
closure ......................................................... 1255


closure .......................................................... 1256

Finite Element Analysis of Inflatable Shells,* by Chin-Tsang Li and John W. Leonard (June, 1973).

by Eberhard Haug ............................................. 1256


by Venkata Narasimharao Tanniru .......................... 1259


by Samuel W. Chung ........................................... 1261


by Palaniappan Meiappan and Pappiah Gopalsamy ............ 1262

by Venkata Narasimharao Tanniru .................................. 1264

Note.—This paper is part of the copyrighted Journal of the Engineering Mechanics Division, Proceedings of the American Society of Civil Engineers, Vol. 100, No. EM6, December, 1974.

*Discussion period closed for this paper. Any other discussion received during this discussion period will be published in subsequent Journals.


*Discussion period closed for this paper. Any other discussion received during this discussion period will be published in subsequent Journals.
The writers wish to thank Strauss for his discussion and valuable comments. Strauss points out that the arc length parameter, $Z$, as defined by either Eq. 55 or Eq. 57 is an inappropriate measure of the mechanical response of graphite. The writers feel that the introduction of such a monotonically increasing function of deformation (i.e., time-scale $Z$) is almost mandatory, for otherwise, two different states of deformation could exist for the same value of $Z$. Furthermore, a positive rate of change of the free energy density, $\phi$ (or the strain energy density, $\epsilon$), with respect to $Z$ (i.e., $d\phi/dZ$) could not be interpreted unambiguously as a process of increasing $\phi$, if $dZ$ could be negative.

Strauss correctly questions the applicability of Eqs. 67 and 56 to predict the mechanical response of graphite under cyclic loading as shown in Fig. 2. The writers concur with Strauss's observations and strongly feel that this shortcoming can be overcome by appropriately defining the response function, $\mu(z)$, of Eq. 67, which would accommodate the loading-unloading processes. From Eq. 63 it is quite clear that even a single term in the response function, $\mu(z)$, appearing in Eq. 62 is sufficient to predict the behavior of polycrystalline graphite for the loading history of continuous straining. In view of this, it is our opinion that it may be fruitful to investigate by including more exponential terms in series representation for $\mu(z)$ to obtain meaningful characterization of the materials behavior under loading-unloading processes. The full implications of the theory will be investigated further in our future work.

It may be mentioned that during the course of this investigation the writers were not aware of Strauss’s work except for a private communications from Snyder (52) that motivated the discussion of this problem. Finally, we thank Strauss for providing us with additional references on the subject.


*Prof., Dept. of Engrg. Mechanics, Univ. of Texas, Austin Tex.; formerly, Prof. and Chmn. of Engrg. Mechanics, Univ. of Alabama, Huntsville, Ala.*

*Sr. Engr., Westinghouse Corp., Pensacola, Fla.*
RECYCLING WASTES FOR STRUCTURAL APPLICATIONS

Closure by Seymour A. Bortz, M. ASCE and Murray A. Schwartz

The writers wishes to thank Rajagopalan for his comments concerning this paper. In the paper we use the term recycle to mean undergo further treatment, change, or use. These are dictionary definitions and under these circumstances recycling as used in the paper appears to be correct.

The strength of the foamed blocks ranged from 2,000 psi (70 pcf)–150 psi (30 pcf) depending on density. It is our intent to consider the use of the 2,000-psi block for post-tensioning. We have built 10-ft beams and precompressed them to 1,000 psi and have kept these beams for demonstrations in the laboratory. The idea of impregnating the block with a resin is a good one and would enhance its properties to the point where thin sections could be used for structural applications.

FINITE ELEMENT ANALYSIS OF INFLATABLE SHELLS

Discussion by Eberhard Haug

The authors introduced a new, curved finite element, developed a theory, and carried out an interesting study on the analysis of pneumatic structures. The writer wishes to raise some questions and to comment on some points concerning the proposed method of analysis.

Does the element satisfy the fundamental requirements of: (1) interelement continuity of displacements; (2) zero strain rigid body modes; and (3) the inclusion of constant strain terms?

Although not all of the requirements may be necessary for a convergent scheme, the displacement interpolations of the element should be designed carefully, especially in a nonlinear analysis for large displacements. It will be interesting to know how the new element behaves for large displacements.

In this paper, the nature of the displacement interpolation is never stated explicitly, but the authors note when discussing sample problems, that it is

---


*b* Supervisory Ceramic Engr., Bureau of Mine, Tuscaloosa Metallurgy Research Lab., University, Ala.

*b* Chin-Tsang Li and John W. Leonard (Proc. Paper 9802).

*b* Engr., Informatique Internationale, Rungis, France.
FIG. 9.—Inflated Uniform Stress Membranes with Different Arrangements of Crossing Cables of Given Cable Tension over 1,200-in. x 1,200-in. Square Opening
reasonable to assume bilinear displacement interpolation functions, associated with a total of 12 displacement degrees-of-freedom at the four corners. The curved geometry of the element, however, is specified by no less than 48 quantities. The element should be termed "superparametric," in the sense that $48 - 12 = 36$ displacement degrees-of-freedom are missing in order to make it an isoparametric element. If such an element were flat before deformation it would remain flat after deformation and its curved geometry capability becomes rather pointless.

