$\boxed{\text{MOMENTUM PRINCIPLE}}$ ← KEY CONCEPT

$\Delta \vec{p} = \vec{F}_{NET} \, \Delta t$

THE CHANGE in MOMEUMTUM ($\Delta \vec{p}$) is EQUAL to the <u>NET</u> <u>FORCE</u> acting on an object ($\vec{F}_{Net}$) times the duration of the INTERACTION (Δt)

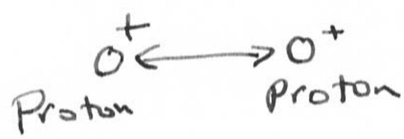
Note: We have defined $\Delta \vec{p}$ before, so this really defines $\vec{F}_{Net}$.

$\vec{F}_{Net} = \frac{1}{\Delta t} \Delta \vec{p}$ [You might guess $\vec{F} = \frac{d\vec{p}}{dt}$]

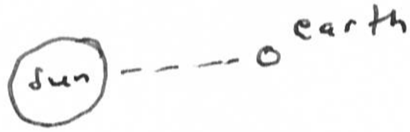
Here we assume $\vec{F}_{Net}$ is a "constant" over the duration time (Δt). Later we will take $\Delta t \to 0$

<u>Forces</u>: Elementary forces - Examples

Electrostatic



GRAVITY

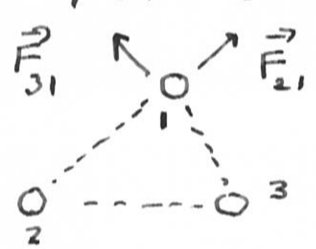


(Other elementary forces are thought to exist. These are the two we will focus on.)

Units: $\Delta \vec{p} \to kg \cdot m/s$ $\quad \Delta t \to s$

$\vec{F}_{Net} = \frac{\Delta \vec{p}}{\Delta t} \to kg \cdot m/s^2 = N$ (Newton)

NET FORCES



$\vec{F}_{NET} = \vec{F}_{21} + \vec{F}_{31}$

↑

On #1

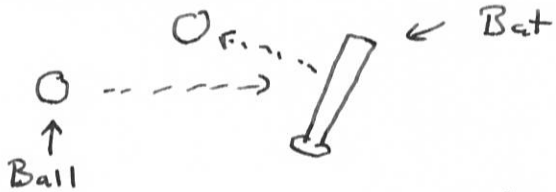
In general

$$\vec{F}_N = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots$$

NEW DEFINITION

IMPULSE ($\Delta\vec{p}$)

$$= \vec{F}\,\Delta t$$



$$\vec{p}_i = \langle 0, 0, 3 \rangle \quad kg\cdot m/s$$

$$\vec{p}_f = \langle -2, 2, -1 \rangle$$

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \langle -2, 2, -1 \rangle \quad kg\cdot m/s$$

The impulse from the bat on the ball $= \Delta\vec{p}$

CHANGE IN MOMENTUM = IMPULSE

Application:

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \vec{F}_{Net}\,\Delta t$$

$$\boxed{\begin{aligned}\vec{p}_f &= \vec{p}_i + \Delta\vec{p} \\ &= \vec{p}_i + \vec{F}_{NET}\,\Delta t\end{aligned}}$$

If we <u>know</u> $\vec{F}_{Net}$ we can predict the change.

Rules in applying "momentum principle"

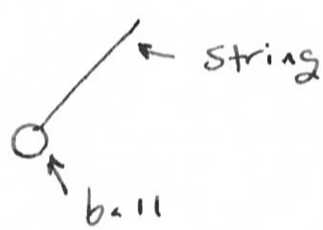
(1) Need to characterize system, e.g. a ball and ~~string~~ string.

(2) Decide what forces are present, e.g., force on the ball.

(3) Figure out duration of force.

(4) Find $\Delta\vec{p}$!

<u>Example</u>:



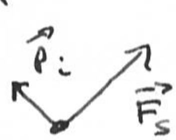
WANT TO FIGURE OUT $\Delta\vec{p}$ for ball.

System: Ball

Surrounding: STRING

(Note: No Gravity here – think deep space)

FORCE DIAGRAM



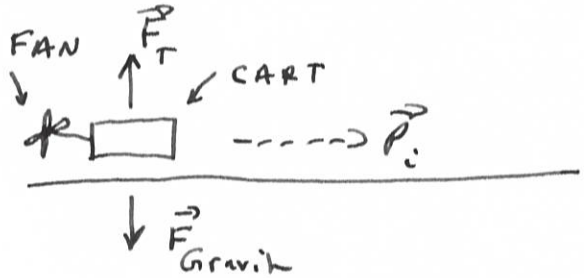
Suppose $\vec{p}_i = \langle -8, 3, 0 \rangle$ kg·m/s

