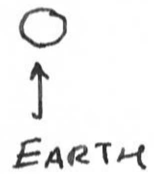
PREDICTING MOTION $\Rightarrow$ GRAVITY

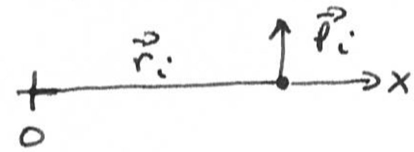
$M_S = 2 \times 10^{30}$ kg

$M_E = 6 \times 10^{24}$ kg



GIVEN:

$$\boxed{\begin{aligned} \vec{r}_i &= \langle 1.5 \times 10^{11}, 0, 0 \rangle \text{ m} \\ \vec{p}_i &= \langle 0, 1.8 \times 10^{29}, 0 \rangle \text{ kg·m/s} \end{aligned}}$$



Finding $\vec{p}_i$:

$r_{SE} = r_i = 1.5 \times 10^{11}$ m

$\cancel{2\pi R}$ $\dfrac{2\pi r_{SE}}{T} = v_E$

One year

Distance $2\pi r_{SE}$

$\cancel{T = \frac{1 \text{ yr}}{365.25 \text{ day}} \times \frac{1 \text{ day}}{24 \text{ hrs}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}}}$

$$T = 1\text{yr} \times \frac{365.25 \text{ days}}{1 \text{ yr}} \times \frac{24 \text{ hrs}}{1 \text{ day}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{60 \text{ sec}}{1 \text{ min}}$$

$$= 3.13 \times 10^7 \text{ sec}$$

$$v_E = \underset{\pi}{2\,(3.14)}\,\underbrace{(1.5 \times 10^{11})\text{ m}}_{r_{SE}} / \underbrace{3.13 \times 10^7 \text{ s}}_{T} = 3 \times 10^4 \text{ m/s}$$

$$p_i = m_E v_E = (6 \times 10^{24}) \text{ kg} \cdot 3 \times 10^4 \text{ m/s} = 1.8 \times 10^{29} \text{ kg m/sec.}$$

USE MOMENTUM RULE $\quad \Delta \vec{p} = \vec{F}_N \Delta t$

Let's take $\Delta t = 1$ month $= \frac{3.13}{12} \times 10^7 \text{s} = 2.6 \times 10^6$ s

$$\vec{F} = -G \frac{M_S M_E}{(r_{SE})^2} \langle 1, 0, 0 \rangle \text{ N}$$

$$= -6.7 \times 10^{-11} \times \frac{(2 \times 10^{30})(6 \times 10^{24})}{[1.5 \times 10^{11}]^2} \langle 1, 0, 0 \rangle$$

$\boxed{\vec{F} = \langle -3.6 \times 10^{22}, 0, 0 \rangle \text{ N}}$ INITIAL FORCE

STEP ONE $\Delta \vec{P} = F_N \Delta t$

$$\vec{P}_f = \vec{P}_i + \vec{F}_N \Delta t$$

$$= \langle 0, 1.8 \times 10^{29}, 0 \rangle + \langle -3.6 \times 10^{22}, 0, 0 \rangle (2.6 \times 10^6)$$

$$= \langle -9.3 \times 10^{28}, \underbrace{1.8 \times 10^{29}}_{\text{No change as No } F_y \text{ component.}}, 0 \rangle \text{ kg·m/s}$$

WHERE IS EARTH
AFTER Δt? $\Delta \vec{r} = \vec{v}_{AVE} \Delta t$

$$\vec{r}_f = \vec{r}_i + \vec{v}_{AVE} \Delta t$$

SAME "TRICK" $\rightarrow$ $\vec{v}_{AVE} \cong \vec{P}_f / m_E$

Note: Since $\vec{F}$ changes over Δt, use $\vec{v}_{AVE} \cong \vec{v}_F$

$$\vec{v}_{AVE} \cong \frac{\langle -9.3 \times 10^{28}, 1.8 \times 10^{29}, 0 \rangle}{6 \times 10^{24}} \text{ m/s} = \langle -1.5 \times 10^4, 3 \times 10^4, 0 \rangle \ \frac{m}{s}$$

$$\vec{r}_f = \underbrace{\langle 1.5 \times 10^{11}, 0, 0 \rangle}_{\vec{r}_i} + \underbrace{\langle -1.5 \times 10^4, 3 \times 10^4, 0 \rangle}_{\vec{v}_F} \underbrace{(2.6 \times 10^6)}_{\Delta t}$$

$$\boxed{\vec{r}_f = \langle 1.1 \times 10^{11}, 7.8 \times 10^{10}, 0 \rangle \text{ m}}$$ AFTER ONE MONTH

