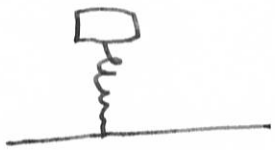
Review

$\Delta \vec{p} = \vec{F}_N \Delta t$ Suppose $\vec{F}_N$ changes?

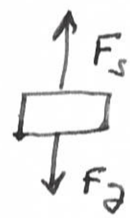
We can break up problem

$\Delta \vec{p} = \Delta \vec{p}_1 + \Delta \vec{p}_2 + \Delta \vec{p}_3 + \cdots$ } Hope that $\vec{F}_N$ does not change over Δt.

SPRING PROBLEM



Compress to 0.1m
$L = 0.1 m$



$L_0 = 0.2m$
$k_s = 8 N/m$
$m = 0.06 kg$

INITIALLY

$F_s = -k_s(L - L_0) = -8(0.1 - 0.2) N$
$= +0.8 N$

$F_g = -mg = -0.588 N$

↳ GRAVITY does NOT change with position.

BREAK up
Problem $\Delta t = 0.3$ sec

STEP 1 $t = 0$ to 0.1 sec
STEP 2 $t = 0.1$ to 0.2 sec
STEP 3 $t = 0.2$ to 0.3 sec.

Find
$\Delta P = (F_s + F_g)\Delta t$

STEP 1 $\Delta P_1 = P_f - P_i$

$P_i = 0$ $P_0 = 0$
$P_f = P_1$

$F_s + F_g = (+0.8 - 0.588) N$
$= 0.212 N$

$F_N \Delta t = 0.0212$ m·kg/s

$v_{AVE} = \frac{\Delta y}{\Delta t}$

$y_1 = y_0 + v_{AVE}\Delta t$

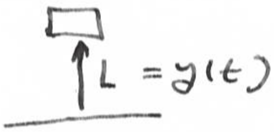
$v_{AVE} \cong v_F$

$v_F = \frac{P_f}{m} = \frac{P_1}{m} = \frac{0.0212}{0.06}$ m/s

$v_F = 0.353$ m/s

$y_1 = \underbrace{0.1}_{y_0} + \underbrace{0.353 \text{ m/s}}_{v_F} \underbrace{(0.1)}_{\Delta t} = 0.135m$

Notation



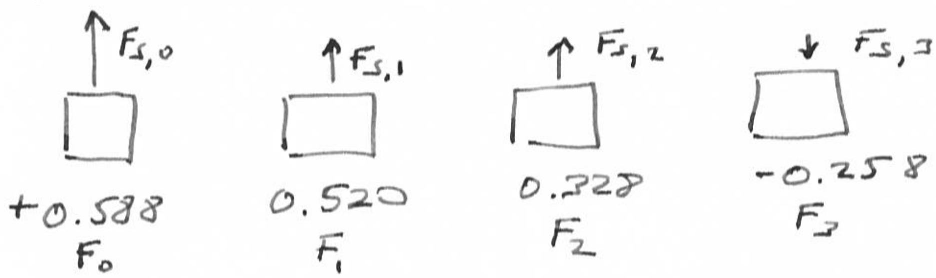
$$\left.\begin{aligned} L_0 &= y_i = 0.1\ m \\ L_1 &= y_f = 0.135\ m \end{aligned}\right\}\ \text{after first step}$$

F_g = CONSTANT
= −0.8 N

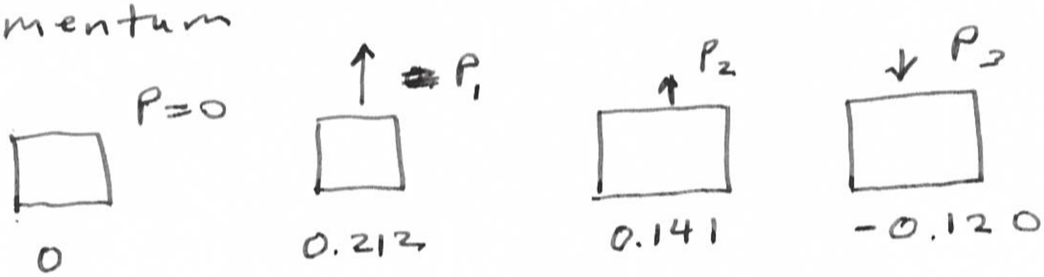
SUMMARY

Δt (s)	y_f (m)	y_i (m)	p_f $\left(\frac{kg\cdot m}{s}\right)$	p_i $\left(\frac{kg\cdot m}{s}\right)$	$F_{f,s}$ (N)	$F_{i,s}$ (N)
0−0.1	0.135	0.100	0.212	0	0.520	0.588
0.1−0.2	0.159	0.135	0.141	0.212	0.328	0.520
0.2−0.3	0.139	0.159	−0.120	0.141	−0.258	0.328

