QUESTION: WHAT IF $\vec{F}$ CHANGES WITH TIME? CAN we still use $\Delta\vec{p} = \vec{F}\Delta t$?

ANSWER: SURE! BREAK up the time intervals →

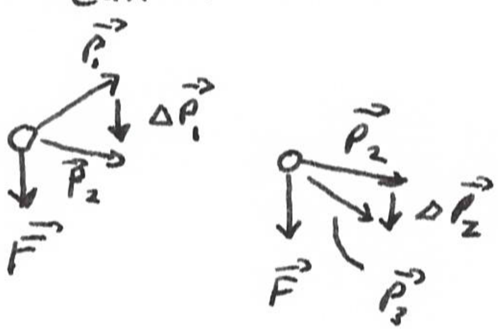
$$\Delta\vec{p} = \Delta\vec{p}_1 + \Delta\vec{p}_2 + \Delta\vec{p}_3 + \dots$$

$$\Delta\vec{p}_1 = \vec{F}_1 \Delta t_1 \quad \Delta\vec{p}_2 = \vec{F}_2 \Delta t_2 \quad \Delta\vec{p}_3 = \vec{F}_3 \Delta t_3$$

where $\vec{F}_1, \vec{F}_2, \vec{F}_3$ are constant over the Δt.

Usually best way is to assume Δt small and take $\Delta t = \Delta t_1 = \Delta t_2 = \Delta t_3$ (UNIFORM TIME "STEP")

$\vec{F}$ can be CONSTANT and still do this.



$$\Delta\vec{p}_1 = \vec{p}_2 - \vec{p}_1 = \vec{F}\Delta t$$

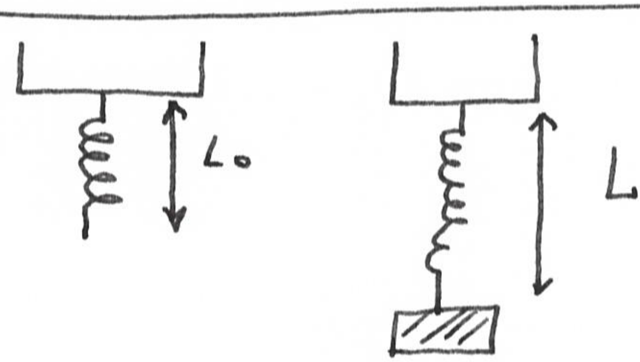
$$\Delta\vec{p}_2 = \vec{p}_3 - \vec{p}_2 = \vec{F}\Delta t$$

$$\vdots$$

and so on!

EXAMPLES

$\boxed{\text{SPRINGS}}$ →



The stretched spring generates a FORCE!

$$|\vec{F}_s| = k_s|s| = k_s|L - L_0|$$

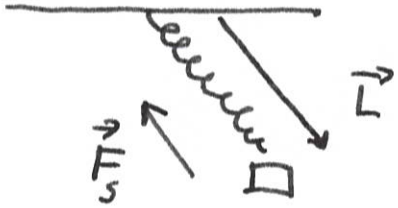
$$s = L - L_0$$

k_s = SPRING CONSTANT

UNITS: $\frac{N}{m}$ in SI.

Note: MAKES No difference if SPRING IS COMPRESSED OR EXPANDED. Force (Magnitude) depends only on $|s|$.

DIRECTION OF FORCE FROM A SPRING

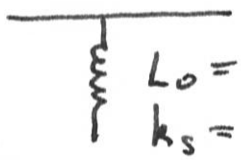


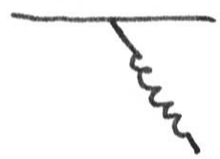
$\vec{L} = L\hat{L} \qquad s = |\vec{L}| - L_0$

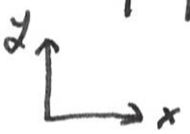
$$\boxed{\vec{F}_s = -k_s\, s\, \hat{L}} = -k_s\left(|\vec{L}| - L_0\right)\hat{L}$$

Note: FORCE opposes stretching or compressing.
Pull on a spring – force wants to pull back.

EXAMPLE:







$\vec{L} = \langle .07, -0.33 \rangle\ m$

$\hat{L} = \dfrac{\vec{L}}{|\vec{L}|}$

$|\vec{L}| = \sqrt{(0.07)^2 + (-0.33)^2}$
$= 0.337\ m$

$\hat{L} = \langle 0.208, -0.979 \rangle$

$s = L - L_0 = 0.337\ m - 0.21\ m = 0.127\ m$

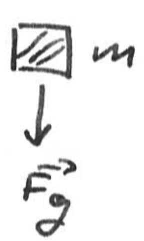
$\vec{F}_s = -k_s s \hat{L} = -(9)(0.127)\langle 0.208, -0.979 \rangle$

$= \langle -0.238, 1.12 \rangle\ N$ (Error in book → Says N/m).