STEP TWO: Now

$$\boxed{\begin{aligned} \vec{r}_i &= \langle 1.1 \times 10^{11}, 7.8 \times 10^{10}, 0 \rangle \text{ m} \\ \vec{P}_i &= \langle -9.3 \times 10^{28}, 1.8 \times 10^{29}, 0 \rangle \text{ kg·m/s} \end{aligned}}$$

NEED

$|\vec{r}_i|$ to find New $\vec{F}_N = -G \dfrac{M_S M_E}{(r_i)^2} \hat{r}_i$

$$r_i = \sqrt{(1.1)^2 + \cancel{(7.8)^2}\,(0.78)^2} \times 10^{11} \text{ m} = 1.34 \times 10^{11} \text{ m}$$

$$\hat{r}_i = \langle 0.82, 0.582, 0 \rangle$$

$$\boxed{\vec{F}_N = \langle -3.6 \times 10^{22}, -2.6 \times 10^{22}, 0 \rangle \text{ N}}$$

$$\vec{P}_f = \vec{P}_i + \vec{F}\Delta t$$

$$= \langle -9.2\times10^{28},\ 1.8\times10^{29},\ 0\rangle$$

$$+ \langle -3.6\times10^{22},\ -2.6\times10^{22},\ 0\rangle\,(2.6\times10^{6})$$

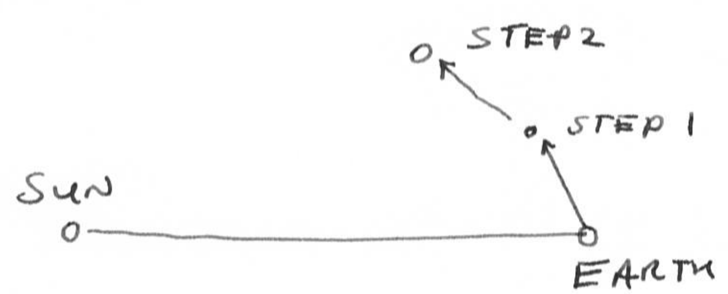
$$\boxed{\vec{P}_f = \langle -1.9\times10^{29},\ 1.1\times10^{29},\ 0\rangle\ \text{kg}\cdot\text{m/s}}$$

$$\vec{v}_f = \vec{P}_f / m_E = \langle -3.2\times10^{4},\ 1.8\times10^{4},\ 0\rangle\ \text{m/s}$$

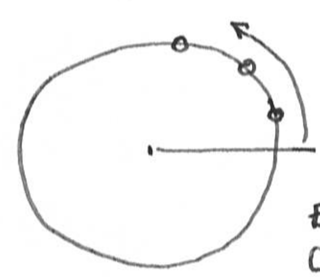
$$\vec{r}_f = \vec{r}_i + \vec{v}_{AVE}\Delta t \cong \vec{r}_i + \vec{v}_{\cancel{AVE}\,F}\,\Delta t$$

$$\vec{r}_f = \underbrace{\langle 1.1\times10^{11},\ 7.8\times10^{10},\ 0\rangle}_{\vec{r}_i} + \underbrace{\langle -3.2\times10^{4},\ 1.8\times10^{4},\ 0\rangle}_{\vec{v}_f}\ \underbrace{(2.6\times10^{6})}_{\Delta t}$$

$$\boxed{\vec{r}_f = \langle 2.9\times10^{10},\ 1.3\times10^{11},\ 0\rangle\ \text{m}}$$ AFTER TWO MONTHS



