Quiz for

You started this quiz near when it was due, so you won't have the full amount of time to take the quiz.

This is a preview of the published version of the quiz

Started: May 18 at 8:40pm

Quiz Instructions

This exam is timed. You have 70 minutes from the time that you start. There are 14 questions total, so do not linger if you have difficulties with one question - you only have 5 minutes for each one.

Some questions require *brief* computations (one or two lines at most).

Several of the questions ask you to identify which of a given number of statements are true or false. Be sure to write "T" or "F" in the given box (without the quotation marks). Do NOT write "t" or "f" or "true" or"false" or anything like that, just T or F.

Question 1 — 1 pts

Let m and n be positive integers.

Let A be a matrix of size $m \times n$, and let b be a vector of size $m \times 1$.

Which of the following statements are necessarily true:

(Write either T or F in the box provided.)

(a) If $m > n$, then the linear system $Ax = b$ has at most one solution.

(b) If $m < n$, then the linear system $Ax = b$ must have at least one solution.

(c) If $\text{rank}(A) = n$, then the linear system $Ax = b$ has at most one solution.

(d) If $\text{rank}(A) = m$, then the linear system $Ax = b$ must have at least one solution.

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Question 2 1 pts

Specify the determinant of the matrix $\begin{bmatrix} 2 & 9 & -3 \\ 0 & -1 & 3 \\ 0 & 0 & 3 \end{bmatrix}$.

Question 3 1 pts

Let V denote the vector space consisting of all polynomials of degree 3 or less, and consider the following three vectors:

$p(x) = 1 + x, \qquad q(x) = 1 + x^2, \qquad r(x) = 2 + x + x^2.$

Form the subspace

$L = \text{span}(p, q, r).$

What is the dimension of L?

Question 4 1 pts

Let$A$ You started this quiz near when it was due, so you won't
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(Write either T or F in the box provided.)

(a) If A and B are both invertible, then AB is invertible.

(b) If A and B are both invertible, then $A + B$ is invertible.

(c) IfAB is invertible, then both A and be B must be invertible.

(d) If A is invertible, then $\left(A^{\mathrm{T}}\right)^{-1} = \left(A^{-1}\right)^{\mathrm{T}}$.

Question 5

1 pts

Let A be a square matrix of size $n \times n$. This question asks what properties in A would be sufficient to guarantee the existence of a factorization of the form $A = VDV^{-1}$ for a diagonal matrix D and an invertible matrix V.

(Write either T or F in the box provided.)

(a) It is sufficient for A to have n different real eigenvalues.

(b) It is sufficient for A to be symmetric (so that $A = A^{\mathrm{T}}$).

(c) It is sufficient for $\mathbb{R}^n$ to have a basis consisting of n eigenvectors of A.

(d) It is sufficient for $det(A) \neq 0$.

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Question 6

1 pts

Let A and$B$ be two matrices. This question concerns properties of the linear system $ABx = y$.

(Write either T or F in the box provided.)

(a) If y is in the range of A, and B is a square invertible matrix, then $ABx = y$ has at least one solution.

(b) If AB is a square matrix and $\det(AB) \neq 0$, then $ABx = y$ has at least one solution.

(c) If $\ker(A) = \{0\}$ and $\ker(B) = \{0\}$, then $ABx = y$ has at at most one solution.

(d) If A and B are both 3×3 matrices, and the rank of B is 2, then there exist y such that $ABx = y$ has no solution.

Question 7

1 pts

Let V be a vector space of dimension 5, let W be a vector space of dimension 4, and let L be a linear map from V to W.

Suppose that there exist two nonzero vector x and y in Vsuch that $\{x, y\}$ is a linearly independent set, and such that $Lx = 0$ and $Ly = 0$.

What is the *maximal* possible value of the dimension of the range of L?

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Question 8

1 pts

The matrix $A = \begin{bmatrix} -4 & -8 & 5 & -5 & -12 \\ -1 & -2 & 1 & -1 & -3 \\ -2 & -4 & 3 & -3 & -6 \end{bmatrix}$ has the RREF

$B = \begin{bmatrix} 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Specify a basis for the range of A.

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Question 9

1 pts

You started this quiz near when it was due, so you won't have the full amount of time to take the quiz.

Let a ... span

$\begin{bmatrix} 1 \end{bmatrix}$ $\begin{bmatrix} 0 \end{bmatrix}$ $\begin{bmatrix} 6 \end{bmatrix}$

a subspace $L = \text{span}(a, b, c)$.

You are asked to build an orthogonal basis for L. You quickly observe that a and b are already orthogonal, so that it is possible to construct a basis of the form $\mathcal{B} = \{a, b, d\}$ where $d = c + sa + tb$ for two scalar numbers s and t.

What value must the scalar s take for $\mathcal{B}$ to be an orthogonal basis?

Question 10

1 pts

Let A be a 3×3 matrix with columns a, b, and c, so that $A = [a, b, c]$.

You know that $\det(A) = 7$.

Form the new matrix $B = [b, -3a, c]$.

What is the determinant of B?

Question 11

1 pts

Let $\{a, b, c, d, e, f\}$ be a collection of six column vectors in $\mathbb{R}^3$.

Set $L_1 = \text{span}(a, b, c)$ and $L_2 = \text{span}(d, e, f)$.

You know that both L_1 and L_2 have dimension 2.

Whic[h]

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(a) The intersection $L_1 \cap L_2$ of the two spaces has dimension one.

(b) The determinant of the matrix with columns $\{a, b, c\}$ is zero. (In other words, $\det[a, b, c] = 0$.)

(c) The dimension of the orthogonal complement of L_1 is 1. (In other words, $\dim(L_1^\perp) = 1$.)

(d) If $d \notin L_1$, then $\text{span}(a, b, c, d) = \mathbb{R}^3$.

Question 12

1 pts

Let V and W be two vector spaces with bases $\mathcal{B}$ and $\mathcal{C}$, respectively.

The dimension of V is n, and the dimension of W is m.

Let $L: V \to W$ be a linear map, and let A be the matrix of L with respect to the bases $\mathcal{B}$ and $\mathcal{C}$.

Which of the following statements are necessarily true?

(Write either T or F in the box provided.)

(a) If $\text{rank}(A) = n$, then L is one-to-one.

(b) If $rank(A) < m$, then L is not onto.

(c) If L is invertible, then the matrix representing L^{-1} with respect to the bases $\mathcal{C}$

and L

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(d) If $\text{rank}(A) = k$, then $\dim(\text{ran}(L)) = n - k$.

Question 13

1 pts

Consider the vectors $a = \begin{bmatrix} 4 \\ 0 \\ 3 \\ 0 \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 2 \end{bmatrix}$.

Let θ denote the angle between a and b.

Specify $\cos(\theta)$.

(Enter the answer as a decimal value, not as a fraction.)

Question 14

1 pts

Consider the matrices $A = \begin{bmatrix} 2 & 1 & -3 \\ 3 & 2 & -1 \\ -8 & -4 & 13 \end{bmatrix}$ and $B = \begin{bmatrix} 22 & -1 & 5 \\ -31 & 2 & -7 \\ x & 0 & 1 \end{bmatrix}$ where x is a real number.

There is one value of x for which $B = A^{-1}$. Specify this value.

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