$\boxed{\text{GRAVITY}}$ NEAR SURFACE OF THE EARTH

$|\vec{F}_g| = mg \qquad g = 9.8\ N/kg = 9.8\ m/s^2$

Direction? Pretty "obvious" TOWARD THE SURFACE.



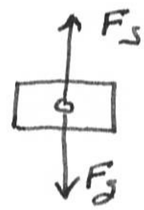
EXAMPLE: SPRING AND GRAVITY

System: BLOCK
SURROUNDINGS: OTHER FORCES! SPRING and EARTH

GIVEN INFORMATION
$L_o = 0.2\,m$ $k_s = 8\,N/m$
$m = 0.06\,kg$

INITIAL CONDITIONS →
COMPRESSED TO ½ length
$L = 0.1\,m$ $P_i = 0$

Note: SIGNS!



<u>No</u> NEED FOR VECTOR ~~NOTA~~ NOTATION

$L = 0.1 m$ $F_s = -k_s(L - L_o)$
$= -8(0.1 - 0.2) = 0.8\ N$

$F_g = -mg = -0.06(9.8) = -0.588\ N$ [NOTE ↓ F_g]

~~In~~ <u>MOTION</u> <u>of</u> <u>BLOCK</u>

WOULD LIKE TO KNOW $X(t = 3\,sec)$.
Let do this $\Delta t = 0.1$ sec [STEP 1]

$\Delta P_1 = F_N \Delta t$ $\Delta P_1 = P_1 - P_o = P_1 = F_N \Delta t$
$(P_i = 0)$ $= [0.8 - 0.588](0.1)$
$= 0.0212\ kg\text{-}m/s$

WANT ~~X(t)~~ $y(t)$

$v_{Avg} = \frac{\Delta y}{\Delta t}$ $y_1 = y_o + v_{Avg} \Delta t$

NOTE: Book TAKES $v_{Avg} \approx v_F$. <u>NOTE</u>: NOT CLEAR THIS IS THE "OPTIMAL" CHOICE!

$v_F = \frac{P_f}{m} = \frac{0.0212}{0.06} = 0.353\ m/s$

$y_1 \overset{\sim}{=} y_o + v_F \Delta t = 0.1 + (0.353)(0.1) = 0.135\ m$

~~At~~ $\Delta y \simeq 0.035\ m$ is a pretty big ~~ch~~ change
Likely Δt is TOO LARGE! (OK for now)

[STEP 2]

$|L| = 0.135\ m$

$F_s = -k_s(L - L_o) = -8(0.135 - 0.2) = {}^{+}0.520\ N$ (SMALLER THAN STEP 1)

$F_g = -mg = -0.588\ N$ (SAME)

$\Delta P = P_2 - P_1 = F_{NET}\,\Delta t$

$P_2 = \underbrace{(0.0212)}_{P_1} + \underbrace{(+0.520 - 0.588)}_{F_s + F_g}\underbrace{0.1}_{\Delta t} = 0.0141$ kg-m/s

$v_2 = \frac{P_2}{m} = 0.236$ m/s $\quad \cancel{\Delta x}$

$y_2 = y_1 + v_2 \Delta t = 0.159$ m $\qquad \boxed{y_2 = 0.159\text{ m}}$

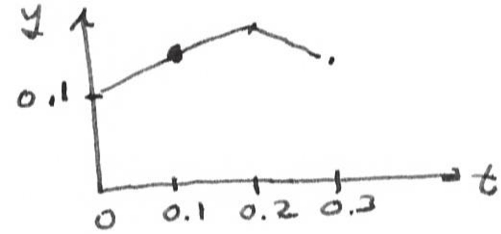
[STEP 3] $\quad L = 0.159$ m $\quad F_s = -k_s(L - L_0) = +0.328$ N

$F_N = (0.328 - 0.580)$ N $= -0.258$ N (Note: SIGN CHANGE!)

$P_3 = P_2 + F_{NET}\,\Delta t = -0.12$ kg-m/s (SIGN!)

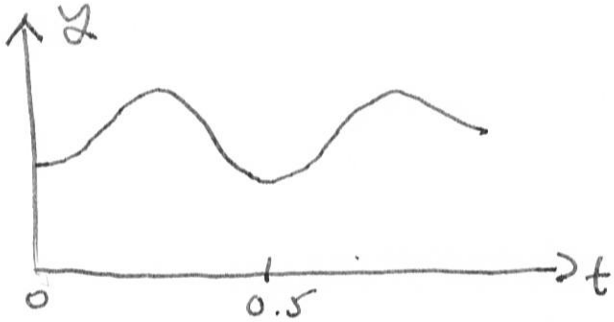
$v_3 = \frac{P_3}{m} = -0.196$ m/s (SIGN!)

