PRACTICE EXAM PROBLEMS

① In deep space, a 500 kg asteroid heading at a velocity $\langle 100, -50, 0\rangle$ m/s collides and <u>sticks to</u> a 1000 kg asteroid with a velocity of $\langle +50, +10, 0\rangle$ m/s. What is the final velocity of asteroid?

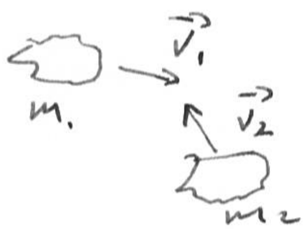
Initially

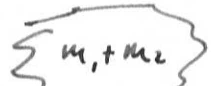
$m_1 = 500$ kg $\quad \vec{v}_1 = \langle 100, -50, 0\rangle$

$m_2 = 1{,}000$ kg $\quad \vec{v}_2 = \langle +50, 10, 0\rangle$

$m_T = 1{,}500$ kg $\quad v_f = ?$

$$\vec{P}_i = \vec{P}_f$$





$\vec{v}_f = ?$

$m_T = m_1 + m_2$

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_T \vec{v}_f$$

$$\vec{v}_f = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_T}$$

$$\vec{v}_f = \frac{500\langle 100, -50, 0\rangle + 1000\langle 50, 10, 0\rangle}{1500}$$

$$= \frac{1}{3}\langle 100, -50, 0\rangle + \frac{2}{3}\langle 50, 10, 0\rangle = \langle 66.66, -10, 0\rangle \frac{m}{s}$$

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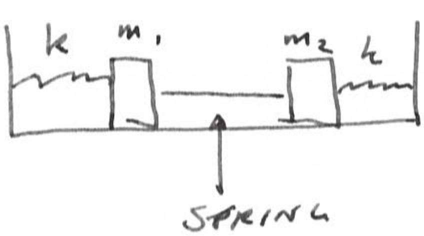
② Consider the following system:



$m_1 = \cancel{5}\ 2$ kg

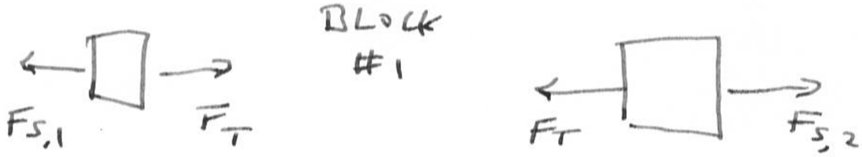
$m_2 = 10$ kg

WHEN THE STRING IS CUT, ~~WHAT~~ Block 2 ~~accelerat~~ accelerates to the right by "a."

WHAT IS THE ACCELERATION OF Block 1 in terms of "a"?

THIS IS A STATIC PROBLEM

NET FORCES = 0



$F_{S,1} = F_{S,2}$

$m_1 a_1 = m_2 a_2$

$a_1 = \frac{m_2}{m_1} a_2$

$= \left(\frac{5}{1}\right) a$

$= \underline{\underline{5a}}$

③ PARTICLE DECAYS

$A \rightarrow B^+ + B^-$ $\quad m_A = 800\ MeV/c^2$

$m_B = 200\ MeV/c^2$

WHAT IS VELOCITY OF B^+?

NOTATION: UNITS ⇒ AT REST

$1\ m_p \Rightarrow E_p = 938\ MeV$

$m_p = \frac{E_p}{c^2} = 938 MeV/c^2$

ENERGY

$E_{BEFORE} = m_A c^2 = 800\ MeV$

$E_{AFTER} = 2\gamma m_B c^2$

Why two (2)? → TWO PARTICLES MUST HAVE SAME VELOCITY

$E_{AFTER} = 2\gamma\,[200\,MeV] = 400\gamma\ MeV$

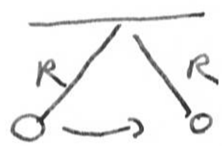
$\gamma = 2 \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = 2$

$\frac{1}{4} = 1 - \frac{v^2}{c^2}$

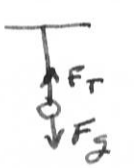
$\frac{v^2}{c^2} = \frac{3}{4} \qquad v = \frac{\sqrt{3}}{2} c$

$v = 0.866\ c$

④ PENDULUM



QUESTIONS: WHAT IS FORCE ON THE STRING WHEN THE PENDULUM IS HANGING STRAIGHT DOWN?



