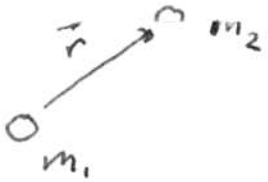
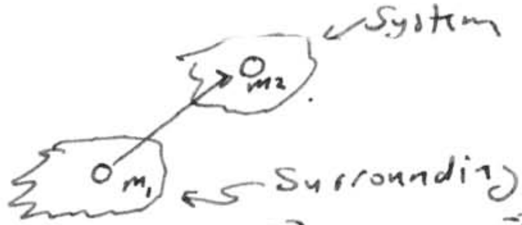
CONSERVATION OF MOMENTUM

$$\Delta \vec{P}_{sys} + \Delta \vec{P}_{surr} = 0$$





$$\Delta \vec{P}_{system} = \Delta \vec{P}_2$$

$$\Delta \vec{P}_{surr} = \Delta \vec{P}_1$$

$$\Delta \vec{P}_1 + \Delta \vec{P}_2 = (\vec{F}_{12} + \vec{F}_{21})\Delta t$$

RECALL $\vec{F}_{12} = -\vec{F}_{21}$
RECIPROCITY

$$\Delta \vec{P}_1 + \Delta \vec{P}_2 = (-\vec{F}_{21} + \vec{F}_{21})\Delta t = 0$$

Rocket
$\vec{P}_i = 0$

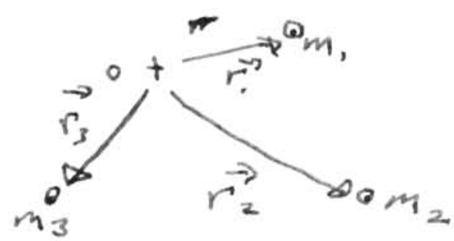
$\vec{P}_f = 0$

$$\vec{P}_{ROCKET} + \vec{P}_{exhaust} = \vec{P}_f$$

$$\vec{P}_{Rocket} = -\vec{P}_{exhaust}$$

CENTER OF MASS

$$\vec{r}_{cm} \equiv \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3}$$



$$m_1 + m_2 + m_3 = M_{TOTAL}$$

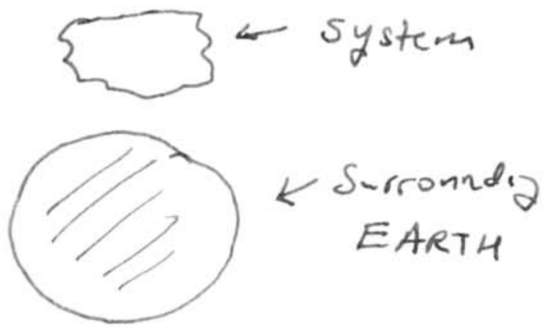
$$M_{TOTAL}\vec{r}_{cm} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3$$

$$M_{TOT}\underbrace{\frac{d\vec{r}_{cm}}{dt}}_{=\vec{v}_{cm}} = m_1\underbrace{\frac{d\vec{r}_1}{dt}}_{=\vec{v}_1} + m_2\underbrace{\frac{d\vec{r}_2}{dt}}_{=\vec{v}_2} + m_3\underbrace{\frac{d\vec{r}_3}{dt}}_{=\vec{v}_3}$$

$$\vec{P}_T = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 = M_T\vec{v}_{cm}$$

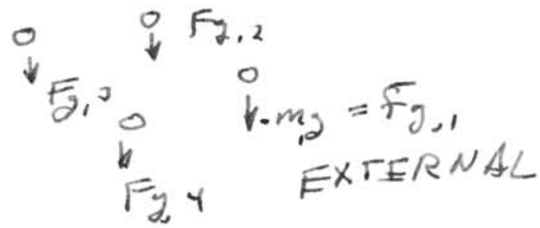
$$\boxed{\vec{P}_{Total} = M_{Total}\vec{v}_{cm}}$$