The authors specify the displacement interpolation functions in the directions of the local, curvilinear element coordinate system, without mentioning any difficulties that may be associated in that case with a proper representation of the rigid body modes. Does a strongly curved element of the considered type behave adequately under certain rigid body displacements?

The writer assumes that the element as described in the paper produces good results if the displacements are small, or if the deformed shape will be geometrically similar to the undeformed shape.

The way the paper derives finite strains is a relic of classical shell theory, using interpolated local, curvilinear surface displacements. This approach seems to be unnecessarily complex, and it is probably inefficient in numerical calculations. It does not profit from the fact that the deformation of the field is kinematically determinate in terms of nodal displacements. Any deformation measure should be expressed as directly as possible in terms of the problem variables (28).

The authors claim that usually two problems arise in the design of inflated structures: (1) The uninflated geometry is known a priori; and (2) the fully pressurized geometry is known a priori. Although this may be true for the simplest known pneumatic forms, in general neither problem arises.

The writer wishes to add another class of problems, which is of great importance in the design of inflatables—the final stress distribution and the loads are given a priori.

Obviously, then, the corresponding inflated shape and the unstressed dimensions of the membrane are sought. This is believed to apply in practical problems and the problem has been solved for given, uniform distributions of stress, e.g., soap films (26,27,28). A modified approach is described in Ref. 29. Fig. 9 shows some uniform tension membranes. The finite elements used are isoparametric 12 degrees-of-freedom quadrilateral elements.

Appendix.—References

CANTILEVER CYLINDRICAL SHELLS UNDER ASSUMED
WIND PRESSURES

Discussion by Venkata Narasimharao Tanniru

The authors should be thanked for their timely work on a problem of topical interest, i.e., the static analysis of clamped-free cylindrical shells by means of bending theory for the case of wind loading. The analysis is done by means of Donnell's shell theory, which requires the determination of eight constants for each harmonic from the eight boundary conditions of the shell. Obviously such an analysis is required for those shells in which the edges of the shell mutually influence each other. The writer wishes to discuss the computation, the results, and the presentation of the results.

Hampe (9) indicates that the bending analysis of clamped-free cylindrical shells can be done in an approximate way for those with \( a/h \) less than 100 and \( l/a \) greater than 10. This results in a reduction of computational labor due to the fact that for these shells the edges do not mutually influence each other. As a result, only four constants arise for each harmonic, which are to be determined from the four boundary conditions at the clamped edge.

It would have been better if the authors had continued the computations beyond the \( l/a \) ratios shown in Fig. 8. The computations are done only up to \( l/a = 5 \) for \( a/h = 300, 400, \) and 500, and only up to \( l/a = 3.2 \) for \( a/h = 200. \) If the computations were done up to the stage of the curves in Fig. 8, becoming parallel to the \( l/a \)-axis (which means that the membrane stress at the root of the windward generator becomes independent of the length of the shell), it would have given an idea of the various \( l/a \) ratios for different values of \( a/h \) beyond which the mutual influence of the edges ceases to exist.

The authors presented the results for the assumed wind pressure, which is defined by the harmonics and their coefficients. The results therefore give the total effect of the various harmonics varying from 0 to 6. The results would be more useful if they were presented separately for each harmonic. Such a presentation would be very valuable for two reasons: (1) The relative differences in the effects of the various harmonics can be gaged; and (2) any designer of oil storage tanks, industrial chimneys, etc., can utilize these results suitably, depending upon the number and coefficients of harmonics of the wind pressure distribution, which govern his design. The wind pressure distributions for the design work vary widely in the number of harmonics and their coefficients, depending upon the code provisions and the experimental results. For example, Hampe (9) gives the wind load as \( p_0 \) \((-0.7 + 0.45 \cos \theta + 1.2 \cos 2 \theta)\), according to the provisions of TGL 10705.

The authors have given, in Eq. 8c, the solution for the uniform component of the wind load. In Eq. 8d, the solution for the semi-infinite shells is given.

\[a\text{October, 1973, by S. Gopalacharyulu and D. J. Johns (Proc. Paper 10085).}\\
\[b\text{Reader, Applied Mechanics Dept., MNR Engrg. Coll., Allahabad, India.}\\
For all the shells computed by the authors the solution of the semi-infinite shell can be very comfortably used. For the shell of \( a/h = 200 \), the length of the decay of bending measured from the rigid base is equal to roughly 0.18 \( a \). That means for \( x > 0.18 \), the bending is absent. For higher ratios of \( a/h \) the decay lengths are still smaller. These decay lengths for the uniform components are taken from the curve given in Ref. 9.