$F_N = \frac{mv^2}{R} = F_T - mg$

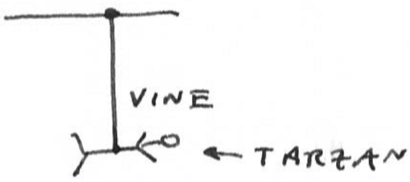
Suppose $\frac{mv^2}{R} = mg$

WHAT IS F_T?

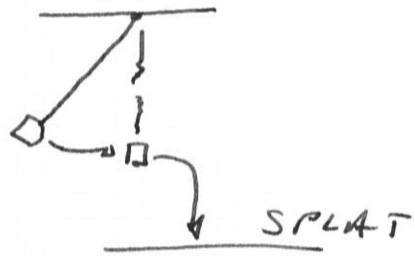
$\boxed{F_T = 2mg}$

REVIEW PROBLEMS:

TARZAN!



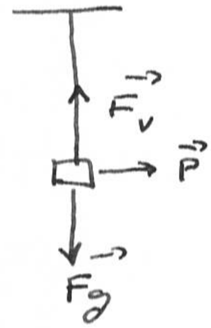
OK. VINE



SYSTEM : TARZAN

FORCES: VINE, EARTH

VINE → CONTACT, EARTH → DISTANCE



Net force

$F_N = F_v - F_g$ [ALONG y-AXIS]

$= F_\perp = \frac{mv^2}{L}$ } Not relativistic

Suppose $m = 90$ kg
$L = 8$ m
$v = 12$ m/s

$F_v = \underbrace{F_g}_{\text{Gravity}} + \underbrace{\frac{mv^2}{L}}_{\text{"DEFLECT" TARZAN'S MOMENTUM}}$

$F_v = (90)(9.8) + \frac{(90)(12)^2}{(8)^2} = 2{,}500$ N

Just F_g

$F_g = 90(9.8) = 882$ N

$F_v = 882$ N

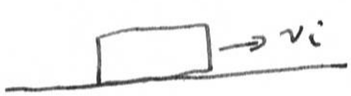
~~$F_v >> F_g$ So VINE~~

Hangs on vine : $F_v = 882$ N

SWINGS ON VINE $F_v = 2{,}500$ N

THIS IS WHY VINE SNAPS!

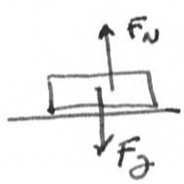
FRICTION:



$\mu_k = 0.4$ $m = 3$ kg

$v_i = 6$ m/s

HOW FAR WILL BLOCK SLIDE?



$f^k = \mu_k F_g = 0.4\, mg = (0.4)(3)(9.8)$ N
$= 11.8$ N

$f^k = \mu_k F_N$

WE HAVE A CONSTANT FORCE.

$$X_f = X_i + v_i \Delta t + \frac{1}{2}\frac{f^k}{m}\Delta t^2$$

P2

$X_f = 6\,\Delta t - \frac{11.8N}{2(3)}\Delta t^2 = 6\,\Delta t - 1.97\,\Delta t^2$

$X_f = 6\,\Delta t - 1.97\Delta t^2$ NEED Δt! $\quad v_f = v_i + \frac{f^k}{m}\Delta t$

