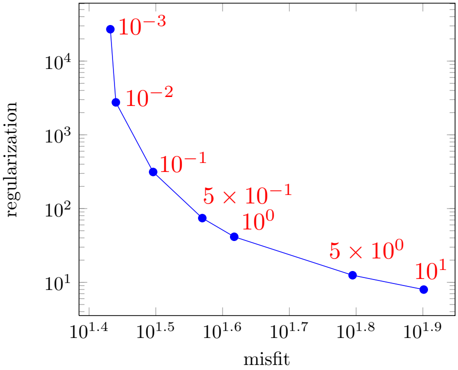
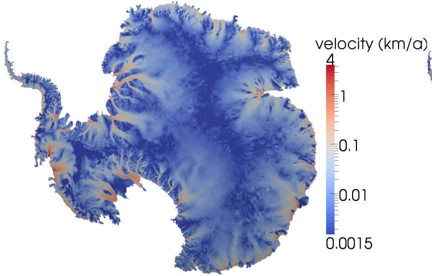
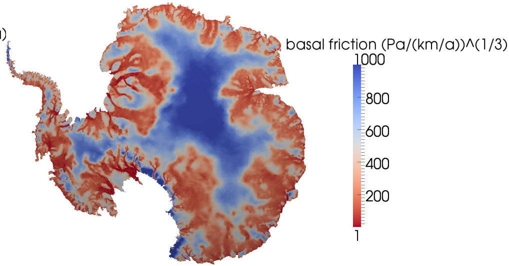
L-curve-based regularization parameter selection



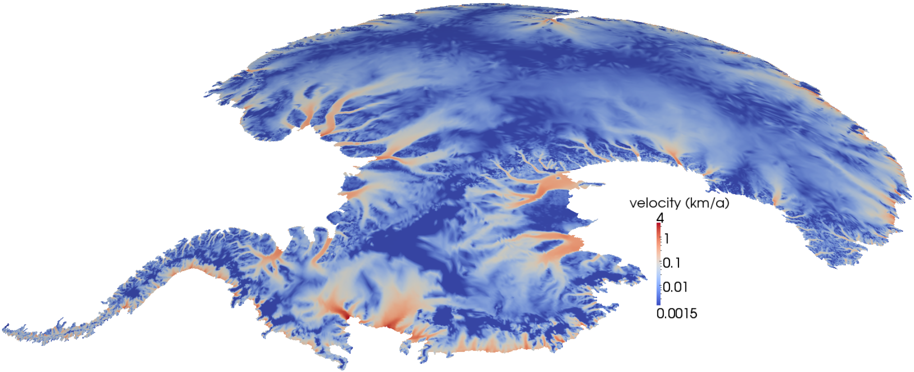
Antarctic ice sheet inversion for basal sliding field: InSAR data





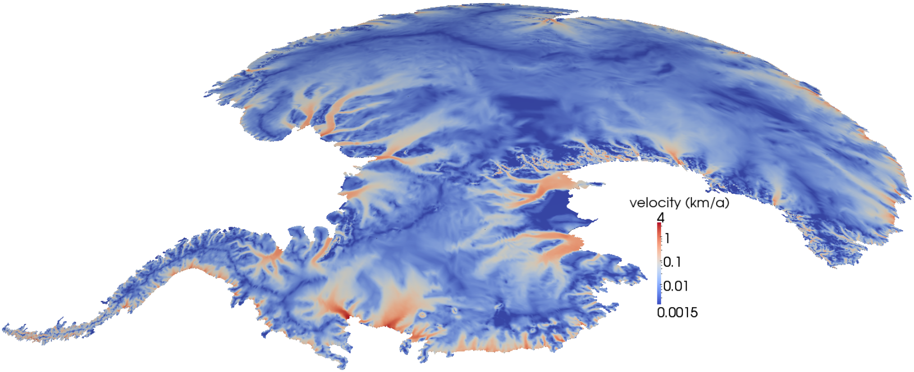
Left: InSAR-based Antarctica ice surface velocity observations
Right: Inferred basal sliding field