The writer calculated the values of \( 1,000 \sigma_x/(E\lambda) \) and \( 1,000 \gamma_{x\theta}/\lambda \) from membrane theory for all the generators, for which the results are given by the authors. There is a common tendency discernible in the results, i.e., the value of \( 1,000 \sigma_x/(E\lambda) \) at the root of the generator from membrane theory is about 30% less than that given by the authors. Similarly the value of \( 1,000 \gamma_{x\theta}/\lambda \) at the rigid base given by the authors is roughly one-half of the value obtained from membrane theory. The membrane equations, taken from Hampe (8), and using the authors’ notation are:

\[
\frac{1,000 \sigma_x}{E\lambda} = -500 \left( \frac{l}{a} \right)^2 \left( \frac{h}{a} \right)^2 \left( -\xi^2 + 2\xi - 1 \right) \sum a_n n^2 \cos n\theta \quad \cdots \cdots \quad (15)
\]
The results of the membrane theory and those of the authors' work (obtained from the membrane strains given by the authors) are shown in Figs. 9 and 10. Fig. 9 pertains to the windward generator, which experiences the maximum $\sigma_z$. Fig. 10 shows the shear strain variation for the generator with $\theta = 30^\circ$. Because the authors did not mention the bending stresses in their results, they obviously must not be considerable. Since the variation of $\sigma_z$ and $\gamma_{zz}$ in the authors' results and in those of membrane theory is similar, the writer is of the opinion that a semimembrane theory can still be developed, which will take into account the shear strains also.

Appendix.—References


ANALYSIS AND STABILITY OF FLOATING ROOFS

Discussion by Samuel W. Chung

The author has carried out useful research for oil storage tank design and has paid attention to the following important factors: (1) The nonuniform lateral pressure distribution assumed in obtaining starting values of dimensionless lateral pressure and deflection of deck plate $P_z$ and $W_0$ in Eqs. 44a and 44b; and (2) the unique compatibility relation between deck plate and pontoon juncture (this is the only practical reference line of the roof because of floatation and distortion) established in Eq. 3a in conjunction with the stability criteria of Eq. 26. However, it seems to the writer that further illustrations need to be made and there may be other treatments for the problem as follows:

1. The pressure distribution on the deck expressed in Eqs. 44a and 44b are classified as uniform part and nonuniform part:

\[ p_z = p_u + p_n \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (70) \]

\[ p_u = -(H_{p_w} + H_{p_o}) + p_z \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (71) \]
\[ p_n = w(\rho_w - \rho_o) \]  

in which \( p_z, p_u, p_n \), and \( p_s \) = dimensioned quantities; and \( p_u \) identifies uniform pressure and \( p_n \) identifies nonuniform pressure. Note that \( p_z \), the unit weight of deck plate under submerged condition, was added to the term used by the author.

2. Since the slope of deck plate at the edge is very important for the purpose of obtaining the load carried by pontoon section we must use accurate deflection mode. The writer observed a case of nonparabolic shape mode of considerably bigger slope than the parabolic shape used by the author in Eq. 51 for starting value. Let us assume a deflection mode of the fourth order, then the dimensionless deflection mode, according to the author's notation, is

\[ W = W_0 \left( 1 + a_1 R^2 + a_2 R^4 \right) \]  

in which \( a_1 \) and \( a_2 \) = constants to be determined by boundary condition and experimental data.

3. If the assumption of infinite bending stiffness of pontoon section is acceptable, we can express the membrane force, \( N_r \), as

\[ N_r = \frac{Q}{2\pi r \left( \frac{dw}{dr} \right)} \]  

in which the total vertical force \( Q \) is

\[ Q = 2\pi \int_0^a p_z r dr \]

If we apply Eqs. 70, 71, 72, and 73 into Eqs. 74 and 75 we can integrate Eq. 3b.

---

**ANISOTROPIC BEAMS BY MOMENT-DIFFERENTIAL METHODS**

**Discussion by Palaniappan Meiappan** and Pappiah Gopalsamy

The authors' contribution to the solution of anisotropic beam problems is an elegant, systematic, direct, and generalized approach. The writers would like to present some remarks on the reports they have presented.

Eqs. 37 and 43 and 38 and 44 are arrived at from Eqs. 14 and 16 and not...
from Eqs. 16 and 14, respectively. The values of $a_{10}$ and $a_{n0}$ in Eqs. 37 and 27 have to be prefixed with negative sign.