$0 = 6 - 3.94\,\Delta t$

$\boxed{\Delta t = 1.52\,s}$

$X_f = [6(1.52) - 1.92(1.52)^2]\,m$

$= \underline{4.6\,m}$

PROJECTILE:

Suppose

$v_i = 6\ m/s$

$\theta = 45°$

$X_f = X_i + v_{i,x}\,\Delta t + \frac{1}{2}\frac{F_N}{m}\Delta t$

$F_N = 0 \quad v_{i,x} = v_i \cos\theta$

$\boxed{X_f = v_i \cos\theta\ \Delta t}$

$X_f = \frac{6}{\sqrt{2}}\Delta t = (4.24)\Delta t\ (m)$

$y_f = y_i + v_{i,y}\,\Delta t + \frac{1}{2}\frac{F_N}{m}\Delta t$

$\hookrightarrow y_f = y_i + v_i \sin\theta\ \Delta t + \frac{1}{2}\frac{F_N}{m}\Delta t^2$

$F_N = -mg$

$y_f = 4.24\,\Delta t - 4.9\,\Delta t^2 \Rightarrow v_f = \frac{dy_f}{dt} = 4.24 - 9.8\,\Delta t$

Two ways: $y_f = 0$

$4.24 = 4.9\,\Delta t \quad \Delta t = 0.865\,s$

$X_f = 3.67\,m$

$\Delta t = 0.432$ sec

$\leftarrow v_f = 0$ at $\Delta t = 0.432$

TAKES ANOTHER Δt TO HIT GROUND

$\underline{\Delta t = 0.864\,s}$

SAME AS BEFORE.

GENERAL RESULT

$y_f = v_i \sin\theta\,\Delta t - \frac{1}{2} g t^2$

$X_f = v_i \cos\theta\ \Delta t$

$y_f = 0 = v_i \sin\theta\ \Delta t - \frac{1}{2} g\,\Delta t^2$

$\boxed{\frac{2 v_i \sin\theta}{g} = \Delta t}$

$\boxed{X_f = \frac{2 v_i^2 \cos\theta \sin\theta}{g}}$

CHECK $\underbrace{\cos\theta = \sin\theta}_{\theta = 45} = \frac{1}{\sqrt{2}}$

$= \frac{v_i^2}{g} = \frac{6^2}{9.8} = 3.67\,m$

$$X_f = \frac{2v_i^2 \cos\theta \sin\theta}{g}$$

MAX occurs WHEN?

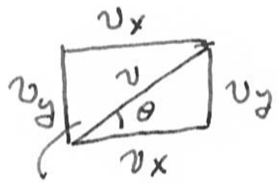
P.3

$f(\theta) = \cos\theta \sin\theta$

$$\frac{df}{dt} = -\sin^2\theta + \cos^2\theta = 0$$

$\sin\theta = \cos\theta$

or $\tan\theta = 1$ $\quad \underline{\underline{\theta = 45^\circ}}$



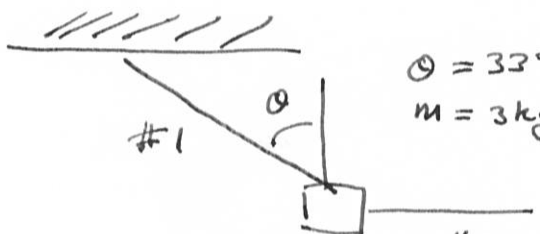
$90-\theta$ $\quad \cos(90-\theta) = \frac{v_y}{v}$

$\sin\theta = \frac{v_y}{v}$

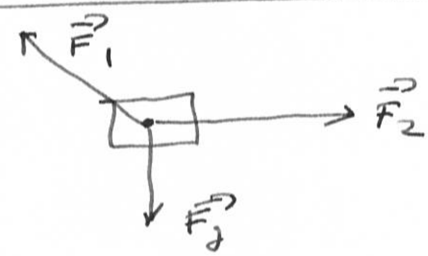
$$X_f = \frac{2v_i^2 \cos(90-\theta)\cos(\theta)}{g}$$

So $\theta = 60$ $\qquad \theta = 30$

$\cos(30)\cos(60)$ $\qquad \left.\cos(60)\cos(30)\right\}$ SAME!