Antarctic ice sheet inversion for basal sliding field: InSAR data



InSAR-based Antarctica ice surface velocity observations

Antarctic ice sheet inversion for basal sliding field: InSAR data



Reconstructed ice surface velocity field

Analysis of data misfit Hessian for a (nonlinear) elliptic coefficient inverse problem (P. Flath, 2013)

Invert for log-coefficient field $\gamma(x)$ in Poisson equation from observations of state $u(x)$ within domain:

$$\min_{\gamma \in \mathcal{M}} \frac{1}{2} \int_0^L \left(b\, u(\gamma) - d_{\mathsf{obs}}\right)^2 dx$$

where the map from γ to u is defined by solution of:

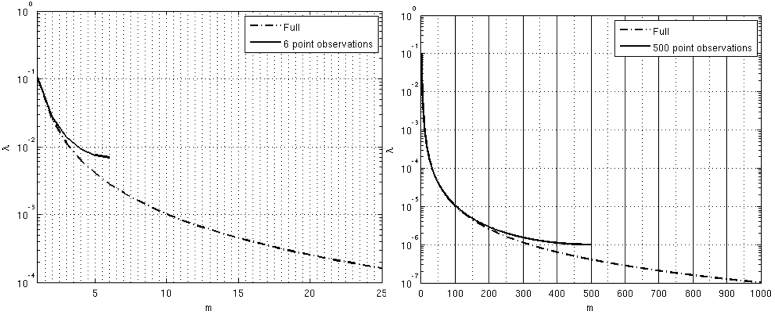
$$-\frac{d}{dx}\left(e^{\gamma} \frac{du}{dx}\right) = 0 \quad \text{for } x \in (0, L)$$
$$u(0) = u_0$$
$$u(L) = u_L$$

Two cases of observation operator:

- full observations: $b(x) = 1$
- pointwise observations: $b(x) = \frac{L}{n} \sum_{j=1}^{n} \delta(x - x_j)$

Analysis of Hessian for elliptic coefficient inverse problem

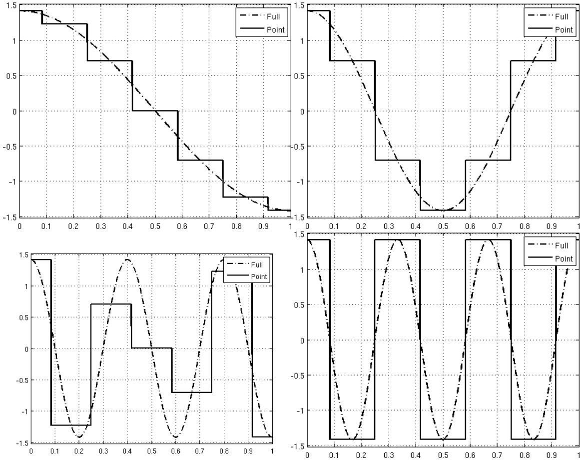
- assume no noise, $\gamma = \gamma_0$ is true log-medium
- Fourier analysis of spectrum of data misfit Hessian, evaluated at γ_0, gives eigenvalues for both cases:
  - full observations: $\lambda_i = \frac{(u_L - u_0)^2}{\pi^2 i^2}$
  - n pointwise observations: $\lambda_i = \frac{(u_L - u_0)^2}{4n^2 \sin(i\pi/2n)^2}$



Spectrum of data misfit Hessian for both full and pointwise observations
Left: 6 observations Right: 500 observations