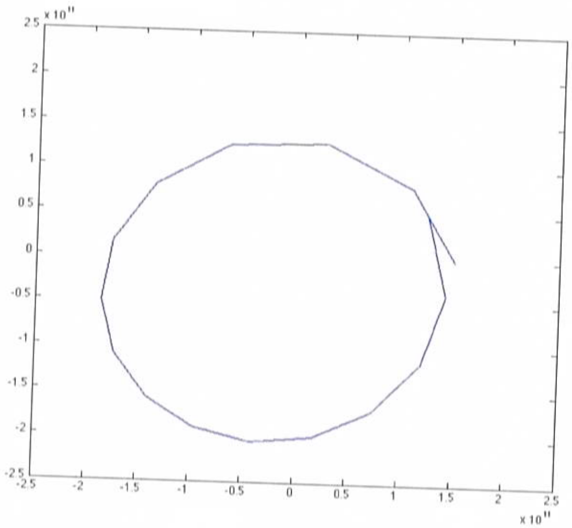
ROUGH PICTURE WITH SMALL ΔT

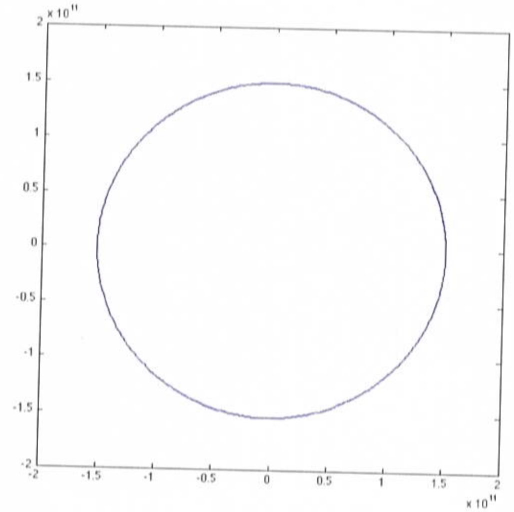


EARTH HAS CIRCULAR ORBIT

PROBLEM Δt TOO BIG.

$$\left.\begin{aligned} r(t=0) &= 1.5\times10^{11}\ \text{m} \\ r(t=1\,\text{month}) &= 1.34\times10^{11}\ \text{m} \\ r(t=2\,\text{month}) &= 1.33\times10^{11}\ \text{m}\end{aligned}\right\}\ \text{NOT CONSTANT}$$





$\Delta t = 1$ month

$\Delta t = 1$ day

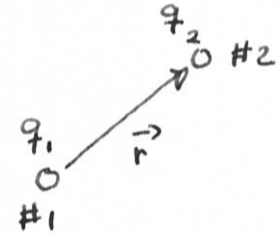
SAME Scheme only difference is the Δt

Newton's LAW OF GRAVITY explains why planets can move in circular orbits.

It also allows them to move in elliptical orbits.

ELECTRICAL FORCES

$$\vec{F}_{\substack{\text{ON } 2\\ \text{BY } 1}} = + \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}|^2} \hat{r}$$

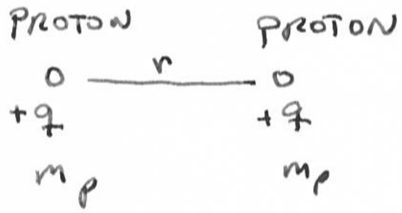


$(q_1, q_2) \Rightarrow$ CHARGE MEASURED IN COULOMBS

q = CHARGE PROTON $= +1.6 \times 10^{-19}$ C
CHARGE ON ELECTRON $= -1.6 \times 10^{-19}$ C
} NOTE SIGN CONVENTION

$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{N \cdot m^2}{C^2}$ ε_0 – PERMITTIVITY OF FREE SPACE

CONSIDER



$m_p = 1.7 \times 10^{-27}$ kg

$$F_g = G \frac{m_1 m_2}{r^2}$$

$$F_E = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

$$\frac{F_E}{F_g} = \frac{1}{4\pi\varepsilon_0} \frac{1}{G} \frac{q^2}{m_p^2} = 1.2 \times 10^{36}$$ ← WOW!

$F_E >> F_g$!!

ATOMS → electrons and Nuclei [Protons & neutrons]

INTERACT STRONGLY OWING TO ELECTRICAL FORCES NOT GRAVITY.

GRAVITY → Planets, stars, galaxies.