Multiparticle Momentum Principle



⇒

$\vec{f}_{i+1,i} = -\vec{f}_{i,i+1}$



$\vec{p}_{sys} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3$

$\vec{F}_N = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$ } External

$$\Delta \vec{p}_{sys} = \vec{F}_{Net} \Delta t$$

$$= [-m_1 g - m_2 g - m_3 g + \dots] \langle 0, 1, 0 \rangle \Delta t$$

$$= M_T g \langle 0, -1, 0 \rangle \Delta t$$

What about internal forces →

$\vec{f}_{12} + \vec{f}_{21} + \vec{f}_{13} + \vec{f}_{31} + \vec{f}_{23} + \vec{f}_{32} + \dots = 0$

$[\vec{f}_{12} + \vec{f}_{21}] + [\vec{f}_{13} + \vec{f}_{31}] + \dots = 0$

$\vec{f}_{21} = -\vec{f}_{12}$, $\quad \vec{f}_{31} = -\vec{f}_{13}$

$$\boxed{\Delta \vec{p}_{sys} = \vec{F}_{N,surr} \Delta t}$$

Calculus:

$$\Delta \vec{p}_{sys} = \vec{F}_{NET} \Delta t$$

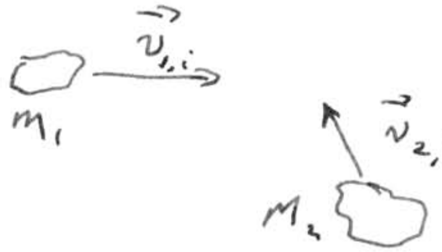
$$\Delta m_T \vec{v}_{cm} = m_T \Delta \vec{v}_{cm} = \vec{F}_{Net} \Delta t$$

$$\boxed{\vec{F}_{NET} = m_T \frac{\Delta \vec{v}_{cm}}{\Delta t}}$$

$$\boxed{\vec{a}_{cm} = \frac{d\vec{v}_{cm}}{dt}}$$

Limit $\Delta t \to 0$: $\quad \vec{F}_{NET} = m_T \dfrac{d\vec{v}_{cm}}{dt} = m_T \vec{a}_{cm}$

Collisions: NEGLIGIBLE
EXTERNAL FORCES



Lumps of clay – hit and stick. No external force

$\Delta \vec{p} = 0$

$\vec{P}_i = m_1 \vec{v}_{1,i} + m_2 \vec{v}_{2,i}$

$\vec{P}_f = (m_1 + m_2)\, \vec{v}_f$

$$\boxed{\vec{v}_f = \frac{m_1 \vec{v}_{1,i} + m_2 \vec{v}_{2,i}}{m_1 + m_2}}$$

EXAMPLE:

$m_1 = 0.2$ kg
$m_2 = 0.5$ kg
$\vec{v}_{1,i} = \langle 6, 0, 0 \rangle$ m/s
$\vec{v}_{2,i} = \langle -5, 4, 0 \rangle$ m/s

$$\vec{v}_f = \frac{0.2 \langle 6,0,0 \rangle + 0.5 \langle -5,4,0 \rangle}{0.2 + 0.5}$$

$= \langle -1.86, 2.86, 0 \rangle$ m/s

CONTACT INTERACTIONS
↓
OBJECTS – "Hitting each other" CHAPT. 4

ATOM THEORY

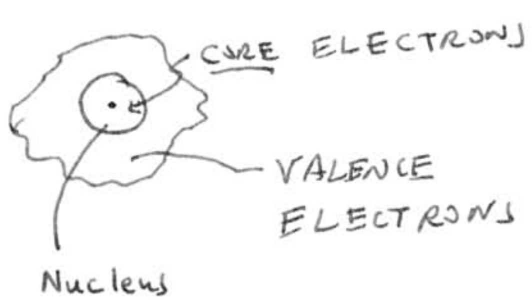
Matter – CONSISTS OF ATOMS. IN GENERAL, ATOMS ATTRACT EACH OTHER.