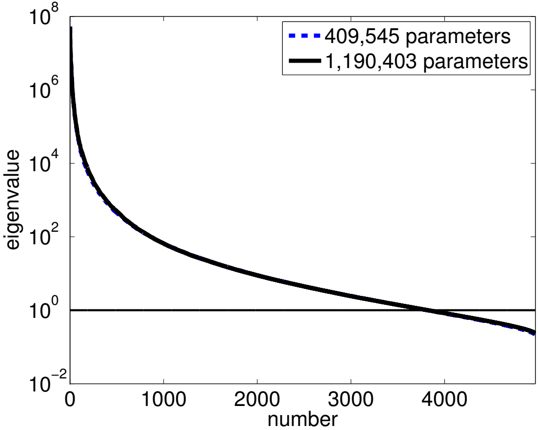
Analysis of Hessian for elliptic coefficient inverse problem

Eigenfunctions of data misfit Hessian are $\cos(i\pi x/L)$ in full observations case, and piecewise-constant interpolants of the cosines in pointwise case



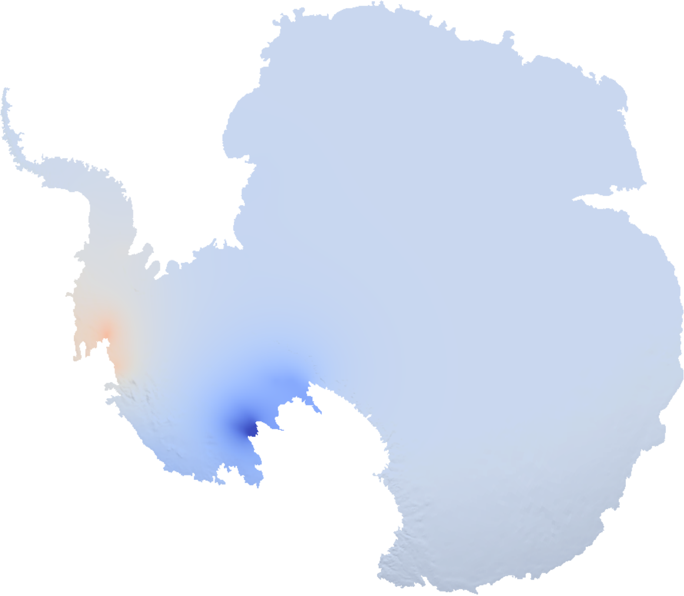
Eigenfunctions 1, 2, 5, and 6 for full and pointwise $(n = 6)$ observations

Spectrum of the prior-preconditioned data misfit Hessian

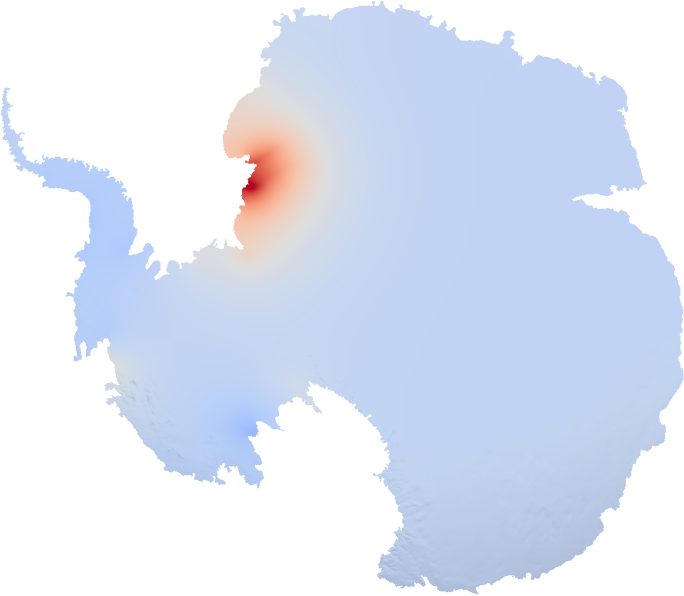


- Spectrum of $\Gamma_{\text{pr}}^{1/2} \boldsymbol{F}^T \Gamma_{\text{noise}}^{-1} \boldsymbol{F}\, \Gamma_{\text{pr}}^{1/2}$ for Antarctica inverse problem with 410K and 1.19M basal sliding parameters (observed to decay like i^{-3})
- 4000 dominant modes, independent of parameter and data dimension
- intrinsic problem dimension depends on information content of data

