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LARGE-SCALE NORTHRIDGE EARTHQUAKE SIMULATION USING OCTREE-BASED MULTIRESOLUTION MESH METHOD

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ABSTRACT

Large scale ground motion simulation in realistic basins can benefit greatly from the use of parallel supercomputing systems in order to obtain reliable and useful results within reasonable elapsed time due to its large problem size. We present a parallel octree-based multiresolution finite element methodology for the elastodynamic wave propagation problem and develop a framework for large scale wave propagation simulations. The framework is comprised of three parts; (1) a mesh generator (Euclid, Tu et al. 2002), (2) a parallel mesh partitioner (ParMETIS, Karypis et al. 2002), and (3) a parallel octree-based multiresolution finite element solver (QUAKE, Kim 2003). The octree-based multiresolution finite element method reduces memory use significantly and improves overall computational performance. The numerical methodology and the framework have been used to simulate the seismic response of the greater Los Angeles basin for a mainshock of the 1994 Northridge Earthquake, for frequencies of up to 1 Hz and domain size of $80 \times 80 \times 30$ km^3. Through simulations for several models, ranging in size from 400,000 to 300 million degrees of freedom on the 3000-processors HP-Compaq AlphaServer Cluster at the Pittsburgh Supercomputing Center (PSC), we achieve excellent performance and scalability.

Keywords: Octree-Based Multiresolution Mesh Method (OBM^3), Northridge Earthquake, Ground Motion Simulation

INTRODUCTION

Wave propagation simulations for earthquake-induced ground motion have been performed over the last 30 years to gain a better understanding of the distribution of the earthquake ground motion in urban regions in space and time. Such insight has contributed to develop building codes in which a seismic prone region is divided into different zones of comparable seismic hazard. The dramatic improvement of supercomputing performance has more recently enabled seismologists and earthquake engineers to more accurately understand the effects of source, wave propagation, and local site conditions on the ground motion.

Parallel computers, consisting of thousands of distributed processors, allow scientists and engineers to simulate larger problems than had been previously impossible. Parallel computing plays a crucial role in large scale simulations in a variety of physical problems. In earthquake

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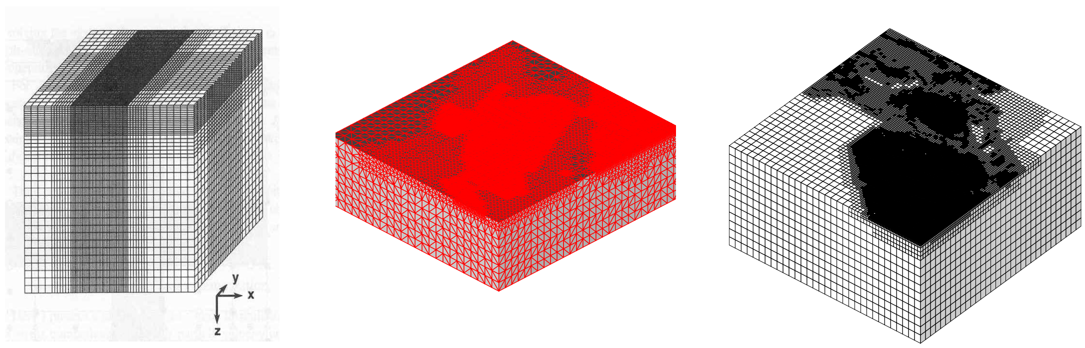


(a) (b) (c)

FIG. 1. (a) Structured Grid (FDM, Pitarka 1999) (b) Unstructured Mesh (FEM, Bao et al. 1998) (c) Octree-Based Multiresolution Mesh (FEM, Kim 2003)

ground motion simulation, it enables one to model ground motion in large, highly heterogeneous basins, such as the Los Angeles (LA) basin with reliable resolution.

An earthquake ground motion simulation entails solving numerically the partial differential equations (PDE) of elastodynamic wave propagation. There are several numerical methods available for ground motion simulations. The finite difference method (FDM) (e.g., Graves 1996; Olsen and Archuleta 1996; Pitarka 1999), the boundary element method (BEM) (e.g., Kawase 1988) and the finite element method (FEM) (e.g., Bao et al. 1998) are commonly used. In seismology and earthquake engineering, the FDM has been the most popular technique due to its satisfactory accuracy and ease of implementation. However, the shortcoming of the FDM is that in its standard form it uses a regular grid even in the presence of highly heterogeneous materials. Considering alternative methods, the main advantage of the BEM is its unique ability to provide a complete solution in terms of boundary values only, with substantial savings in modeling effort. But the BEM is not an adequate technique for ground motion simulations in large scale and highly heterogeneous domains, because the systems of algebraic equations become dense as it requires that the domain should be divided into a large number of smaller homogeneous subdomains within each domain. On the other hand, the FEM is a very common numerical technique for solving PDEs in boundary value problems. FEM has the advantage of sparseness and adaptivity, but it has several disadvantages: meshing effort, computation time and memory use. In this study, the Octree-Based Multiresolution Mesh Method (OBM^3) is introduced in an attempt to make improvements in all these areas.