$y_3 = 0.139$ m



THE POSITION "JERKS" BECAUSE $\Delta t = 0.1$ TOO BIG!

$\Delta t = 0.01$



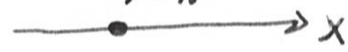
MOTION IS PERIODIC!

Note: $v_{Avg} = v_f$ ONLY WORKS BECAUSE Δt IS LARGE

MOTION CAN EASILY BE DONE WITH A COMPUTER!

CONSTANT FORCE

$\rightarrow F_N$



Suppose F_N CONSTANT

$$\Delta p = F_N \Delta t \qquad p_f = p_i + F_N \Delta t$$

DIVIDE BY m

$$v_f = v_i + \frac{F_N}{m}\Delta t$$

NOW CONSIDER

$$v_{avg} = \frac{\Delta x}{\Delta t} \qquad x_f = x_i + v_{avg}\Delta t$$

$$v_{avg} = \frac{v_f + v_i}{2}$$

$$v_{avg} = \frac{\left(v_i + \frac{F_N}{m}\Delta t\right) + v_i}{2} = v_i + \frac{1}{2}\frac{F_N}{m}\Delta t$$

$$\boxed{x_f = x_i + v_i \Delta t + \frac{1}{2}\frac{F_N}{m}\Delta t^2}$$

VALID FOR ANY Δt! Why?

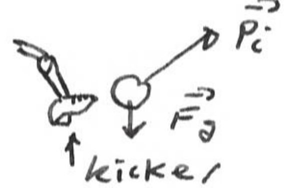
ANALYTICAL SOLUTION

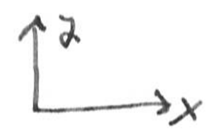
$$\frac{\Delta p}{\Delta t} = F = \text{CONSTANT} \qquad \frac{dp}{dt} = F \qquad \frac{dv}{dt} = F/m$$

$$v = v_i + \frac{F}{m}t \qquad t = 0 \quad v = v_i, \quad x = x_i$$

$$\frac{dx}{dt} = v_i + \frac{F}{m}t \qquad x_f = x_i + v_i t + \frac{F}{m}\frac{t^2}{2}$$

EXAMPLE: Kicked ball





GIVEN:

$m = 500\,g$

$\vec{r}_i = \langle 0,0,0 \rangle$

$\vec{v}_i = \langle 3,7,0 \rangle\ m/s$

System: THE BALL

SURROUNDING: EARTH (ONLY SOURCE OF FORCE)

No force in the x direction!

$$\boxed{x_f = x_i + v_{x,i}\Delta t} \leftarrow F = 0$$

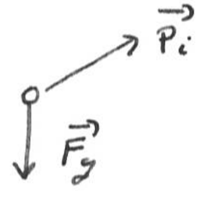
$F_y = -mg$

$$\boxed{y_f = y_i + v_{y,i}\Delta t + \frac{1}{2}\frac{F_y}{m}\Delta t^2}$$

Where is ball $t = 0.5\,s$?

When does ball hit ground?

IMPORTANT: THESE expressions ARE VALID SAVE THE NEGLECT OF AIR FRICTION



Where is ball at $t = 0.5$ s (or $\Delta t = 0.5$)

$$x_f(t=0.5) = \underset{x_i}{0} + \underset{v_{x,i}}{3}\,\underset{\Delta t}{(0.5)} = \underline{\underline{1.5\ m}}$$

$$y_f = \underset{y_i}{0} + \underset{v_{y,i}}{7}(0.5) - \tfrac{1}{2}\underset{\frac{mg}{m} = F_g}{(9.8)}(0.5)^2 = (0.5)(7) - 4.9(0.25)$$

$$= \underline{\underline{2.275\ m}}$$

$\vec{r}_f = \langle 1.5, 2.275, 0 \rangle$ m at $t = 0.5$ s

Ball hits ground when $y_f = 0$

$$y_f = y_i + v_{y,i}\Delta t - \tfrac{1}{2} g \Delta t^2$$

$$0 = 0 + v_{y,i}\Delta t - \tfrac{1}{2} g \Delta t^2 \Rightarrow \Delta t = \frac{2 v_{y,i}}{g}$$

$$\Delta t = \frac{2(7)}{9.8}\ s = \underline{\underline{1.43\ s}}$$

Where is ball at $\Delta t = 1.43$ s

$y_f = 0$

$$x_f = x_i + v_{x,i}\Delta t = 0 + 3(1.43)\ m = 4.29\ \underline{\underline{m}}$$

$$\vec{r}_f = \langle 4.29, 0, 0 \rangle\ m$$