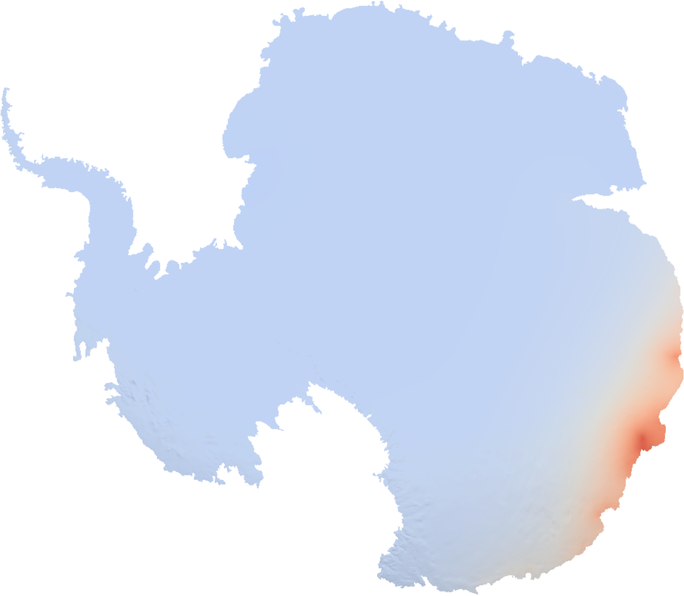
Eigenvector 1 of prior-preconditioned data misfit Hessian



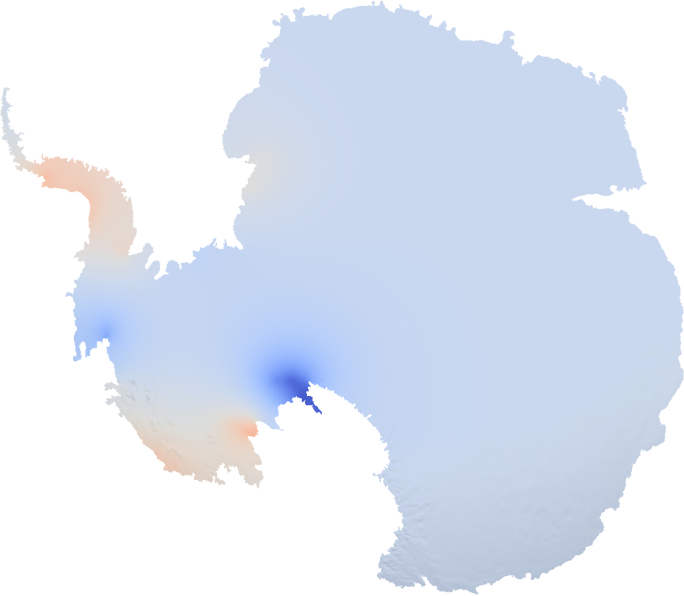
Eigenvector 2 of prior-preconditioned data misfit Hessian