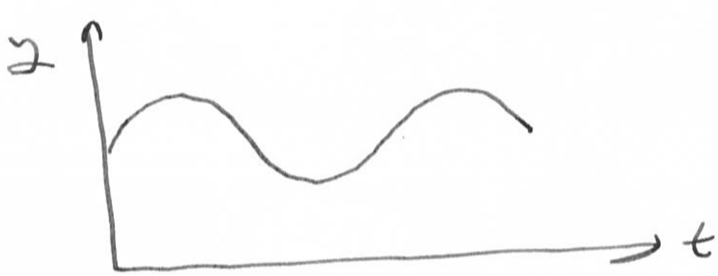
SUMMARY – FORCES



Momentum



CRUDE SOLUTION
$\Delta t = 0.1$ sec



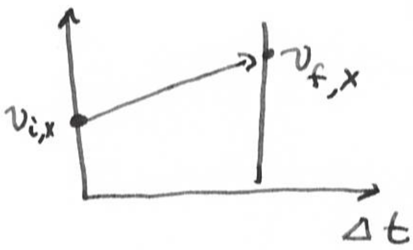
Accurate Solution.

CONSTANT FORCE

$$\Delta \vec{P} = \vec{F}_N \, \Delta t \qquad \boxed{\vec{P}_f = \vec{P}_i + \vec{F}_N \Delta t}$$

ALWAYS TRUE, NOT DEPENDENT ON Δt

$$\vec{v}_f = \vec{v}_i + \left(\frac{\vec{F}_N}{m}\right) \Delta t$$



Consider $\vec{F} = |\vec{F}|\,\hat{F}$ where $\hat{F}$ is along x-axis.

$$\vec{F} = F_x \hat{i}$$

$$v_{f,x} = v_{i,x} + \frac{F_x}{m} \Delta t$$

IN THIS CASE $\Rightarrow$

$$v_{AVE} = \frac{v_{i,x} + v_{f,x}}{2}$$

~~ALWAY~~ ALWAYS TRUE!

$$v_{AVE} = \frac{v_{i,x} + \left[v_{i,x} + \frac{F_x}{m}\Delta t\right]}{2} = v_{i,x} + \frac{1}{2}\frac{F_x}{m}\Delta t$$

$$v_{AVE} = \frac{\Delta x}{\Delta t} \Rightarrow x_f = x_i + v_{AVE}\,\Delta t$$

$$x_f = x_i + \left[v_{i,x} + \frac{1}{2}\frac{F_x}{m}\Delta t\right]\Delta t$$

$$\boxed{x_f = x_i + v_{i,x}\Delta t + \frac{1}{2}\frac{F_x}{m}\Delta t^2}$$

ALWAYS TRUE <u>IF</u> F_x = CONSTANT

GENERAL SOLUTION

$$\boxed{\vec{r}_f = \vec{r}_i + \vec{v}_i \Delta t + \frac{1}{2}\frac{\vec{F}}{m}\Delta t^2}$$

GENERAL SOLUTION: CALCULUS

CONSIDER one-dimensional problem →

$\vec{F} = F_x \hat{i}$ $\Delta p = F_x \Delta t$ Take $\Delta t \to 0$

$\frac{dp}{dt} = F_x$ $\frac{dv_x}{dt} = \frac{F_x}{m}$ INTEGRATE BOTH SIDES

$v_x = v_x^\circ + \frac{F_x}{m} \Delta t$

VERIFY

$$\frac{dv_x}{dt} = \frac{dv_x^\circ}{dt} + \frac{F_x}{m}\frac{d}{dt}(\Delta t) \qquad \Delta t = t - t_0 \qquad \frac{d\Delta t}{dt} = 1$$

$$\boxed{\frac{dv_x}{dt} = \frac{F_x}{m}} \qquad v_x = v_x^\circ + \frac{F_x}{m}\Delta t \qquad t = t_0 \quad \Delta t = 0$$

v_x° = ~~INITIA~~ INITIAL VELOCITY

$$\boxed{v_x^f = v_{x,i} + \frac{F_x}{m}\Delta t}$$

Now recall $v_x = \frac{dx}{dt}$

$$\frac{dx}{dt} = v_{x,i} + \frac{F_x}{m}\Delta t$$

INTEGRATE

$$x = x_0 + v_{x,i}\Delta t + \frac{F_x}{m}\frac{1}{2}(\Delta t)^2$$

Again $\Delta t = 0$ $t = t_0 \Rightarrow$ initial

$$\boxed{x_f = x_i + v_{x,i}\,\Delta t + \frac{1}{2}\frac{F_x}{m}\Delta t^2}$$

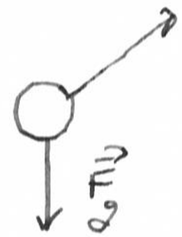
Note: $a_x = \frac{dv_x}{dt}$ so $a_x = \frac{F_x}{m}$ $\boxed{F_x = m a_x}$