PARALLEL OCTREE-BASED MULTIRESOLUTION MESH METHOD

The FEM based on an unstructured mesh has many advantages when modeling numerically various physical phenomena on complex domains in terms of geometry and material properties. Since the mesh in the computational domain is created adaptively according to the geometry and material properties, the numerical methods for unstructured meshes, such as FEM, enable one to improve the resolution and/or enlarge the computational domain retaining the computer capacity. Even though the FEM with its unstructured mesh capabilities is superior to the FDM that employs uniform grid structure, there is need to store large sparse matrices in order to solve the governing PDEs. Having to handle a large sparse matrix results in reducing either the resolution of the problem or the computational domain size due to the memory limitation of computers in large scale simulations.

To resolve this problem, the OBM^3 is presented here for the spatial discretization of the domain into regular hexahedral elements. An 'octree' is a tree data structure in 3-D analogous to the quadtree in 2-D. The basic characteristic of the octree-based multiresolution mesh is to recursively subdivide an element in the domain into 8 subelements until the criterion for refinement is satisfied while building the octree data structure. This criterion is taken to be the shear wave velocity within the medium.

Compatibility: Constraint Equations

$$\overline{\mathbf{u}}_s = \frac{\mathbf{u}_m^i + \mathbf{u}_m^j}{2} \tag{1}$$

$$\widehat{\mathbf{u}}_s = \frac{\mathbf{u}_m^i + \mathbf{u}_m^j + \mathbf{u}_m^k + \mathbf{u}_m^l}{4} \tag{2}$$

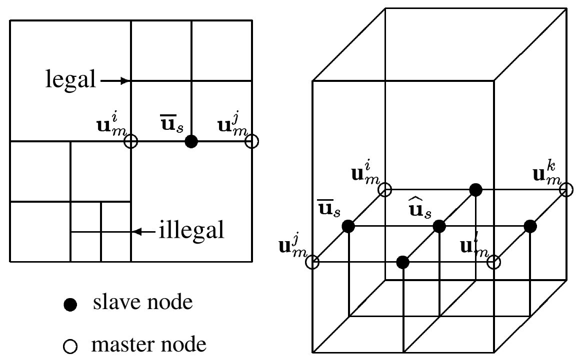


FIG. 2. Quadtree and Octree-Based Mesh

Since the OBM^3 is based on the octree algorithm, all the elements in OBM^3 are geometrically similar; except that they vary in their size. This geometric similarity of each element is a key advantage of OBM^3. Instead of assembling the global stiffness matrix in FEM with an unstructured mesh, the geometric similarity of OBM^3 allows every element to be represented by a single normalized element stiffness matrix $\mathbf{K}^e$ through element-by-element calculation of explicit time integration. $\mathbf{K}^e$ is a pre-calculated 24×24 element matrix of a cubic element with 3 degrees of freedom on each node. $\mathbf{K}^e$ factors out terms related to material properties and is normalized with respect to the element size; it is then applicable to all elements. We emphasize that an important characteristic of this approach is that it is unnecessary to store individual stiffness matrices for each element within the mesh. Furthermore, frequent reference of $\mathbf{K}^e$ by each element during element-by-element calculation at every time step leads $\mathbf{K}^e$ to reside in cache rather than memory. This cache-friendly algorithm improves overall computational performance considering two issues; (1) performance of cache (6ns) is 10 times faster than that of memory (60ns) and (2) matrix-vector product operation consumes most of computation time in solver.