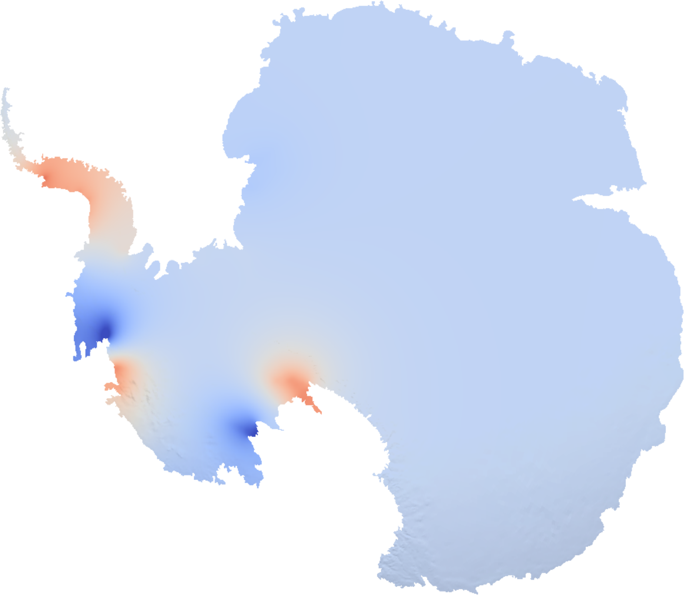
Eigenvector 3 of prior-preconditioned data misfit Hessian



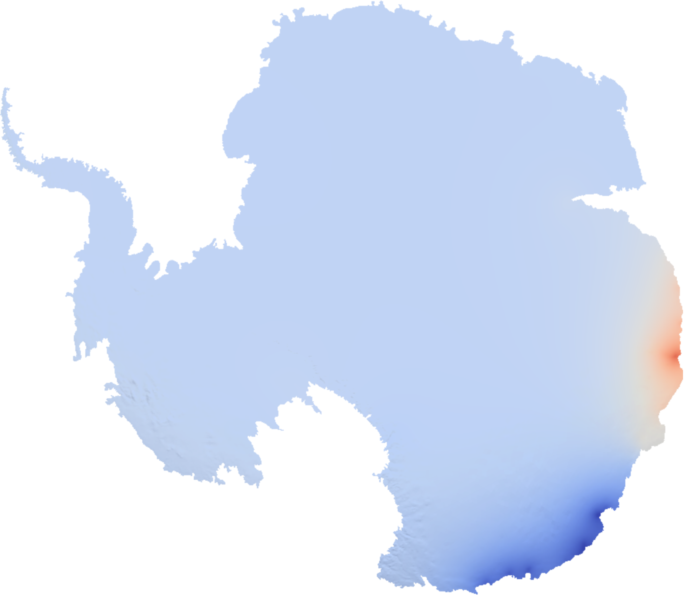
Eigenvector 4 of prior-preconditioned data misfit Hessian



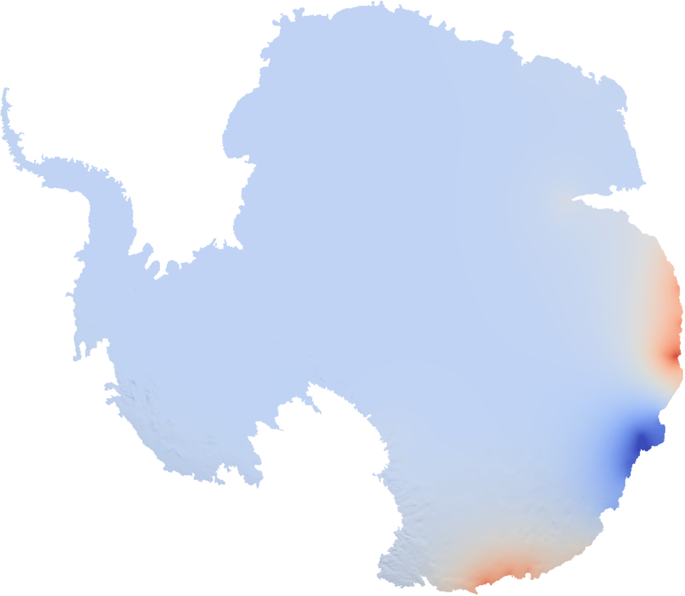
Eigenvector 5 of prior-preconditioned data misfit Hessian



