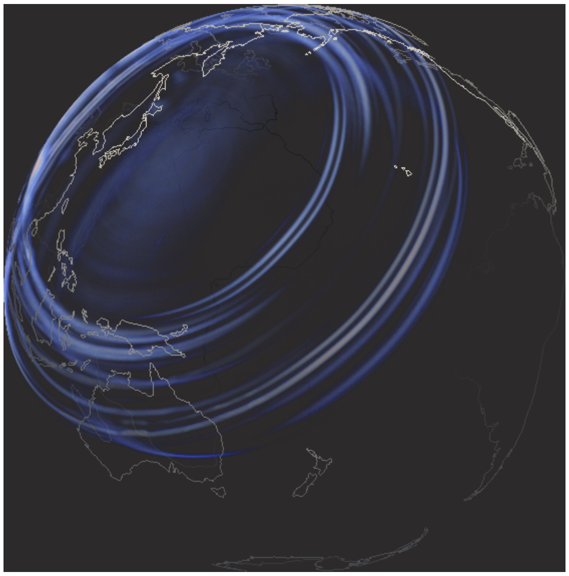
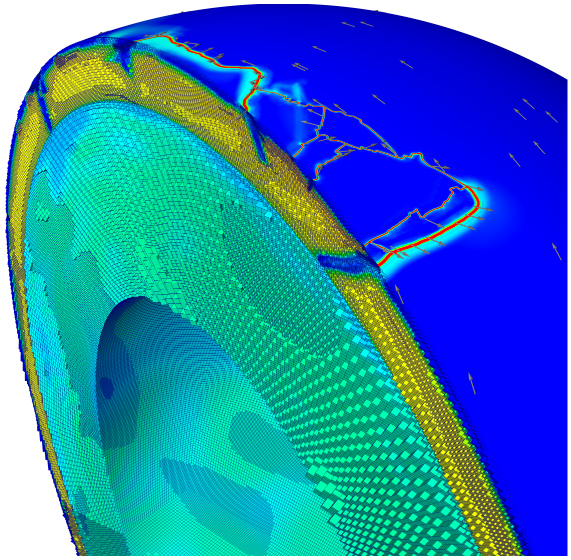
**Fall 2019**
**Computational and Variational Methods for Inverse Problems**
**Cross-listed as CSE 397, GEO 391, ME 397, ORI 397**





**Unique numbers:** 64769 (CSE), 26801 (GEO), 18319 (ME), 18534 (ORI)

**Lectures:** Mon/Wed 17:00–19:00, GDC 1.406

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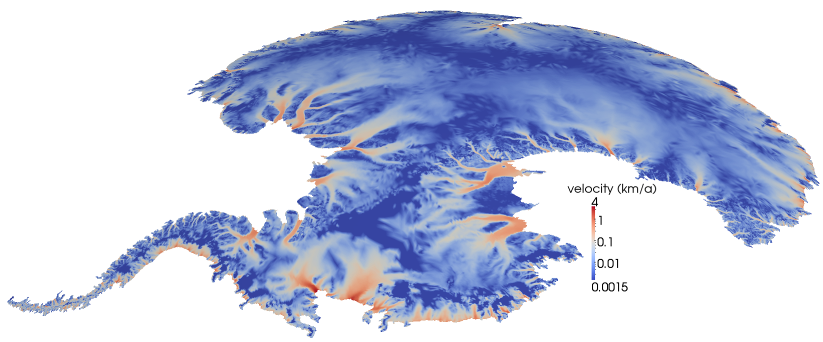
**Slack:** We'll be using Slack (`https://slack.com`) for all messaging, discussion, handouts, assignments, etc. Instructions will be distributed during the first class.

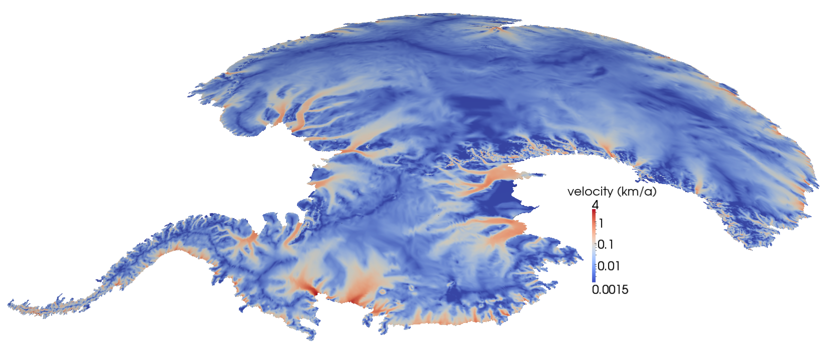
**Description:** This course provides an introduction to the numerical solution of inverse problems that are governed by systems of partial differential equations (PDEs). The focus of the course is on variational formulations, ill-posedness, regularization, adjoint methods for gradients and Hessians, variational discretization, and efficient large-scale optimization algorithms for inverse problems. Students will develop numerical implementations for model problems using the inverse problems library hIPPYlib (`https://hippylib.github.io`), which builds on the high-level finite element toolkit FEniCS (`https://fenicsproject.org`) for discretization and the HPC library PETSc (`https://www.mcs.anl.gov/petsc`) for scalable and efficient linear algebra operations and solvers. These implementations will allow us to study the influence of data noise, regularization, the observation operator, the choice of the parameter field, and the nature of the underlying PDE model on the identifiability of the model parameters, as well as facilitating experimentation with different solution algorithms. Time permitting, the course will also provide a brief introduction to the Bayesian framework and draw connections between the classical and the Bayesian interpretations. Examples will be drawn from different areas of science and engineering, including continuum fluid and solid mechanics, geophysics, and image processing.