EXAMPLES OF INTERACTING ATOMS ⇒

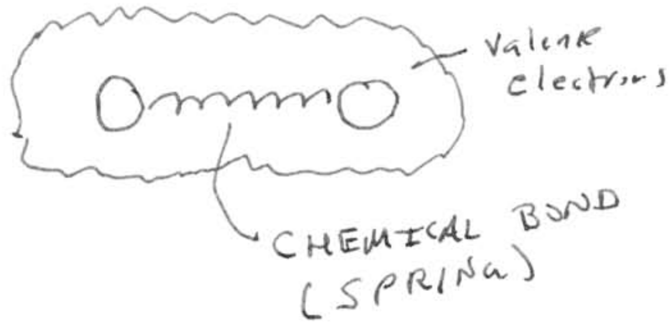
$\underbrace{\text{SOLIDS, LIQUIDS}}_{\text{CONDENSED MATTER}}$

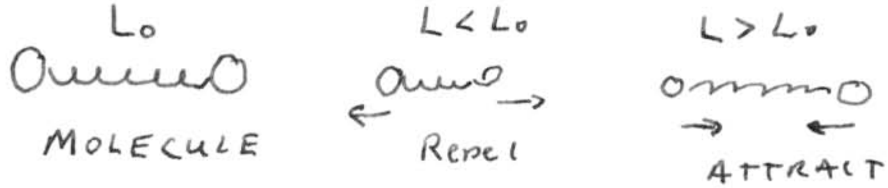
GASES ⇒ ATOMS STILL INTERACT, BUT WEAKLY

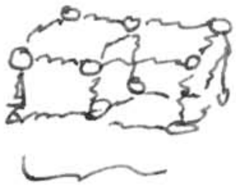
SIMPLE MODEL FOR INTERACTIONS ⟷ ATOMS



Ball and spring model



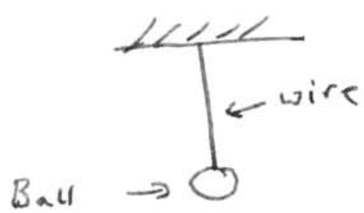




ATOMS – VIBRATE BACK and FORTH
Add HEAT – atoms vibrate even more! IF they vibrate too much, SOLID WILL MELT!

SOLID MATTER (TAKES UP SPACE, SUPPORTS SHEAR)

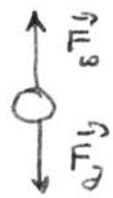
UNDER TENSION



SYSTEM: BALL

SURROUNDINGS: WIRE & EARTH

FORCE DIAGRAM



IF THE BALL DOES NOT MOVE →

$\vec{p}_i = 0 \quad \vec{p}_f = 0$

$\vec{F}_w + \vec{F}_g = 0$

$\vec{F}_w = -\vec{F}_g$

$F_g = -m_b g$

$F_w = +m_b g$

$\Delta\vec{p} = \underbrace{(\vec{F}_g + \vec{F}_w)}_{\vec{F}_N} \Delta t$

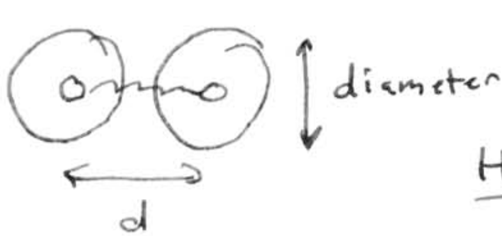
Suppose BALL

$m_b = 2\,kg \qquad F_w = (2)(9.8) = \underline{19.6\,N}$

WIRE – atoms acting like springs ⇒ "bond length" increases as WIRE STRETCHES.

How long is the chemical bond "Spring"?

SPRING LENGTH ≈ SIZE OF ATOM



How BIG ARE ATOMS?

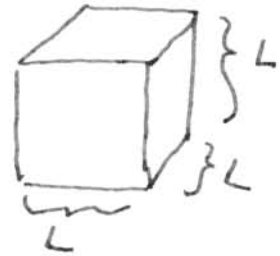
RECALL 1 mole = 6.02×10^{23} atoms ⇒

Atomic weight ⇒ 64 grams/mole } Copper [Cu] metal

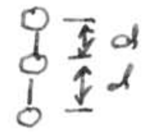
DENSITY $\Rightarrow \frac{\text{MASS}}{\text{Volume}} = \rho$

