NEW MODELS AND THEORIES OF
DYNAMIC FRICTION

J. T. ODEN and J. A. C. MARTINS
Texas Institute for Computational Mechanics
Department of Aerospace Engineering and
Engineering Mechanics
The University of Texas at Austin

Abstract. Normal contact and friction models appropriate for a class of dry dynamic friction problems are discussed. Computational results obtained using these models indicate that they are capable of describing important dynamic friction phenomena.

INTRODUCTION

Classical notions of dynamic friction between sliding rigid bodies are described in every introductory textbook on mechanics and have influenced experimental work on dynamic friction for most of this century: Sliding of bodies in equilibrium which touch on a contact surface begins when the friction force $F$ reaches a critical value proportional to the net normal force $N$, the constant of proportionality $\mu_s$ being the static coefficient of friction; once sliding begins, the friction force attains a value $F = \mu_k N$, where $\mu$ is the kinetic coefficient of friction and $\mu_k > \mu_s$.

Unfortunately, the results of literally hundreds of experiments spanning nearly a half-century have failed to establish that such a friction law is valid locally, at points of contact of metallic bodies. The fact is that this classical notion of dry dynamic friction is, at best, a rough global law which applies only to gross forces developed on essentially rigid bodies. Significant experimental and theoretical results indicate that to capture the essential features of dynamic friction in a continuum model it is necessary to model adequately the normal compliance of the contact interface. Once this is done, a model dynamic friction acceptable for describing such phenomena as stick-slip motion and friction damping, can be derived which involves a constant
coefficient of friction $\mu = \mu_s$.

In [1], a critique and assessment of a large volume of experimental literature is given and used as a basis for proposals of nonlinear constitutive laws of the normal deformation of metallic interfaces. These laws are complemented with tangential friction conditions to produce an interface model for studying a class of dynamic friction problems.

In the present note, we review features of our dynamic friction model and present variational principles for contact problems in elasto-dynamics in which nonlinear contact-friction laws are in force. In addition, finite element models and algorithms are described for the numerical solution of problems of friction damping and stick-slip motion. Also, conclusions drawn from numerical experiments are summarized.

2. NORMAL CONTACT AND FRICTION MODELS

We consider a metallic body $B$ in contact with a second body $B'$ along a contact surface $\Gamma_C$ of $B$. Without loss in generality, we can assume for our purposes that $B'$ is rigid, but it can slide relative to $B$ with some prescribed velocity $\dot{U}_T$. The actual surface of $B$ is a rough interface composed of oxides, impurities, work-hardened material, etc., and is of a different constitution than that of the parent material of which $B$ is composed. The boundary surface $\Gamma_C$ of $B$ can be defined as the surface passing through the highest asperities (or some average asperity height) and its orientation is defined by a unit vector $\vec{n}$ normal to $\Gamma_C$. The initial gap $g$ between $B$ and $B'$ is the distance along $\vec{n}$ between the highest asperities of $B$ and the flat surface of $B'$. If $t_0$ and $t$ denote the thickness of the interface before and after deformation, respectively, and if $u_n = u \cdot \vec{n}$ is the component of the displacement of material particles on $\Gamma_C$ in the direction of $\vec{n}$ then the approach $a$ of the material contact surfaces is defined by

$$a = t_0 - t = (u_n - g)_+$$

where $(\cdot)_+ = \max (0, \cdot)$. If $\dot{u}_T = \dot{u} - u_n \vec{n}$ denotes the tangential velocity of particles on $\Gamma_C$, then the relative sliding velocity between $B$ and $B'$ is equal of $\dot{u}_T - \dot{U}_T$.

Based on our earlier study [1], we consider cases in which the following normal and frictional conditions hold:

$$\sigma_n = c_n (u-g)_+ \vec{n} + b_n (u-g)_+ u_n$$

$$u_n < g \Rightarrow \sigma_T = 0$$
Here $\sigma_n$, $\sigma_T$ are normal and tangential stress components ($\sigma_n = \sigma_{ij} n_i n_j$, etc.) and $c_n$, $c_m m_n$, $b_n$, $f_n$ are material constants. Detailed interpretations and justification of these results are given in [1]. This friction law is a slight generalization of Coulomb's law with a coefficient of friction of $\mu = (c_n/c_m m_n/m_n) \cdot |\sigma_n|^a$, $a = (m_n/m_n) - 1$.

3. VARIATIONAL PRINCIPLE FOR DYNAMIC FRICTION

The following variational principle holds for dynamic friction problems in which conditions (1) hold and body $B$ is linearly elastic (with $\sigma_{ij} = E_{ijk\ell} \varepsilon_{k\ell}$, $\varepsilon_{k\ell} = (U_{k,\ell} + U_{\ell,k})$ (2):