This method of approach can be extended to nonprismatic members, e.g., dams, retaining walls subjected to triangular bending load and direct load. As an extension to the beams of nonuniform sections, stress expressions $\sigma_{11}$ with corresponding beam and load figures are shown in Fig. 7. The stress expression for the beam in Fig. 7(a) is

$$\sigma_{11} = \frac{P(L - x_i)x_i}{I_{2x_i}} + 2P\eta \left( \frac{1}{I_{0x_i}} - \frac{x_i^2}{I_{2x_i}} \right) \quad (60)$$

and the expression of stress, $\sigma_{11}$, for the beam in Fig. 7(b) is

$$\sigma_{11} = \frac{w(L - x_i)^3}{6L I_{2x_i}} x_i - \frac{w}{L} \left( L - x_i \right)^2 \left( \frac{x_i^2}{I_{2x_i}} - \frac{1}{I_{0x_i}} \right)$$

$$+ \frac{w}{L} (L - x_i)^3 \beta x_i \left( \frac{x_i^2}{3I_{2x_i}} - \frac{3}{5I_{0x_i}} \right)$$

$$- \frac{w}{L} \left[ \frac{-K_1 I_{2x_i}}{10I_{0x_i}^2} + \frac{16(\alpha_1 - 3\eta^2)}{5I_{0x_i}} x_i^2 + \frac{k_2 x_i^4}{6I_{2x_i}} \right] \quad (61)$$
in which $I_{0x_1}$ and $I_{2x_1} = \int dA$ and $\int x_2^2 dA$ at a distance, $x_1$, from the origin, respectively; and

\[
k_1 = (-48\eta^3 + 4\eta\alpha_1 + 9\alpha_2) \tag{62}\]

and

\[
k_2 = (16\eta^3 - 8\eta\alpha_1 + 3\alpha_2) \tag{63}\]

Note the modification in the stress expression, $\sigma_{11}$, for a beam of variable depth (2$B$-depth at the origin of the coordinate system, 2$b$-depth at free end; and $t$ thickness) over a similar problem with a constant depth as in Example 1. Terms $I_{0x_1}$, $I_{2x_1}$, $I_{4x_1}$, and $B_x_1$ are substituted for $I_0$, $I_2$, $I_4$, and $b$, respectively. Beam column problems could be studied with an advantage over the classical strength of materials approach. Possible extensions could be evolved for three-dimensional anisotropic, elastic beams through which interesting solutions for the cases of biaxial bending with or without thrust could be obtained.

Discussion by Venkata Narasimharao Tanniru

The authors have presented a simplified method for obtaining the elasticity solution for anisotropic beams. The writer would like to comment on the derivation and the simplicity of the method.

The authors state in their Introduction: "In working with Hashin's method to obtain the solution of a variety of beam problems, it was observed that the solutions followed a specific pattern, i.e., all solutions could be expressed as sums of moment differentials. Extending the observation, two theorems were obtained which will be presented in the paper." The observation about the solutions is correct. The theorems were proved by showing that the undetermined coefficients of the series for $\sigma_{11}$ can be uniquely determined. In the writer's opinion the proof is not direct in obtaining the result for $\sigma_{11}$ at the end of the derivation. The result is simply verified to be correct. The writer wishes to suggest a direct proof, which follows from the equations of Hashin (1).

Using Hashin's (1) notation, the solution for the stress function is

\[
F = \sum_{m=0}^{M} \sum_{n=0}^{N} C_{mn} x_1^m x_2^n \tag{64}
\]

The solution, therefore, for $\sigma_{11}$ is

\[
\sigma_{11} = \sum_{m=0}^{M} \sum_{n=0}^{N-2} D_{mn} x_1^m x_2^n \tag{65}
\]

in which $D_{mn}$ = arbitrary constants.

Differentiating twice Eq. 9 with respect to $x_2$, the following governing differential equation for $\sigma_{11}$ is obtained

\[
-S_{1111} \frac{\partial^4 \sigma_{11}}{\partial x_1^2 \partial x_2^2} - 2(2S_{1212} + S_{1122}) \frac{\partial^4 \sigma_{11}}{\partial x_1^2 \partial x_2^2} + 4 S_{1112} \frac{\partial^4 \sigma_{11}}{\partial x_1 \partial x_2^3}
\]

The solution for $\sigma_{11}$ given by Eq. 65 can be uniquely determined from the recursion relationships, which can be developed from the differential equation, Eq. 66, and from the conditions of statics pertaining to the axial force, $P$, and the bending moment, $M$. In principle the problem of giving the solution for $\sigma_{11}$ is solved. This is just an extension of the work of Hashin (1) with the concentration on the solution for $\sigma_{11}$. The number of constants in the method given by the authors and the aforementioned extension of Hashin’s work for obtaining $\sigma_{11}$ is the same. In the authors’ notation, the number of constants to be determined is $2(n + 1) + [n(n + 1)/2] = 1/2 (n + 1)(n + 4)$. The number of constants for the solution of $\sigma_{11}$ in Hashin’s method $= 1/2(M + 1)(M + 4)$. Term $M$ in Hashin’s notation is equal to $\eta$ in the authors’ notation. Then writing the solution for $\sigma_{11}$ in the moment-differential or force-differential form as done by authors in Eqs. 10 and 17 is simply a matter of algebra and is basically a matter of personal preference. In Hashin’s method, $N$ — the highest power of $x_2$ in the solution for stress function — is equal to $M + 3$. In the solution for $\sigma_{11}$, the highest power of $x_2$ will then be equal to $M + 1$. Term $M$, the highest power of $x_1$ in the solution for stress function, is equal to the highest power of $x_1$ in the polynomial for $\sigma_{11}$ and therefore in the polynomial for the bending moment or axial force. Therefore the solution given by Hashin (1) can be extended for the solution for $\sigma_{11}$ by suitably developing the recursion relationship of coefficients $D_{mn}$ and considering the statical conditions of the axial force and bending moment. The step domain of coefficients, $D_{mn}$, will be smaller than that of $C_{mn}$ by two diagonals. In the solution for the stress function the restriction is $m + n \leq M + 3$. In the solution for the stress, $\sigma_{11}$, the restriction will be $m + n \leq M + 1$. With this restriction the solution for $\sigma_{11}$ can be immediately written as a summation of products of the polynomials of $x_2$ and differentials of the bending moment or axial force as done by the authors. In the writer’s opinion, getting the polynomial solution for $\sigma_{11}$ directly and then writing it in the form given by the authors is a direct and more convincing proof of the theorems.