$\rho = 8.94\ g/cm^3$

CONSIDER CUBIC METER OF Copper



$\Rightarrow$ Blow up



$L = N_a d$

MASS $= N_a^3 m_a = M$

Volume $= N_a^3 d^3$

$$\frac{[N_a^3 m_a]}{[N_a^3 d^3]} = \frac{m_a}{d^3} = \rho$$

Mass of Copper atom

$$m_a = \frac{64\ g}{\text{mole}} \cdot \frac{1\ \text{mole}}{6.02 \times 10^{23}\ \text{atom}}$$

$$= 1.07 \times 10^{-22}\ g/\text{atom}$$

$$d^3 = \frac{m_a}{\rho}$$

$$= \frac{1.02 \times 10^{-23}\ g/\text{atom}}{8.94\ g/cm^3} = 1.19 \times 10^{-23}\ cm^3/\text{atom}$$

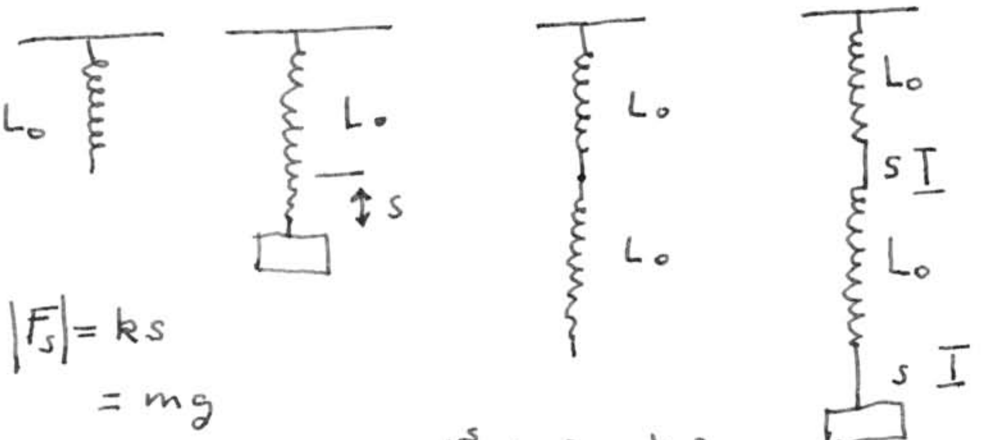
STIFFNESS OF COUPLED AND LINKED SPRINGS

"SIZE" OF Copper ATOM

$d = 2.2 \times 10^{-08}$ cm/atom

$d = 2.2 \times 10^{-10}$ m/atom

$d = 0.22$ nm $\quad d = 2.2$ Å



~~THE~~ THINK ABOUT THIS!

$|F_s| = k s$

$= mg$

$|F_s| = k_2^s (2s)$

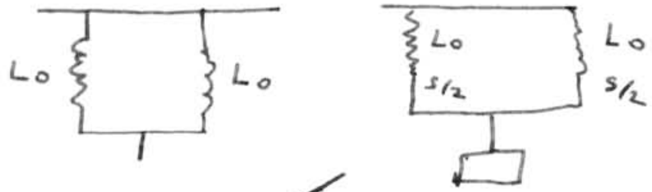
$= mg$

[SERIES OF SPRINGS]

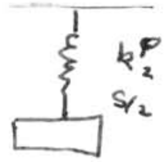
$k_2^s (2s) = k s$

$\boxed{k_2^s = \frac{1}{2} k}$

A LONGER SPRING OF TWO IDENTICAL SPRINGS ~~IS~~ IS ONLY HALF AS STIFF AS EACH OF THE SHORTER ~~SPRINGS~~ SPRINGS!



$$\underbrace{k(s/2)}_{\frac{mg}{2}} + \underbrace{k(s/2)}_{\frac{mg}{2}} = mg$$



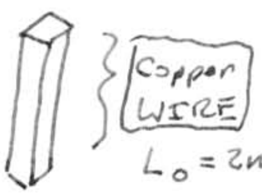
$k^P_2 = 2k$ $\qquad k^P_2 \underbrace{(L-L_0)}_{(L-L_0)} = \underbrace{2k(s/2)}_{ks = mg} = mg$