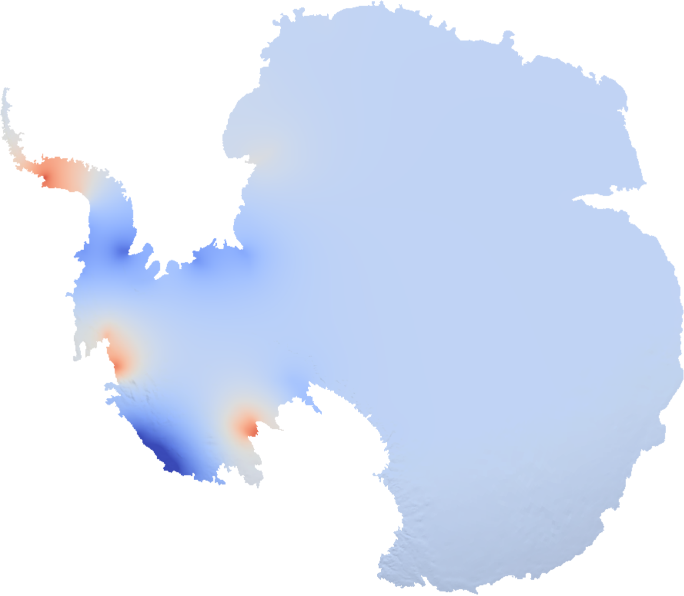
Eigenvector 6 of prior-preconditioned data misfit Hessian



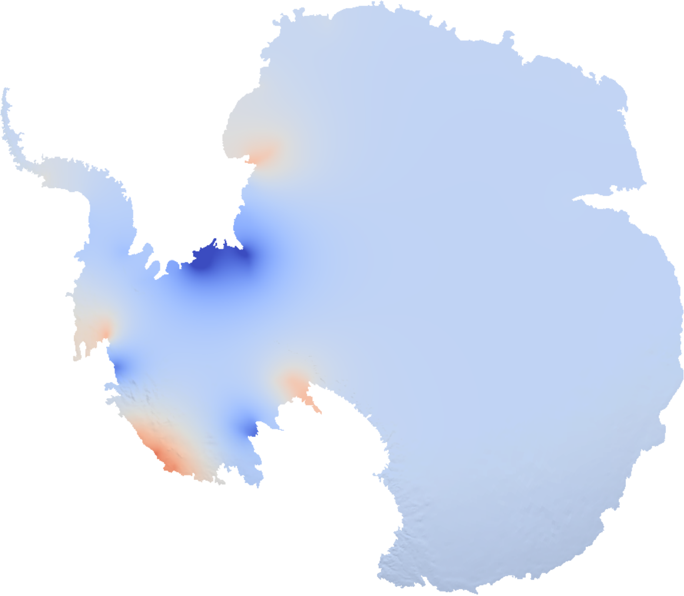
Eigenvector 7 of prior-preconditioned data misfit Hessian



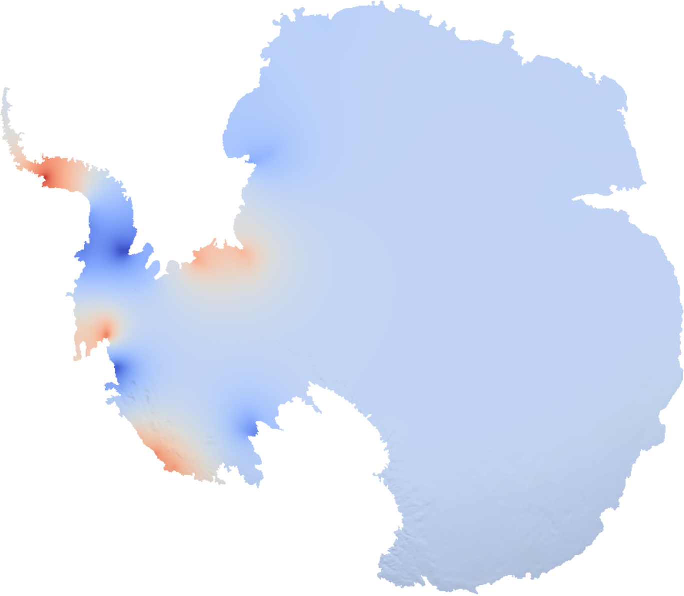
Eigenvector 8 of prior-preconditioned data misfit Hessian