Our goal in wave propagation simulation is to solve the Navier's equation,

$$\nabla \cdot [\mu(\nabla \mathbf{u} + \nabla \mathbf{u}^{\mathbf{T}}) + \lambda(\nabla \cdot \mathbf{u})\mathbf{I}] + \mathbf{f} = \rho \frac{\partial^{\mathbf{2}} \mathbf{u}}{\partial \mathbf{t}^{\mathbf{2}}}, \tag{3}$$

where $\mathbf{u}$ is the displacement vector, ρ is the density and μ and λ are Lamé constants. We discretize this equation based on the Octree-Based Multiresolution Mesh Method (OBM^3). When OBM^3 is constructed, a potential discontinuity in the displacement occurs at the interface between two elements having different size in adjacency as shown in Figure 2. To ensure continuity, we apply an additional constraint condition via Equation 1 and 2. For parallel implementation of QUAKE, MPI is used for message passing library. Figure 3 shows the framework for large scale ground motion simulations. It consists of an out-of-core mesher, a parallel partitioner and a parallel solver. It thus enables us to perform any ground motion simulations that meet total memory size of available computer systems.

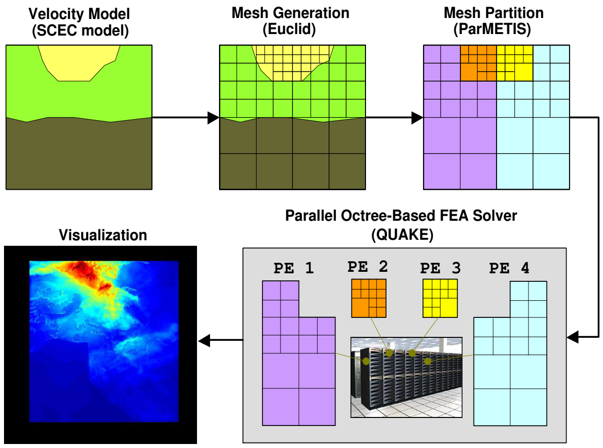


FIG. 3. Framework for large scale wave propagation simulation

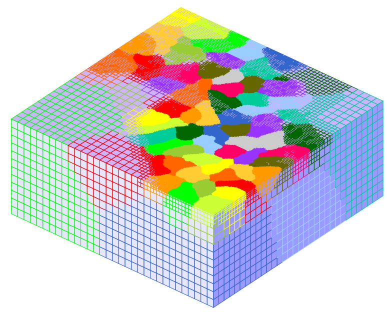


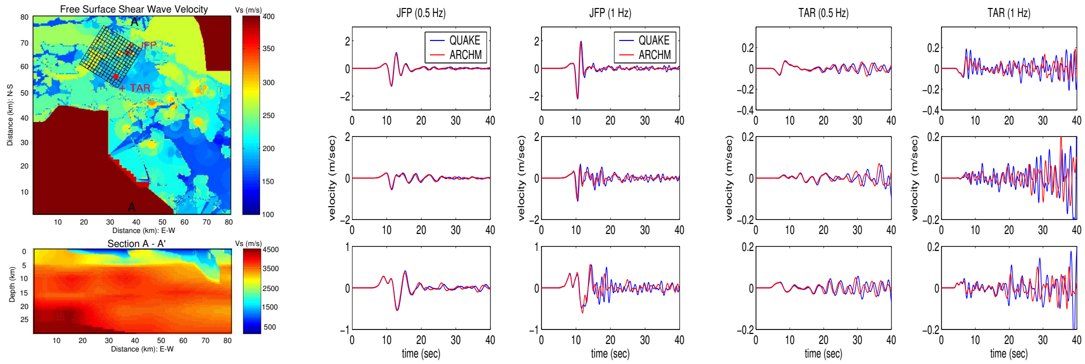
FIG. 4. Element partitioning (64 PEs)

GROUND MOTION SIMULATION

As an illustration of the OBM^3 methodology, we next apply it to the simulation of the 1994 Northridge earthquake (Wald et al. 1996). The model size is 80 × 80 × 30 km^3 covering the greater LA area (Magistrale et al. 2000). The numerical model has approximately 80 million hexahedral elements of OBM^3 and 100 million grid points with 1 Hz resolution and minimum shear wave velocity of 100 m/s. The simulation has been run on 2048 processors of HP-Compaq AlphaServer Cluster at the PSC. Figure 5(a) shows the distribution of the shear wave velocity on the free surface and on a vertical cross-section of the model. Figures 5(b) and 5(c) show simulation results of OBM^3 compared with Archimedes (ARCHM) with target resolution of 0.5 Hz. Archimedes (Shewchuk and O'Hallaron 1998), developed by QUAKE group at Carnegie Mellon University, is a parallel tool for wave propagation simulation employing unstructured mesh FEM with tetrahedral elements, that has already been verified with several FDM codes. Even though Archimedes shows good performance in ground motion simulation, difficulty of handling extremely large scale models in term of meshing, partitioning and storing large sparse matrices in core of solver, motivated the development of OBM^3. We can notice why high resolution simulation is required through Figures 5(b) and 5(c). Figure 6 shows the rupture propagation of velocity on the fault plane. Figure 7 shows wave propagation on the top surface with time history. The red dot in Figures 6 and 7 represents the hypocenter and epicenter, respectively. Figure 8 shows the snapshot of wave propagation on the top surface at 11 sec after the initiation of the event. The black lined rectangle in Figures 7 and 8 represents the projection of the fault plane onto the top surface.

