of $t^2 - 2t - 3 = 0$ are

$t = 1 \pm \sqrt{1+3}$

0 words

Question 13

1 pts

Specify the determinant of the matrix A = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \\ -1 & 0 & 7 \end{bmatrix}$.

Add the first row to the last, to get

$A' = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 9 \end{bmatrix}$ then $\det(A) = \det(A') = 1 \cdot 3 \cdot 9 = 27$

27

OR: Use the "diagonals" rule: $\det(A) = 1 \cdot 3 \cdot 7 - 2 \cdot 3 \cdot (-1) = 21 + 6 = 27$

Question 14

1 pts

Specify the largest eigenvalue of the matrix A = $\begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$.

$p(t) = \det(tI - A) = \det \begin{bmatrix} t & -3 & 0 \\ 0 & t & 2 \\ 0 & 0 & t \end{bmatrix} = t^3$

0

So the only evl is $\lambda = 0$.

Quiz saved at 8:04am

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