$\theta = 33^\circ$
$m = 3kg$



IF BLOCK IS STATIC

$\vec{F}_1 + \vec{F}_2 + \vec{F}_g = 0$

OR

$F_{1,x} + F_{2,x} + F_{g,x} = 0$

$F_{1,y} + F_{2,y} + F_{g,y} = 0$

$F_{1,x} = -F_{2,x}$

$F_{1,y} = +mg$

$F_{g,x} = 0$

$F_{g,y} = -mg$

$F_{2,y} = 0$

$F_{1,x} = F_1 \cos(90+\theta)$

$F_{1,y} = F_1 \cos(\theta)$

$\cos(90+\theta) = \cos 90 \cos\theta - \sin 90 \sin\theta$

$= -\sin\theta$

$$F_1 = \frac{mg}{\cos\theta} = \frac{3(9.8)}{\cos(33)}$$

$\boxed{F_1 = 35.1\ N}$

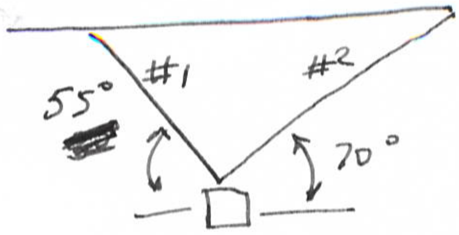
$-F_1 \sin\theta = \cancel{mg} - F_{2,x}$

$F_1 \cos\theta = mg$

$$-\tan\theta = \frac{-F_2}{mg}$$

$\boxed{F_2 = mg\tan\theta} = (3)(9.8)\tan(33)$

$\boxed{F_2 = 19.1\ N}$



$m = 350$ kg

#1, #2 WIRES

Radius = 1.2 mm

$Y = 6.9 \times 10^{10}\ N/m^2$

Find F_1, F_2 and STRAIN in each wire.

$\vec{F}_1 = F_1 \langle \cos(180-55), \cos(90-55), 0 \rangle$

$\vec{F}_2 = F_2 \langle \cos(70), \cos(90-70), 0 \rangle$

$\vec{F}_g = -mg \langle 0, 1, 0 \rangle$ Again $\Rightarrow$ $\vec{F}_1 + \vec{F}_2 + \vec{F}_g = 0$

X - COMPONENTS

$F_1 \cos(125) + F_2 \cos(70) = 0$ $\quad F_1 = 0.6 F_2$

y - COMPONENTS

$F_1 \cos(35) + F_2 \cos(20) - mg = 0$

$-\cancel{0.574 F_1 + 0.3202348 F_2 = 0}$

$0.812 F_1 + 0.940 F_2 = 3430$ $\quad 0.491 F_2 + 0.94 F_2 = 3430$

$\boxed{\begin{array}{l} F_2 = 2400\ N \\ F_1 = 1438\ N \end{array}}$

STRESS —

$\frac{F}{A} \Rightarrow F_1/A = 5.3 \times 10^8\ N/m^2$

$F_2/A = 3.2 \times 10^8\ N/m^2$

$A = \pi r^2$

$= (3.14)(1.2 \times 10^{-3})^2\ m^2$

$= 4.5 \times 10^{-6}\ m^2$

$\frac{\Delta L}{L} = Y^{-1} \frac{F}{A}$

$\left(\frac{\Delta L}{L}\right)_2 = \frac{1}{Y} \frac{F_1}{A} = 0.8 \times 10^{-2} \Rightarrow 0.8\%$

$\left(\frac{\Delta L}{L}\right)_1 = \frac{1}{Y} \frac{F_2}{A} = 0.5 \times 10^{-2} \Rightarrow 0.5\%$