GENERAL SOLUTION

$$\boxed{\vec{r}_f = \vec{r}_i + \vec{v}_i\,\Delta t + \frac{1}{2}\frac{\vec{F}}{m}\Delta t^2}$$

ALWAYS TRUE IF $\vec{F}$ = CONSTANT.

EXAMPLE:



$\vec{r}_i = \langle 0, 0, 0 \rangle$

$\vec{p}_i = \langle \frac{3}{2}, \frac{7}{2}, 0 \rangle$ kg·m/s

$m = 500$ g $= \frac{1}{2}$ kg

FORCES

$F_x = 0$ CONSTANT! $\quad x_f = x_i + v_{i,x} \Delta t$

$F_y = -mg$ CONSTANT! $\quad y_f = y_i + v_{i,y} \Delta t + \frac{F_y}{2m} \Delta t^2$

$$x_f(t) = 0 + \underbrace{3}_{v_{i,x} = \frac{p_{i,x}}{m} = 3\text{ m/s}} \Delta t = 3\Delta t \quad (m)$$

$$y_f(t) = 0 + \underbrace{7}_{v_{i,y} = \frac{p_{i,y}}{m} = 7\text{ m/s}} \Delta t + \frac{1}{2} \frac{\overbrace{(-mg)}^{F_g}}{m} \Delta t = 7\Delta t - \underbrace{4.9}_{(g/2)} \Delta t^2$$

QUESTIONS: (a) where is ball at $t = 0.5$ s
(b) When does ball hit ground?

(a)

$x_f(t = 0.5) = 3(\frac{1}{2}) = 1.5$ m

$\cancel{y_f = 7(\frac{1}{2}) - (4.9)(0.5)^2}$

$y_f = 7(\frac{1}{2}) - 4.9(\frac{1}{2})^2$
$= 2.275$ m

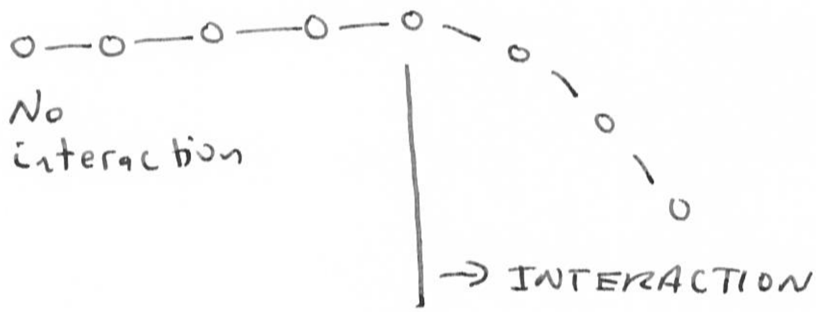
$$\boxed{\vec{r}_f = \langle 1.5, 2.275, 0 \rangle \text{ m}}$$

(b) $y_f(t_b) = 0 = 7\Delta t_b - 4.9 \Delta t_b^2$

t_b = time to hit ground

$\Delta t_b = \frac{7}{4.9}$ s
$= \underline{1.43 \text{ sec}}$

FUNDAMENTAL INTERACTIONS [Chpt. 3]



NATURE OF INTERACTION

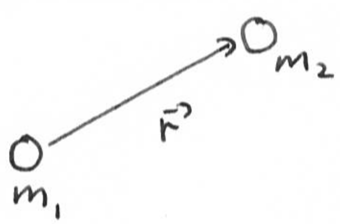
⇒ FORCE

Types of forces

(1) GRAVITY
(2) ELECTROMAGNETIC
(3) STRONG (Nuclear)
(4) WEAK (Elementary PARTICLES)

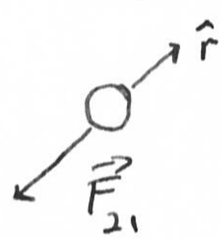
} Focus on (1) and (2) ORDINARY Experience.