Find a displacement field $\dot{U} = U(x,t)$ in a space $V$ of admissible displacements such that

$$\begin{align*}
&\langle \rho \ddot{u}(t), v - \dot{u}(t) \rangle + a(u(t), v - \dot{u}(t)) \\
&+ \langle P(u(t), \ddot{u}(t)), v - \dot{u}(t) \rangle + j(u(t), v) \\
&- j(u(t), u(t)) \geq \ell(t), v - \dot{u}(t)\rangle
\end{align*}
$$

for all $v$ in $V$

Here $\langle \cdot, \cdot \rangle$ denotes duality paring on $V'$ x $V$ (e.g. $\langle u, v \rangle = \int u \cdot v \, dx$, with $\Omega$ the interior of $B$ and $dx = dx_1 dx_2 dx_3$ is a volume element), $v$ is an arbitrary virtual displacement, $a(\cdot, \cdot)$ is the vertical work,

$$a(u,v) = \int_{\Omega} E_{ijk\ell} u_{k,\ell} v_i \, dx$$

$$P(u,\ddot{u}), v = \int_{\Gamma_c} [c_n(u_n - g)_+ m_n v_n + b_n(u_n - g)_+ f_n n_n v_n] \, ds$$

$$j(u,v) = \int_{\Gamma_c} c_T(u_n - g)_+ v_T \dot{U}_T \, ds$$
and \((\mathbf{f}, \mathbf{y} - \mathbf{u}_1)\) is the vertical work of the external forces. Here,

\[ V = \{ \mathbf{v} = (v_1, v_2, v_3) \mid v_1 \in H^1(\Omega) \}, \]

\[ v_1 = 0 \text{ on } \Gamma_D \subseteq \partial\Omega \]

We must also impose initial conditions, \(v(0) = v_0, \dot{v}(0) = \dot{v}_1\). The fact that dynamic friction leads to a variational inequality rather than an equality is due to the presence of the non-differentiable frictional functional \(j(\cdot, \cdot)\). In practice, we use standard regularization ideas and replace \(j\) by a differentiable approximation \(j_\epsilon\), depending on a positive parameter \(\epsilon\).

4. **FINITE ELEMENT MODELS AND NUMERICAL RESULTS**

Standard conforming finite element approximations of (2) (with \(j\) replaced by \(j_\epsilon\)) lead to a discrete dynamical system of the form

\[
\begin{align*}
M \ddot{\mathbf{r}} + K \mathbf{r} + P(\mathbf{r}, \dot{\mathbf{r}}) + J(\mathbf{r}, \dot{\mathbf{r}}) &= \mathbf{F} \\
\mathbf{r}(0) &= \mathbf{r}_0, \dot{\mathbf{r}}(0) &= \dot{\mathbf{r}}_1
\end{align*}
\]

Here \(M\) and \(K\) are the usual mass and stiffness matrices, \(P\) is a vector of nonlinear functions of the nodal displacement vector \(\mathbf{r}\) and velocity vector \(\dot{\mathbf{r}}\), \(J\) is the nonlinear friction-damping matrix arising from the friction functional \(j\) (or \(j_\epsilon\)), and \(\mathbf{F}\) is the force vector.

We employ both central difference temporal approximations of (3) with a lumped mass matrix and an implicit Newmark-\(\beta\) method to study transient dynamic problems. Our results indicate that our friction model is capable of describing a variety of dynamical frictional phenomena, including stick-slip motion, friction damping, and steady sliding friction.

Because of space limitations, we shall record here only the principal conclusions drawn from a series of numerical experiments.

We developed a two-dimensional finite-element model of an elastic block resting on a moving belt while being restrained by a linear spring, as indicated in Fig. 1 (the model consisted of a 4 x 4 mesh of nine-node biquadratic elements). Equations (3) were integrated for a choice of parameters and material constants representative of friction experiments and for a given belt velocity. Computed results indicate that the body maintains a stable sliding equilibrium position for given \(\dot{U}\) for a coefficient of friction \(\mu\) sufficiently, small. As \(\mu\) is increased, for fixed \(\dot{U}\), a dynamic situation is reached in which small oscillations about an equilibrium position occur. By linearizing dynamical equations about an equilibrium position and performing a
standard eigenvalue analysis of the resulting system, one can detect the onset of such oscillations by the emergence of imaginary eigenvalues for sufficiently large $\mu$. If $\mu$ is increased further, eigenvalues with positive real parts are obtained which signal dynamic instability, is manifested in rotational and normal oscillations of growing magnitude and is associated with stick-slip motion.

In particular, other observations derived from our numerical results for the case of $\mu$ sufficiently large are:

- The variation of the normal force on the contact surface produces changes in the sliding friction force which in turn produce a tangential oscillation;
- The tangential oscillation may then become sufficiently large that, for small values of the belt velocity, the points of the body on the contact surface attain the belt velocity and the body sticks for short intervals of time;
- With the increase in magnitude of the normal oscillations, actual normal jumps of the body may occur;
- The repeated periods of adhesion have the result of decreasing the average value of the friction force on the contact surface and, due to the absence of equilibrium with the restoring force on the tangential spring, the tangential displacement $U_{GX}$ of the center of mass decreases;

It follows that for given geometry and material data, one of the two following situations may occur:

(a) for values of $x$-velocity component $U_{GX}$ of the center of mass larger than some critical value, the normal, rotational and tangential oscillations evolve to what appears to be a steady oscillation with successive periods of adhesion and sliding, the average values of the friction force and of the spring elongation being smaller than those corresponding to the steady sliding equilibrium position.

(b) for values of $U_{GX}$ lower than the critical value, and at a sufficiently small value of the spring elongation, the normal (and rotational) damping is able to damp out the normal (and rotational) oscillation and the body sticks, since the restoring force of the spring is then smaller than the maximum available friction force.

Thus, monitoring the spring elongations, as is often done in friction experiments, case (a) would be perceived as an apparently smooth sliding with a coefficient of kinetic friction smaller than the coefficient of static friction and case (b) would be perceived as stick-slip motion.
Acknowledgement: The support of the Air Force Office of Scientific Research through Contract F4950-84-0024 is gratefully acknowledged.

Reference


Figure 1
Figure 1