~~SPACE~~ STRONG INTERACTION

H atom 1p
Deuterium atom 1p + 1n
Tritium atom 1p + 2n
He atom 2p + 2n

p = PROTON
n = NEUTRON

Holds p & n together!

NEWTON'S LAWS

- WORK WELL FOR "ORDINARY" Experiences
- FAIL → High velocity (RELATIVITY)
  → SMALL DISTANCES (QUANTUM THEORY)

PROBLEMS ⇒ MANY BODIES (OBJECTS) INTERACTING

No SIMPLE SOLUTIONS USING NEWTON'S LAWS

(ALSO TRUE FOR QUANTUM THEORY)

DETERMINISM & QUANTUM THEORY ⇒

DOES GOD ROLL DICE? (READ 3.9 & 3.10 Book)

$\boxed{\text{CONSERVATION OF MOMENTUM}}$

$$\Delta \vec{P}_{system} + \Delta \vec{P}_{surrounding} = 0$$

KEY LAW!

RECIPROCITY

$$\Delta \vec{P}_1 + \Delta \vec{P}_2 = 0$$

RECALL $\Delta \vec{P}_1 = \vec{F}_{12} \Delta t$

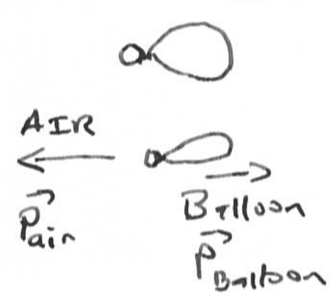
$$\Delta \vec{P}_2 = \vec{F}_{21} \Delta t$$

$$\Delta \vec{P}_1 + \Delta \vec{P}_2 = (\vec{F}_{12} + \vec{F}_{21}) \Delta t$$

BUT $\vec{F}_{12} = -\vec{F}_{21}$ $\quad \Delta \vec{P}_1 + \Delta \vec{P}_2 = 0$

~~Ball~~ BALLOON ROCKET

$\vec{P}_i = 0$



SYSTEM: BALLOON PLUS AIR

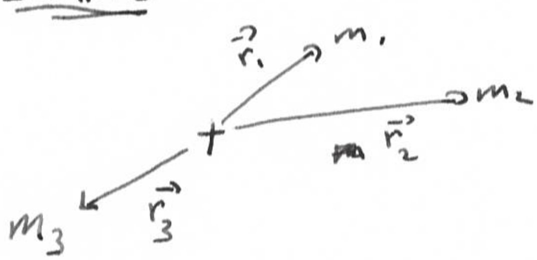
SURROUNDING: EVERYTHING ELSE

$$\vec{P}_f = \vec{P}_{AIR} + \vec{P}_{Balloon} = \vec{P}_i = 0$$

$$\vec{P}_{BALLOON} = -\vec{P}_{AIR}$$

CENTER OF MASS

Define



$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + M_3}$$

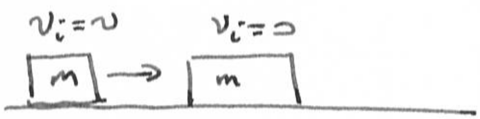
$$\boxed{\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m\vec{r}_3}{M_{TOTAL}}}$$

$$\vec{v}_{cm} = \frac{1}{M_{TOTAL}}\left\{m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + m_3\frac{d\vec{r}_3}{dt}\right\}$$

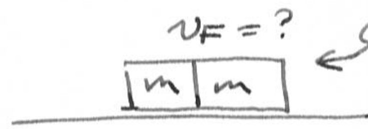
$$= \frac{1}{M_{TOTAL}}\left\{m_1\vec{v}_1 + m_2\vec{v}_1 + m_3\vec{v}_3\right\}$$

$$\boxed{\vec{P}_{TOTAL} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 = M_{TOTAL}\vec{v}_{cm}}$$

EXAMPLE



AFTER COLLIDING



STICK TOGETHER

[NO FRICTION]

WHAT IS v_F?

$P_i = [mv] + [m0] = mv$

$P_f = 2mv_f$

$P_i = P_f$

$mv = 2mv_F$

$$\boxed{v_F = \frac{1}{2}v}$$