Section exam 1 for M341: Linear Algebra and Matrix Theory

8:00am - 9:15am, Feb. 27, 2020. Closed books. No notes.

Question 1: (20p) For this question, please write only the answer, no motivation. 4p per question.

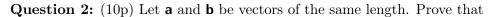
(a) Specify the solution set to the linear system $\begin{bmatrix} 1 & 0 & -2 & 0 & 3 \\ 0 & 1 & 5 & 0 & 2 \\ 0 & 0 & 0 & 1 & 7 \end{bmatrix}$

(b) Specify the solution set to the linear system $\begin{bmatrix} 1 & 0 & -2 & 0 & 3 \\ 0 & 1 & 5 & 0 & 2 \\ 0 & 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$

(c) Let t be a real number and consider the matrix $\mathbf{A} = \begin{bmatrix} 1 & t \\ 3 & 2 \end{bmatrix}$. For which values of t does \mathbf{A} have an inverse?

(d) Let \mathbf{x} and \mathbf{y} be two vectors in \mathbb{R}^7 . State the definition of the projection of \mathbf{x} onto the vector \mathbf{y} .

- (e) Circle the statements that are necessarily true.
 - (i) If the square matrix \mathbf{A} is invertible, then $(\mathbf{A}^T)^{-1} = (\mathbf{A}^{-1})^T$.
 - (ii) Every square matrix can be written as a sum of a symmetric and a skew-symmetric matrix.
 - (iii) If \boldsymbol{x} and \boldsymbol{y} are vectors of the same length, then $\boldsymbol{x}\cdot\boldsymbol{y}\geq -\|\boldsymbol{x}\|\,\|\boldsymbol{y}\|.$
 - (iv) Let **A** and **B** be square matrices of the same size. If **AB** is invertible, then both **A** and **B** must be invertible.



$$\|\mathbf{a} + \mathbf{b}\|^2 + \|\mathbf{a} - \mathbf{b}\|^2 = 2\|\mathbf{a}\|^2 + 2\|\mathbf{a}\|^2.$$

Question 3: (20p) Let **A** be a square matrix for which I-A is invertible. Prove that for $n=1, 2, 3, \ldots$ it is the case that

$$\mathbf{A}^0 + \mathbf{A}^1 + \mathbf{A}^2 + \dots + \mathbf{A}^n = \left(\mathbf{I} - \mathbf{A}\right)^{-1} \left(\mathbf{I} - \mathbf{A}^{n+1}\right).$$

Question 4: (18p) Let $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ be a 3×2 matrix. Let \mathbf{E}_1 , \mathbf{E}_2 , and \mathbf{E}_3 be 3×3 matrices such that

$$\mathbf{E}_1\mathbf{A} = \left[\begin{array}{ccc} a_{11} & a_{12} \\ a_{21} - 3a_{11} & a_{22} - 3a_{12} \\ a_{31} & a_{32} \end{array} \right], \qquad \mathbf{E}_2\mathbf{A} = \left[\begin{array}{ccc} a_{11} & a_{12} \\ a_{21} & a_{22} \\ 5a_{31} & 5a_{32} \end{array} \right], \qquad \mathbf{E}_3\mathbf{A} = \left[\begin{array}{ccc} a_{31} & a_{32} \\ a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right].$$

- (a) Specify the matrices \mathbf{E}_1 , \mathbf{E}_2 , and \mathbf{E}_3 .
- (b) Specify the inverses of the matrices \mathbf{E}_1 , \mathbf{E}_2 , and \mathbf{E}_3 .

No motivations are required.

Hint: \mathbf{E}_1 and \mathbf{E}_2 are both standard elementary row operations. However, \mathbf{E}_3 is not.

Problem 5: (22p) Solve the linear system

$$\begin{array}{cccccc} x_1 & -x_2 & +x_3 & = & 5 \\ x_1 & -x_2 & +2x_3 & = & 7 \\ -2x_1 & +5x_2 & +4x_3 & = & -7 \end{array}$$

Hint: Remember to verify your solution once you have computed it!

Problem 6: (10p) Specify the inverses of the two matrices

$$\mathbf{A} = \begin{bmatrix} 2 & -8 \\ -2 & 7 \end{bmatrix}, \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix}.$$

No motivation is required, just give the answers.

Hint: Remember to verify your answers.