At the end of Example 2, the authors state: “It can be shown, after solving 15 simultaneous equations with 15 unknowns, using Hashin’s method, we will arrive exactly at the same solution.” In this connection, the writer wishes to point out that Hashin (1) gave a solution for the stress function, which necessarily has more constants. The entire solution of the problem, i.e., obtaining $\sigma_{11}$, $\sigma_{22}$, and $\sigma_{12}$ is Hashin’s (1) aim. The authors have concentrated only on $\sigma_{11}$. If this is the aim, the solution can be sought for $\sigma_{11}$ in exactly the same way as Hashin obtained the solution for the stress function.

The writer has shown in the preceding paragraphs that the number of constants in the method given by the authors and the solution for $\sigma_{11}$ as an extension of Hashin’s work is the same. Each recursion relationship of coefficients involves five coefficients, whether by Hashin’s method or by the method presented by the authors.
\[ + 4 S_{212} \frac{\partial^4 \sigma_{11}}{\partial x_1^3 \partial x_2} - S_{222} \frac{\partial^4 \sigma_{11}}{\partial x_1^4} = 0 \]  

\[(66)\]

The solution for \( \sigma_{11} \) given by Eq. 65 can be uniquely determined from the recursion relationships, which can be developed from the differential equation, Eq. 66, and from the conditions of statics pertaining to the axial force, \( P \), and the bending moment, \( M \). In principle the problem of giving the solution for \( \sigma_{11} \) is solved. This is just an extension of the work of Hashin (1) with the concentration on the solution for \( \sigma_{11} \). The number of constants in the method given by the authors and the aforementioned extension of Hashin's work for obtaining \( \sigma_{11} \) is the same. In the authors' notation, the number of constants to be determined is \( 2(n + 1) + [n(n + 1)/2] = 1/2 (n + 1)(n + 4) \). The number of constants for the solution of \( \sigma_{11} \) in Hashin's method = \( 1/2(M + 1)(M + 4) \). Term \( M \) in Hashin's notation is equal to \( n \) in the authors' notation.

Then writing the solution for \( \sigma_{11} \) in the moment-differential and force-differential form as done by authors in Eqs. 10 and 17 is simply a matter of algebra and is basically a matter of personal preference. In Hashin's method, \( N \)—the highest power of \( x_2 \) in the solution for stress function—is equal to \( M + 3 \). In the solution for \( \sigma_{11} \), the highest power of \( x_2 \) will then be equal to \( M + 1 \). Term \( M \), the highest power of \( x_1 \) in the solution for stress function, is equal to the highest power of \( x_1 \) in the polynomial for \( \sigma_{11} \), and therefore in the polynomial for the bending moment or axial force. Therefore the solution given by Hashin (1) can be extended for the solution for \( \sigma_{11} \) by suitably developing the recursion relationship of coefficients \( D_{mn} \) and considering the statical conditions of the axial force and bending moment. The step domain of coefficients, \( D_{mn} \), will be smaller than that of \( C_{mn} \) by two diagonals. In the solution for the stress function the restriction is \( m + n \leq M + 3 \). In the solution for the stress, \( \sigma_{11} \), the restriction will be \( m + n \leq M + 1 \). With this restriction the solution for \( \sigma_{11} \) can be immediately written as a summation of products of the polynomials of \( x_2 \) and differentials of the bending moment or axial force as done by the authors. In the writer's opinion, getting the polynomial solution for \( \sigma_{11} \) directly and then writing it in the form given by the authors is a direct and more convincing proof of the theorems.

At the end of Example 2, the authors state: "It can be shown, after solving 15 simultaneous equations with 15 unknowns, using Hashin's method, we will arrive exactly at the same solution." In this connection, the writer wishes to point out that Hashin (1) gave a solution for the stress function, which necessarily has more constants. The entire solution of the problem, i.e., obtaining \( \sigma_{11} \), \( \sigma_{22} \), and \( \sigma_{12} \) is Hashin's (1) aim. The authors have concentrated only on \( \sigma_{11} \). If this is the aim, the solution can be sought for \( \sigma_{11} \) in exactly the same way as Hashin obtained the solution for the stress function.