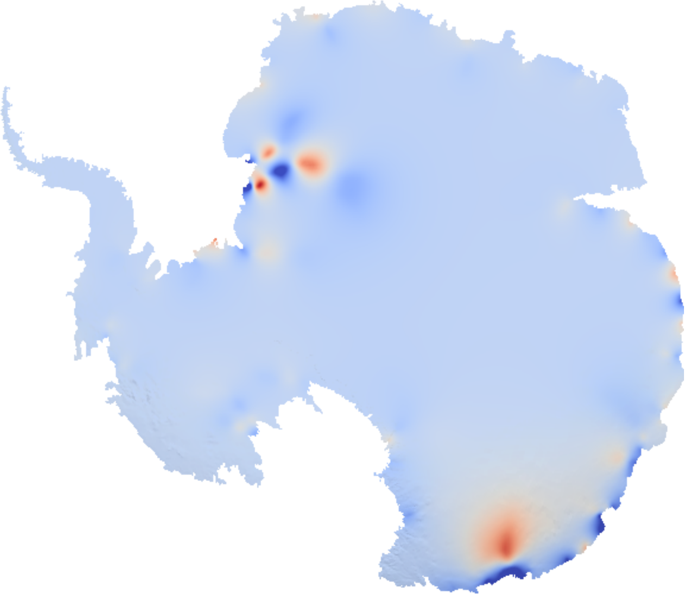
Eigenvector 9 of prior-preconditioned data misfit Hessian



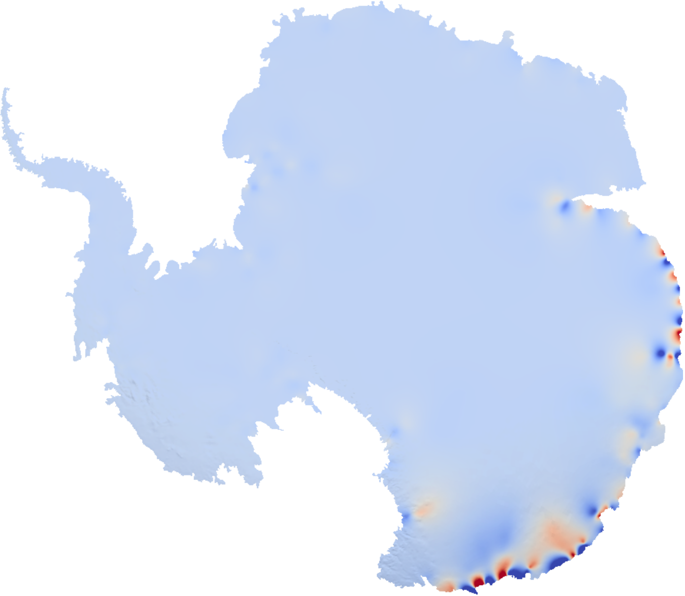
Eigenvector 10 of prior-preconditioned data misfit Hessian