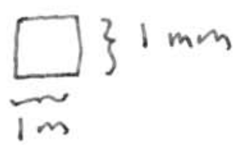
k^P_2 PARALLEL COMBINATION STIFFENS SPRING CONSTANT

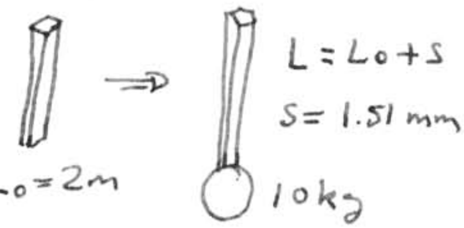
SUMMARY $\Rightarrow$ $k^P_n = nk \quad k^S_n = k/n$ } Rule in general

HOW STIFF IS THE CHEMICAL BOND?

CROSS SECTION OF WIRE:





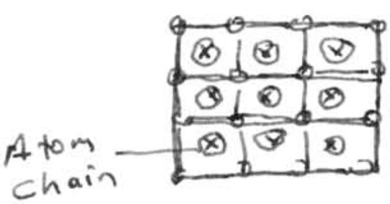


FOR THE WIRE

$F_g = mg = F_s = k_{s,w}\, s \qquad k_{s,w} = \frac{mg}{s} = \frac{(10)(9.8)\ kg\cdot m^2/s}{0.00151 m}$

$$\boxed{k_{s,w} = 6.5 \times 10^4\ N/m}$$

Number of chains



$$N_{chains} = \frac{\text{CROSS-SECTION AREA}}{\text{AREA PER ATOM}}$$

Copper WIRE : $d_{cu,atom} = 2.2 \times 10^{-10}$ m

$$N_{CHAINS} = \frac{(1 \times 10^{-3})^2\ m^2}{(2.2 \times 10^{-10})^2\ m^2} \Leftarrow \frac{\text{Area}}{\text{Area per Atom}}$$

$\approx 2 \times 10^{13}$

WHAT ABOUT LENGTH OF CHAIN? How many atoms?

$L_o = 2m \qquad L_o = N_a d$

$$N_a = \frac{L_o}{d} = \frac{2\ m}{2 \times 10^{-10} m} = 10^{10} \text{ atoms}$$

$$k_{S,WIRE} \cong \frac{k_{S,atom} \times N_{CHAINS}}{N_{ATOMS} \text{ in CHAIN}}$$

$$k_{S,atom} \cong \frac{k_{S,wire} \times N_{atoms} \text{ in Chain}}{N_{CHAINS}}$$

$$\cong \frac{(6.5 \times 10^{4})(10^{10})}{2 \times 10^{13}} \text{ N/m}$$

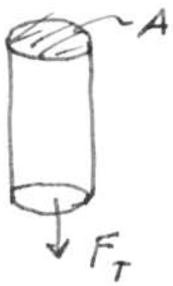
$$\boxed{k_{S,atom} \approx 3 \times 10^{1} \text{ N/m}}$$

QUESTION!

WHY DOES BOOK GIVE $\underline{\underline{3}}$ SIGNIFICANT FIGURES IN ITS DERIVATION?

STRESS, STRAIN & YOUNG'S MODULUS

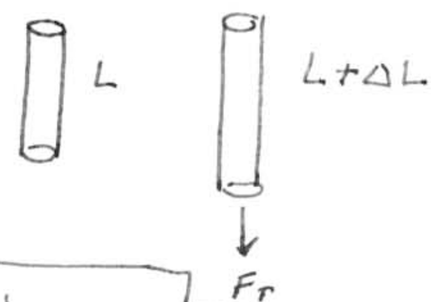
STRESS $\equiv F_T/A$



F_T = FORCE IN TENSION

A = AREA

STRAIN $\equiv \frac{\Delta L}{L}$



YOUNG'S MODULUS

~~STRAIN~~ STRESS $\frac{F_T}{A} \propto$ STRAIN $\frac{\Delta L}{L}$ $\qquad \frac{F_T}{A} = Y \frac{\Delta L}{L}$

Y = YOUNG'S MODULUS