GRAVITY



FORCE OF m_1 on m_2:

$$\vec{F}_{2\text{ by }1} = -G\,\frac{m_1 m_2}{|r^2|}\,\hat{r}$$



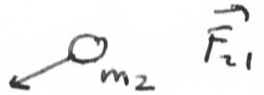
Note $|\vec{F}| \propto \dfrac{m_1 m_2}{r^2}$

GRAVITY INCREASES as PRODUCT $m_1 \times m_2$

DECREASES as $\frac{1}{r^2}$

Note on SIGN

GRAVITY IS ALWAYS ~~ATT~~ ATTRACTIVE

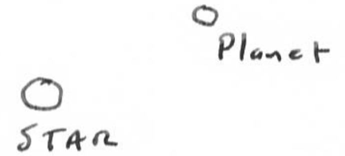


one (m_1) wants two (m_2) to move TOWARD it.

UNITS:

$$\boxed{G = 6.7 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}}$$

EXAMPLE



$\vec{r}_s = \langle 2\times10^{11}, 1\times10^{11}, 1.5\times10^{11}\rangle$ m

$m_s = 4\times10^{30}$ kg

$\vec{r}_p = \langle 3\times10^{11}, 3.5\times10^{11}, -0.5\times10^{11}\rangle$ m

$m_p = 3\times10^{24}$ kg

Find $\vec{F}_{gravity}$ on planet by STAR and on star by planet.

$$r = |\vec{r}| = |\vec{r}_p - r_s| = \sqrt{(r_p-r_s)_x^2 + (r_p-r_s)_y^2 + (r_p-r_s)_z^2}$$

$$= \sqrt{(1\times10^{11})^2 + (2.5\times10^{11})^2 + (2\times10^{11})^2}\ \text{m}$$

$$= 3.35\times10^{11}\ \text{m}$$

$$|F| = G\frac{m_p m_s}{r^2} = (6.7\times10^{-11})\frac{(3\times10^{24})(4\times10^{30})}{[3.35\times10^{11}]^2}\ \text{N}$$

$$= 7.16\times10^{21}\ \text{N}$$

Note: $|F_{ps}| = |F_{sp}|$

Direction of Force

$$\hat{r} = \frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|} = \frac{\langle 1\times10^{11}, 2.5\times10^{11}, -2\times10^{11}\rangle}{3.35\times10^{11}}$$

$$\hat{r} = \langle 0.299, 0.746, -0.597\rangle$$

$\vec{r}_s + \vec{r} = \vec{r}_p$

$\vec{r} = \vec{r}_p - \vec{r}_s$

$$\vec{F}_{ps} = -\frac{G m_p m_s}{|r|^2}\hat{r}$$

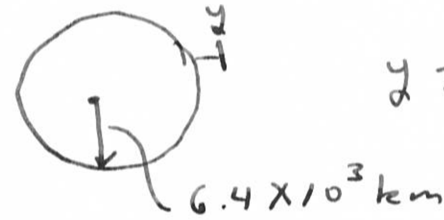
$$= (7.16\times10^{21})\langle -0.299, -0.746, +0.597\rangle\ \text{N}$$

$$\vec{F}_{ps} = \langle -2.14\times10^{21}, -5.34\times10^{21}, +4.27\times10^{21}\rangle\ \text{N}$$

Note: $\vec{F}_{sp} = -\vec{F}_{ps}$?

NEAR EARTH'S SURFACE

$$F_g = G \frac{M_E m}{(R_E + y)^2}$$



$y \approx 1\,m$

$R_E + y \approx R_E$

$$F_g = \left(\frac{GM_E}{R_E^2}\right) m = \frac{(6.7 \times 10^{-11})(6 \times 10^{24})}{(6.4 \times 10^6)^2}\, m \quad (N)$$

$$= \left(9.8 \frac{m}{s^2}\right) m = g\, m \qquad \boxed{g = 9.8\, m/s^2 = 9.8 \frac{N}{kg}}$$

RECIPROCITY:

$$\vec{F}_{1 \text{ on } 2} = - \vec{F}_{2 \text{ on } 1}$$

TRUE for GRAVITY AND ELECTROSTATIC FORCES. (Not MAGNETIC)

Note: Seems odd - force exerted by earth on us is <u>same</u> as us on the earth.

$$\Delta P_{us} = F_{\text{earth on us}}\, \Delta t \qquad \Delta P_{earth} = F_{\text{us on earth}}\, \Delta t$$

$$\Delta v_{us} = \frac{F_{\text{earth on us}}}{m_{us}} \Delta t \qquad \Delta v_{earth} = \frac{F_{\text{us on earth}}}{m_{EARTH}} \Delta t$$

$\Delta v_{us} \gg \Delta v_{EARTH}$ as $m_{EARTH} \gg m_{us}$!