**Prerequisites:** Graduate standing or consent of instructor. Background in numerical linear algebra, partial differential equations, and nonlinear optimization is desirable. However, the required mathematical background will be covered when needed—albeit quickly. A mathematically mature student will be able to acquire from the lectures the necessary mathematical and computational background. If in doubt, contact me.

**Required work:** Five or six assignments involving a mix of theory, implementation, and computational experiments. No exams.

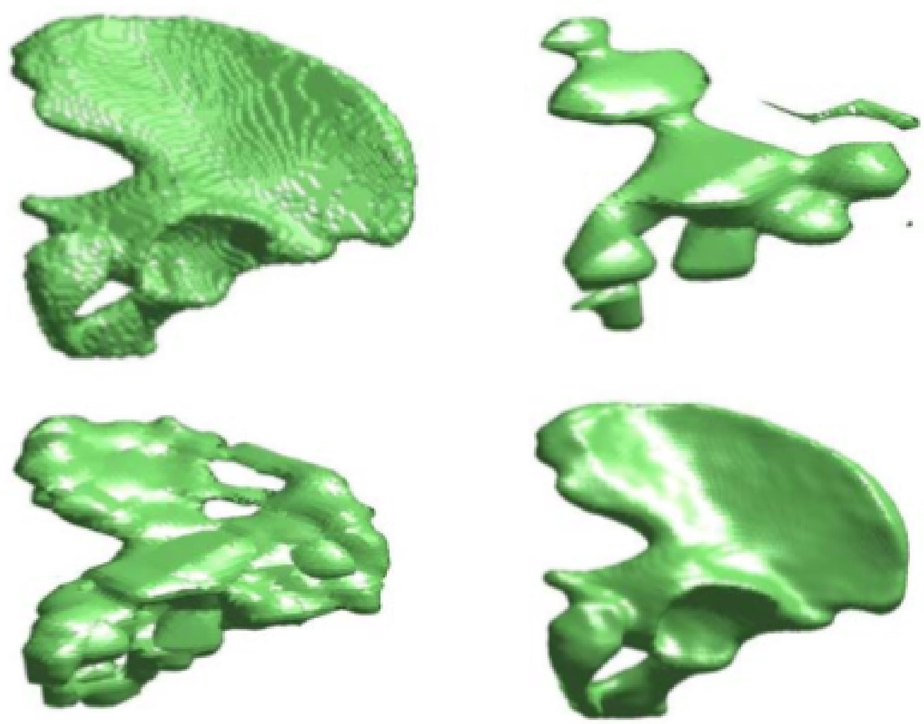
**Collaboration Policy:** Students are encouraged to discuss among themselves the course material and assignments. However, all turned-in material must be the work of the individual student. For further information on the University's Scholastic Dishonesty policy, see [http://deanofstudents.utexas.edu/sjs/scholdis.php](http://deanofstudents.utexas.edu/sjs/scholdis.php).



## Course Topics

- introduction and examples of inverse problems with PDEs
- ill-posed problems and regularization
  - theoretical aspects
  - different regularization methods
  - choice of regularization parameter
- variational methods, weak forms, Galerkin methods
- computing derivatives via adjoints
  - steady and unsteady problems
  - discrete vs. continuous
  - linear and nonlinear PDEs
  - distributed, boundary, and finite-dimensional parameters and measurements
- numerical optimization methods
  - line search globalization
  - steepest descent
  - Newton method
  - Gauss-Newton method
  - inexact Newton-conjugate gradient method
- inequality constraints on parameters
- Bayesian approach to inverse problems



## Books on Inverse Problems

No textbook required, but several good references for variational inverse problems include:

**Theory and computational methods for inverse problems:**

- Heinz Engl, Michael Hanke, and Andreas Neubauer, *Regularization of Inverse Problems*, Dordrecht, 2nd edition, 1996.
- Curtis R. Vogel, *Computational Methods for Inverse Problems*, SIAM, 2002.
- Guy Chavent, *Nonlinear Least Squares for Inverse Problems*, Springer, 2009.
- Per Christian Hansen, *Discrete Inverse Problems: Insight and Algorithms*, SIAM, 2010.
- Jennifer Mueller and Samuli Siltanen, *Linear and Nonlinear Inverse Problems with Practical Applications*, SIAM, 2012.

**Numerical optimization background:**

- Jorge Nocedal and Stephen J. Wright, *Numerical Optimization*, Springer-Verlag, 1999.
- C. Tim Kelley, *Iterative Methods of Optimization*, SIAM, 1999.

**Optimization of systems governed by PDEs:**

- Max D. Gunzburger, *Perspectives in Flow Control and Optimization*, SIAM, 2003.
- M. Hinze, R. Pinnau, M. Ulbich, and S. Ulbrich, *Optimization with PDE constraints*, Springer, 2009.
- Fredi Tröltzsch, *Optimal Control of Partial Differential Equations: Theory, Methods and Applications*, Graduate Studies in Mathematics Vol. 112, AMS, 2010.
- Alfio Borzì and Volker Schulz, *Computational Optimization of Systems Governed by Partial Differential Equations*, SIAM, 2012.

**Probabilistic approach to inverse problems:**

- Albert Tarantola, *Inverse Problem Theory and Methods for Model Parameter Estimation*, SIAM, 2005.
- Jari Kaipio and Erkki Somersalo, *Statistical and Computational Inverse Problems*, Springer, 2005.
- Luis Tenorio, *An Introduction to Data Analysis and Uncertainty Quantification for Inverse Problems*, SIAM, 2017.
- Johnathan Bardsley, *Computational Uncertainty Quantification for Inverse Problems*, SIAM, 2018.