PERFORMANCE AND SCALABILITY

We achieve excellent performance and scalability of QUAKE with 1 to 2048 processors on HP-Compaq AlphaServer Cluster at PSC. Table 1 shows details about the performance for the Northridge earthquake simulation in the LA basin with highest resolved frequencies ranging from 0.1 Hz to 1 Hz, corresponding to a range of problem sizes from 134,500 to over 100



(a) (b) (c)

FIG. 5. (a) S-wave velocity distribution in the LA basin model (b) Result comparison of 0.5 Hz and 1 Hz model at JFP (c) Result comparison of 0.5 Hz and 1 Hz model at TAR

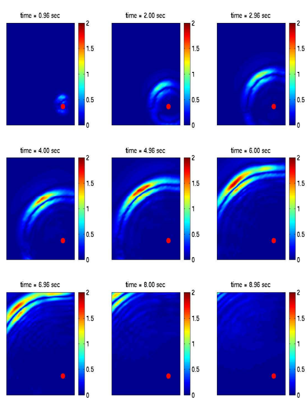


FIG. 6. Rupture Propagation: Velocity

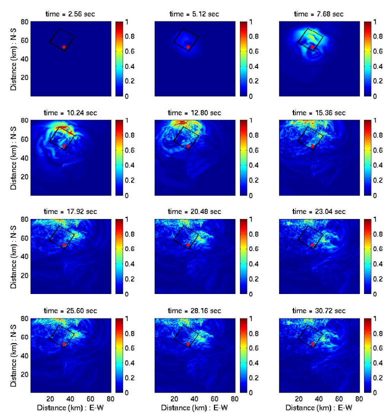


FIG. 7. Wave Propagation on the Top Surface

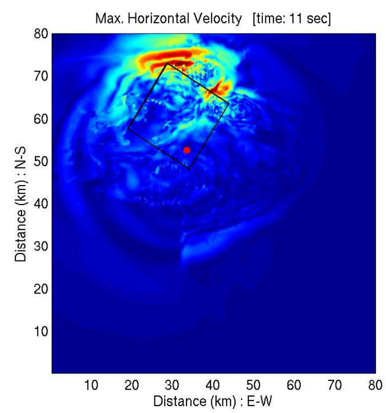


FIG. 8. Max. Wave Propagation on the Top Surface

million grid points. A relative efficiency of 88 % is achieved with 2048 PEs. The relative efficiency is calculated based on the sequential performance of LA10S and CPU efficiency is determined based on the theoretical peak performance (2 GFLOPS/s) of a processor of HP-Compaq AlphaServer Cluster.

TABLE 1. Performance Analysis and Comparison of QUAKE

Model	LA10S	LA5S	LA2S	LA1H$_A$	LA1H$_B$	LA1H$_B$
PEs	1	16	128	512	1024	2048
Grid Pts	134,500	618,672	14,792,064	47,556,096	101,939,200	101,939,200
MFLOPS/s	505	491	469	451	450	443
Rel. Effcny	100 %	97 %	93 %	89 %	89 %	88 %
CPU Effcny	25.3 %	24.6 %	23.5 %	22.6 %	22.5 %	22.2 %

CONCLUDING REMARKS

We have introduced a multiresolution finite element methodology for large-scale ground motion simulations and successfully performed the Northridge earthquake simulation in the Greater Los Angeles basin of $80 \times 80 \times 30$ km^3 domain size with approximately 300 million DOF, frequency range of 0 Hz - 1 Hz and minimum shear wave velocity of 100 m/s. The framework developed in this study allows us to perform extremely large-scale ground motion simulations and to get more accurate and reliable results with a higher resolution simulation. Our next target is to perform a ground motion simulation in the LA basin with frequency range of 0 Hz - 2 Hz and bigger computational domain (e.g. $150 \times 150 \times 50$ km^3), which will involve a billion DOF problem. We believe that the simulation allows us to better understand the earthquake-induced ground motion with higher frequency level becoming closer to the realistic ground motion.

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