The writer has shown in the preceding paragraphs that the number of constants in the method given by the authors and the solution for \( \sigma_{11} \) as an extension of Hashin's work is the same. Each recursion relationship of coefficients involves five coefficients, whether by Hashin's method or by the method presented by the authors.
ANALYSIS OF VIBRATION OF HOLLOW-CONE VALVES

Errata

The following corrections should be made to the original paper:

Page 1152, Eq. 10: Should read
\[
\exp\left(\frac{\pi b}{2R} x_0\right) - 1 \quad \text{instead of} \quad \exp\left(\frac{\pi b}{2R} x_0\right) - \frac{1}{2R}
\]
\[
\exp\left(\frac{\pi b}{2R} x_0\right) + 1
\]

Page 1158, Fig. 12: Should read “2.90 m” instead of “29.0 m” and “valve \( M = 1:62 \)” instead of “valve \( M = 1:25 \)”

COHERENCE OF GRID-GENERATED TURBULENCE

Discussion by M. H. Abdul Khader, K. Elango, and S. Sadasivan

The authors have presented an interesting model for coherence function in grid turbulence with the objective of using the same for simulating the natural wind in dynamic response problems. Further improvements in this approach should envisage the inclusion of factors not taken care of by the assumption of idealized isotropy of the turbulence field. One such important factor is the existence of considerably higher energy density in the low frequency ranges in the actual spectrum of the atmospheric turbulence.

The writers have measured the characteristics of grid turbulence obtained in a water stream with significant low frequency Free Stream Turbulence (FST). The scope of the experiments included measurements behind three different grids at two different values of FST. Ref. 19 furnishes the details of the experimental apparatus and procedure.

1Asst. Prof., Hydraulics Engrg. Lab., Indian Inst. of Tech., Madras, India.
2Lect., Hydraulics Engrg. Lab., Indian Inst. of Tech., Madras, India.
3Sr. Research Asst., Hydraulics Engrg. Lab., Indian Inst. of Tech., Madras, India.
Typical spectra obtained for a FST intensity of 15% and a mean velocity of 17.50 cm/s are shown in Fig. 19. In these plots only the spectral function of the FST is normalized and other spectra are scaled in terms of the intensity of 15%.

Although Eq. 2 is strictly valid only for the case of isotropic turbulence, as mentioned by the authors, Eq. 3 can be satisfactorily applied to compute the coherence function when the spectral density function decays more or less exponentially at large frequencies. Computation of coherence function numeri-
in the low and intermediate wave number ranges, bringing out the important role of the integral scale parameter in the analytical model for coherence. It is also seen that the computed coherence values are higher than one would predict from a Von Karman spectrum. This implies that more realistic models should be considered than the conventional isotropic idealization when simulating atmospheric turbulence. However, the writers found that the microscale does not prove a suitable parameter for unifying the coherence function.

Appendix.—Reference

The normal mode analysis is a powerful tool for approximating soil-structure interaction. The author has made a significant contribution to the use of modal analysis of the discrete parameter system with his method of computing the modal damping. The writers would like to comment on the author's approach and suggest an alternate approach that accomplishes essentially the same end.

The author computed the modal damping by matching the transfer functions defined in Eqs. 32 and 33 for the "rigorous" and normal mode solutions at a predetermined point of the structure. The matching is done for harmonic base motions with frequencies equal to all the natural frequencies of the system that are in the range deemed important. Since the impedances of the system are actually frequency dependent and since only harmonic inputs are considered, it would appear to be a simple matter to account for the frequency dependence of the dashpots in the matching process. The frequency dependence of the stiffnesses, however, would be more difficult to include in that the natural frequencies of the system are dependent upon these stiffnesses and thus an iterative procedure would be necessary. Furthermore, the modal method is not applicable when a slightly different structure is used for each mode. It is recognized that incorporating any frequency dependence in the interaction model would make a comparison of the exact and normal mode solutions for a particular base input, such as is given in Fig. 5(a), a difficult task. However, there is no apparent need to do this since the applicability and accuracy of the author's method was well demonstrated.

The author's matching was done at one predetermined location on the structure; in the example problem, the top mass was chosen because of its sensitivity to damping. The writers would like to suggest a somewhat more general approach of simultaneously matching or best-fitting, by some weighted average, at more than one location. Various weighting factors could be used, e.g., the height above the base and the story shear. Also, any measure of the response deemed to be important could be matched, e.g., displacement or velocity amplifications.

The author found that the methods of Biggs (13) and Johnson and McCafferty (6) significantly overestimate the modal damping in many cases. The writers come to the same conclusion using the following slightly different method to arrive at the modal damping. Since the incorporation of structural damping in the interaction model makes the analysis of this system cumbersome, the writers arrive at adjusted modal damping values by first looking at the case
of no structural damping. In this way, no assumption about the nature of the structural damping matrix need be made. Including only radiation damping in the analysis gives a diagonal damping matrix.