$\vec{F}_{Net} = \langle 10, 25, 0 \rangle$ N

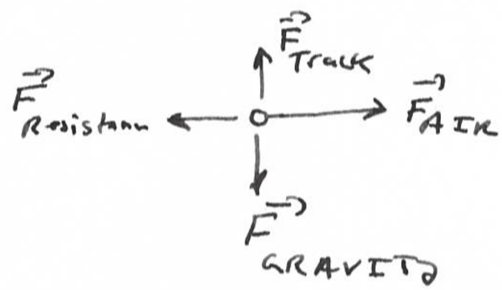
$\Delta t = 0.5$ s what is $\vec{p}_{final}$?

$$\vec{p}_f = \underbrace{\langle -8, 3, 0 \rangle}_{\vec{p}_i} + \underbrace{\langle 10, 25, 0 \rangle}_{\vec{p}_f} \underbrace{0.5}_{\Delta t} = \langle -3, 15.5, 0 \rangle$$

Issues we need to think about: FRICTION, GRAVITY, OTHER OBJECTS.



$\vec{F}_{TRACK}$, $\vec{F}_{GRAVITY}$, $\vec{F}_{AIR}$, $\vec{F}_{Air\ Resistance}$



$$\vec{p}_f = \vec{p}_i + (\vec{F}_{Track} + \vec{F}_{GRAVITY} + \vec{F}_{air} + \vec{F}_{air\ Resistance}) \Delta t$$

Note: $\vec{F}_{TRACK} + \vec{F}_{GRAVITY} = 0$

Why? Not moving up or down.

$$\vec{F}_{Net} = \vec{F}_{air} - \vec{F}_{resistance} = \langle 0.2, 0, 0 \rangle \text{ N} \quad \text{(Given)}$$

$$\vec{p}_i = \underbrace{0.5 \text{ kg}}_{\text{mass of cart}} \times \underbrace{\langle 0.3, 0, 0 \rangle}_{\text{velocity}} \text{ m/s} = \langle 0.15, 0, 0 \rangle$$

$$\vec{p}_f = \underbrace{\langle 0.15, 0, 0 \rangle}_{\vec{p}_i} + \underbrace{\langle 0.2, 0, 0 \rangle}_{\vec{F}_{Net}} \underbrace{(0.6 \text{ s})}_{\Delta t \text{ (given)}} = \langle \cancel{0.54}\ 0.27, 0, 0 \rangle \frac{\text{kg·m}}{\text{s}}$$

What is $\vec{v}_f$? $\vec{v}_f = \dfrac{\vec{p}_f}{m} = \langle 0.54, 0, 0 \rangle \dfrac{\text{m}}{\text{s}}$

What happens if Momentum Changes?

Where is the cart?

$$\vec{r}_f = \vec{r}_i + \vec{v}_{AVE}\,\Delta t$$

But what is ~~...~~ $\vec{v}_{AVE}$?

We do know $\vec{v}_i$ and $\vec{v}_f$ from $\vec{p}_i$ and $\vec{p}_f$. We could guess!

$$\vec{v}_{AVE} \approx \frac{1}{2}\left\{\frac{\vec{p}_i + \vec{p}_f}{m}\right\}$$

OK if $\vec{v}_i \approx \vec{v}_f$ but in general we will need something better!

Momentum with Changing <u>forces</u>!

Suppose $\vec{F}_{N,1}, \vec{F}_{N,2}, \vec{F}_{N,3}$ where the force changes

We could do the following

$$\Delta\vec{p} = \Delta\vec{p}_1 + \Delta\vec{p}_2 + \Delta\vec{p}_3$$

$$= \vec{F}_{N,1}\Delta t_1 + \vec{F}_{N,2}\Delta t_2 + \vec{F}_{N,3}\Delta t_3$$

Example

Δt (s)	$\vec{F}$ (N)
0–2	⟨0.2, 0.2, 0⟩
2–4	~~⟨0.2,~~ ⟨0, −0.3, 0⟩
~~2~~ 4–6	⟨−0.3, 0.4, 0⟩

$$\Delta\vec{p}_1 = \underbrace{\langle 0.2, 0.2, 0\rangle}_{\vec{F}_{N,1}}\,\underbrace{2}_{\Delta t} = \langle 0.4, 0.4, 0\rangle$$

$$\Delta\vec{p}_2 = \langle 0, -0.3, 0\rangle\, 2 = \langle 0, -0.6, 0\rangle$$

$$\Delta\vec{p}_3 = \langle -0.3, 0.4, 0\rangle\, 2 = \langle -0.6, 0.8, 0\rangle$$

$$\Delta p = \Delta\vec{p}_1 + \Delta\vec{p}_2 + \Delta\vec{p}_3 = \langle -0.2, 0.6, 0\rangle \ \text{kg}\cdot\frac{\text{m}}{\text{s}}$$

<u>ITERATION</u>

(1) Find $\vec{F}$

(2) Update $\vec{p}_f = \vec{p}_i + \vec{F}_{Net}\,\Delta t$

(3) Update $\vec{r}_f = \vec{r}_i + \vec{v}_{AVE}\,\Delta t$

(4) REPEAT