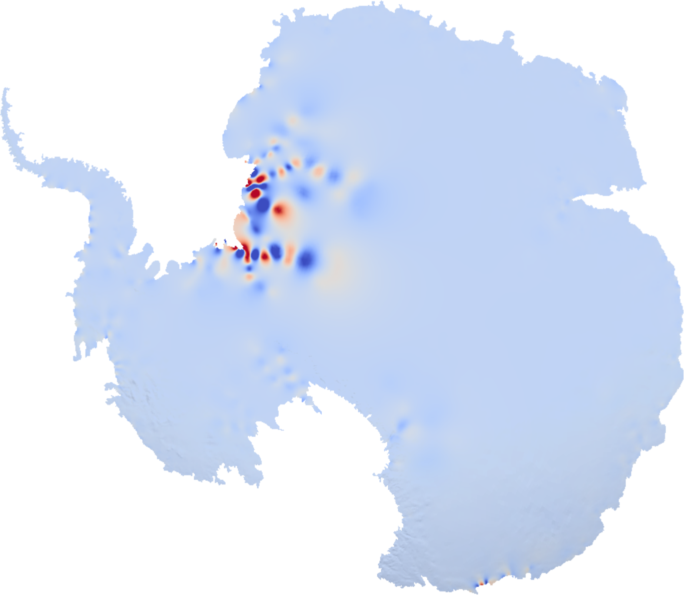
Eigenvector 100 of prior-preconditioned data misfit Hessian



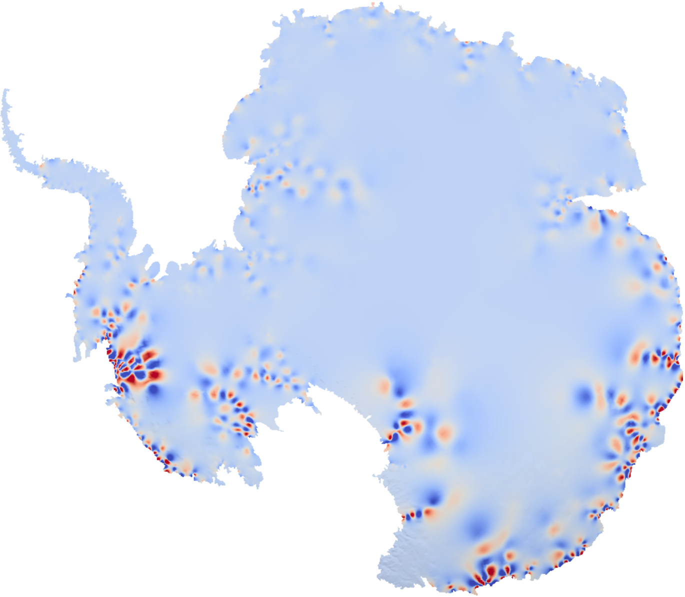
Eigenvector 200 of prior-preconditioned data misfit Hessian



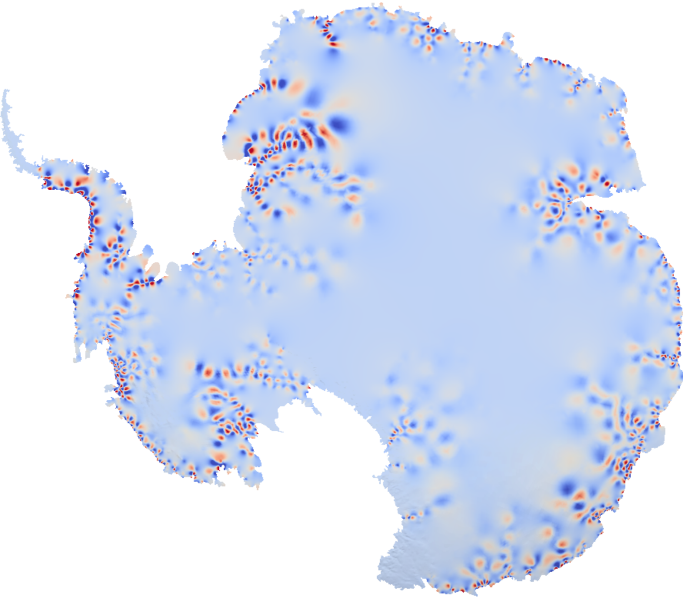
Eigenvector 500 of prior-preconditioned data misfit Hessian



E-vector 1000 of prior-preconditioned data misfit Hessian



E-vector 2000 of prior-preconditioned data misfit Hessian



E-vector 4000 of prior-preconditioned data misfit Hessian