In Biggs' method, the modal damping for the kth mode is expressed as a weighted average of the various damping factors in the system. If this basic assumption is retained, then, for the case of no structural damping, an adjusted damping value can be defined as

$$\beta_k = \alpha \left[ \frac{\beta_x (E_x)_k + \beta_y (E_y)_k}{(E_x)_k + (E_y)_k + (E_z)_k} \right]$$

(38)

in which $\alpha$ is a factor to be evaluated. The bracketed term can be easily amended to include other modes of deformation where applicable.

The factor, $\alpha$, is determined by matching any desired response of the interaction model with the same quantity obtained by the modal method. More than one response can be matched by minimizing the differences between the two solutions. The base input for which this matching is accomplished can be taken to be either the actual input to which the structure is to be subjected or a contrived input containing all the frequencies of interest. Since no structural damping is included, finding the rigorous solution for even a complicated base input presents little computational difficulty.

In Eq. 38, $\alpha$ is considered to be mode independent which means that the damping factors for all modes are "adjusted" by the same percentage. Since only one input is used (which automatically includes all frequencies of interest) it is a simple matter to find the appropriate value for $\alpha$ that matches (or minimizes the differences of) the responses. The accuracy of this process can be obtained at this point by comparing the rigorous and modal solutions.

Once $\alpha$ is determined, the final values for $\tilde{\beta}_k$ (which include structural damping) can be obtained from

$$\tilde{\beta}_k = \frac{(\beta_x)_k (E_x)_k + \alpha [\beta_x (E_x)_k + \beta_y (E_y)_k]}{(E_x)_k + (E_y)_k + (E_z)_k}$$

(39)

Using Eq. 39, the assumption of Eq. 26 is unnecessary, also the effect of structural damping on the response of the system can easily be seen.

The writers have used this technique in several instances and found in all cases that Biggs' method overestimated the damping, i.e., $\alpha < 1$. For cases where interaction effects were significant, $\alpha$ was found to be approx 0.7.

Discussion by Michael J. O'Rourke,\textsuperscript{5} A. M. ASCE

The writer would like to congratulate the author on an interesting article presenting a new method for determining modal damping values for soil-structure interaction systems. The writer would like to mention four points about the paper:

1. In Fig. 8, it would have been useful to have plotted the envelopes of

\textsuperscript{5}Asst. Prof. of Civ. Engrg., Rensselaer Polytechnic Inst., Troy, N.Y.
floor acceleration, shear, and moment using modal damping values from Biggs' method and the Johnson-McCaffery method. What is the error introduced in these typical design parameters by using the preceding methods?

2. The writer would be interested in knowing the computer time required to calculate the exact response as compared to the time required to calculate the modal response, since the modal response requires iteration to determine the modal damping values before the numerical integration.

3. Defining a matrix \([\tilde{C}]\) as

\[
[\tilde{C}] = ([\Phi])^T [\tilde{C}] [\Phi] 
\]

in which \([\Phi]\) is defined by Eq. 23; and \([\tilde{C}]\) is defined by Equation 15. Are the modal damping values associated with the diagonal elements of the matrix \([\tilde{C}]\) close to those obtained by the author's, Biggs, or the Johnson-McCaffery methods?

4. Note that the author numerically integrates the modal equations of motion after using his new method to determine the modal damping values. Using this approach, the engineer would be required to choose a particular acceleration time history as the basis for the earthquake analysis. On the other hand, if a response spectrum-modal superposition approach were taken, the engineer could pick a design response spectrum (16) that gives a better representation of typical earthquakes than any particular acceleration time history. Another approach, of course, would be to combine the author’s method for determining the modal damping values with the response spectrum-modal superposition technique. If this approach were followed the accuracy for the envelope of floor accelerations, shears, and moments would decrease because of the errors inherent in modal superposition (17).

Appendix.—References


Discussions by Chang Chen

The writer wishes to extend his compliments to the author for the rigorous derivation of composite modal damping values by matching the transfer functions. Even though the author’s method was compared favorably with the exact solution, the appeal of the normal mode method's simplicity is lost. When the author’s method is followed, one has to solve Eq. 31 first, then the simultaneous nonlinear algebraic Eq. 35.

Engineers have the tendency to use the simplified method with results close to the rigorous solution. Thus, it is the intention of this discussion to investigate...
the possible simplified alternative of calculating the composite modal damping values.

The Biggs method as described in Eq. 25 was intended for hysteretic damping only. Roesset, et al. (18) extended this weighted damping method to hysteretic and viscous dampings. As indicated in Eq. 35 of Ref. 18, the viscous damping term should be modified by a factor that is the ratio of modal frequency over the reference frequency. The radiation dampings in Eq. 37 are of viscous type and thus should be modified by the factor. The reference frequencies are implicitly shown in Eq. 27, in which \( \omega_0 = \frac{1}{2\pi} \sqrt{K/M_0} \) for translation; and \( \omega_\theta = \frac{1}{2\pi} \sqrt{K_\theta/I_\theta} \) for rocking. Making use of the values in Tables 1 and 3 for the case of \( V_s = 1,000 \) fps, we have \( \omega_0 = 10 \) cps and \( \omega_\theta = 9.26 \) cps. Comparing these values with the modal frequencies in Fig. 3, we see that the radiation dampings in Eq. 37 should be divided by a factor of about three in the calculation of weighted damping value for the first mode. Thus, the comparison of the transfer functions in Fig. 4 and the modal damping values in Table 4 will not be as dramatic as they are shown. Consequently, the modified Biggs method or the weighted damping method will not underestimate the structural responses and has the advantages of being simple and straightforward.

Appendix.—Reference


NONSTATIONARY RESPONSE OF STRUCTURAL SYSTEMS\(^a\)

Discussion by Ross B. Corotis, \(^3\) M. ASCE

The authors have provided a valuable extension of nonstationary response analysis for multidegree systems subjected to segmented forcing functions. Fig. 2, which is also related to the authors' earlier work (8), is in agreement with a similar approach taken by the writer (12). That article indicated that time-varying fluctuations in the response spectral density were directly influenced by the frequency content of the forcing function substantially above the undamped resonant frequency of the oscillator.

In Fig. 5(a), the authors present \( |I_\omega(t, \omega)|^2 \) during the die-down phase (following a segmented forcing function). The writer wishes to point out that this is different from the evolutionary spectrum, which is perhaps a more common mixed time-frequency description of a nonstationary process. Since the forcing function

\(^a\)April, 1974, by Robert E. Holman and Gary C. Hart (Proc. Paper 10502).

\(^3\)Asst. Prof. of Civ. Engrg., the Technological Inst., Northwestern Univ., Evanston, Ill.
has been modulated by a rectangular time pulse, the system will behave, in the absence of a subsequent pulse, as an unforced oscillator for \( t > t_r \), with initial conditions determined by the forced response at \( t = t_r \). Physically then, the response will be a decaying sinusoid at the natural damped frequency of the oscillator. A time-varying “instantaneous” frequency domain representation of this motion would be a time-decreasing dirac delta function at \( \omega = \omega_j \sqrt{1 - \xi_j^2} \), with the integral under the spike shown in Fig. 5(b). What the authors present is a frequency decomposition of the entire (nonstationary) decaying process.

Appendix.—Reference


Discussion by Loren D. Lutes, 4 M. ASCE

The authors’ analysis of multidegrees of freedom systems subjected to nonstationary excitation is certainly a valuable extension of the current literature. However, the results presented for the response to time-modulated white noise seem questionable to the writer. One can give a definition of \( I_j(t, \omega) \) by an integral in the time domain which is more useful in this regard. By substitution of Eq. 18 into Eq. 21 one obtains

\[
I_j(t, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H_j(\omega_t) e_j(\tau) \exp \left[ i(\omega - \omega_t) \tau + i \omega_t \tau \right] d\tau d\omega
\]  

(31)

but the “impulse response function” for mode \( j \) of the system is given by

\[
a_j^{-1} D_j(t - \tau) u(t - \tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_j(\omega_t) \exp \left[ i \omega_t (t - \tau) \right] d\omega
\]  

(32)

so that

\[
I_j(t, \omega) = \frac{1}{a_j} \int_{-\infty}^{t} D_j(t - \tau) e_j(\tau) \exp (i \omega \tau) d\tau
\]  

(33)

Noting then, that

\[
\int_{-\infty}^{\infty} \exp \left[ i \omega (\tau_1 - \tau_2) \right] d\omega = 2\pi \delta(\tau_1 - \tau_2)
\]  

(34)

in which \( \delta(\cdot) \) is the Dirac delta function gives

\[
\int_{-\infty}^{\infty} I_j(t, \omega) I_{k^*}(t, \omega) d\omega
\]

\[
= \frac{2\pi}{a_j a_k} \int_{-\infty}^{t} e_j(\tau) e_{k^*}(\tau) D_j(t - \tau) D_k(t - \tau) d\tau
\]  

(35)

For the time-modulated white noise

4Assoc. Prof. of Civ. Engrg., Rice Univ., Houston, Tex.
\[ e_p(\tau) = \mathcal{G}_p \left[ u(t - t_{r_1}) - u(t - t_r) \right] \]  
(36)

so that \[ e_p(\tau) e_{ks}(\tau) = 0 \quad \text{for} \quad r \neq s \]  
(37)

Thus it appears that the quantity shown in Fig. 13 should be identically zero, for \( j \neq k \) as well as for \( j = k \), as found by the authors.

One may also note that the ordinates in Figs. 11-13 all have units of frequency, rather than being dimensionless as in the other figures. Perhaps this is simply the result of a printing error.