RIGHT HAND RULE

$\vec{\omega} \Rightarrow \omega$ as a VECTOR

$$\vec{L}_{ROT} = I\vec{\omega}$$

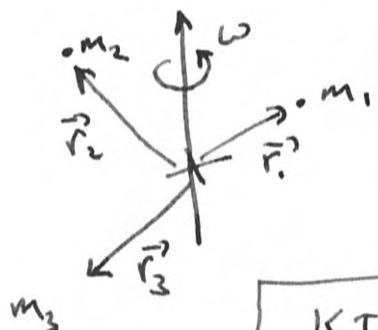
$$L_{ROT} = I_1 \omega$$

ONE PARTICLE

$$v = r_{1,\perp}\,\omega$$

$$|\vec{r}_1 \times \vec{p}\,| = r_{1,\perp} m v$$
$$= r_{1,\perp} m\, r_{1,\perp}\,\omega$$
$$= m\, r_{1,\perp}^2\, \omega$$
$$= I_1 \omega$$

MANY PARTICLES



$$L_{rot} = [m_1 r_{1,\perp}^2 + m_2 r_{2,\perp}^2 + m_3 r_{3,\perp}^2 + \cdots]\,\omega$$
$$= I\omega \qquad (I = \text{Moment of INERTIA})$$

KINETIC ENERGY

$$K_{rot} = \tfrac{1}{2} I\omega^2 = \frac{1}{2}\frac{(I\omega)^2}{I} = \frac{L_{rot}^2}{2I}$$

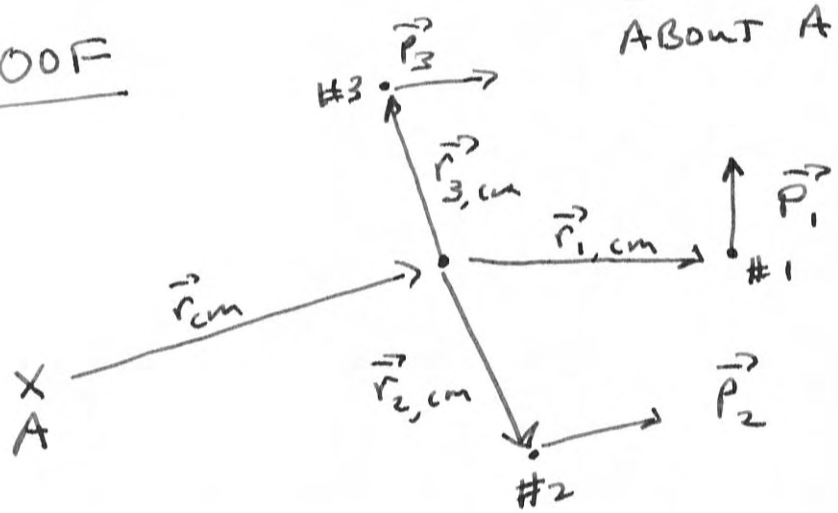
REMINDER $K = K_{TRANS} + \underbrace{K_{rot} + K_{vib}}_{K_{rel}}$

$K_{rel} \leftarrow$ RELATIVE TO CENTER OF MASS

CAN WE DO THE SAME for $\vec{L}$?

$$\vec{L}_A = \underbrace{\vec{L}_{TRANS,A}}_{\text{ABOUT } A} + \underbrace{\vec{L}_{ROT}}_{\text{ABOUT CENTER OF MASS}}$$

PROOF



$$\vec{L}_A = (\vec{r}_{cm} + \vec{r}_{1,cm}) \times \vec{p}_1 + (\vec{r}_{cm} + \vec{r}_{2,cm}) \times \vec{p}_2 + (\vec{r}_{cm} + \vec{r}_{3,cm}) \times \vec{p}_3 = \underbrace{(\vec{r}_{cm} \times \underbrace{(\vec{p}_1 + \vec{p}_2 + \vec{p}_3)}_{\vec{p}_{cm}})}_{\text{TRANSLATIONAL}} + \underbrace{(\vec{r}_{1,cm} \times \vec{p}_1 + \vec{r}_{2,cm} \times \vec{p}_2 + \vec{r}_{3,cm} \times \vec{p}_3)}_{\text{ROTATIONAL}}$$

$$\boxed{\vec{L}_A = \vec{L}_{TRANS,A} + \vec{L}_{ROT}}$$

ANGULAR MOMENTUM PRINCIPLE

RECALL $\frac{d\vec{P}}{dt} = \vec{F}_N$ WHAT ABOUT $\frac{d\vec{L}}{dt} = ?$

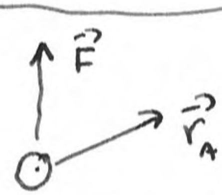
PARTICLE (NO "SPIN")

$\vec{L}_A = \vec{r}_A \times \vec{p}$ $\quad \frac{d\vec{L}_A}{dt} = \frac{d(\vec{r}_A \times \vec{p})}{dt} = \frac{d\vec{r}_A}{dt} \times \vec{p} + \vec{r}_A \times \underbrace{\frac{d\vec{p}}{dt}}_{= \vec{F}}$

$$\frac{d\vec{r}_A}{dt} \times \vec{p} = \vec{v}_A \times \vec{p} = \vec{v}_A \times m\vec{v}_A = m(\vec{v}_A \times \vec{v}_A) = 0$$

(ALSO TRUE FOR $\vec{p} = \gamma m \vec{v}$)

$$\boxed{\frac{d\vec{L}_A}{dt} = \vec{r}_A \times \vec{F} = \vec{\tau}_A} \qquad \boxed{\vec{\tau}_A = \text{TORQUE}}$$



RH Rule AGAIN!

CONSERVATION OF ANGULAR MOMENTUM

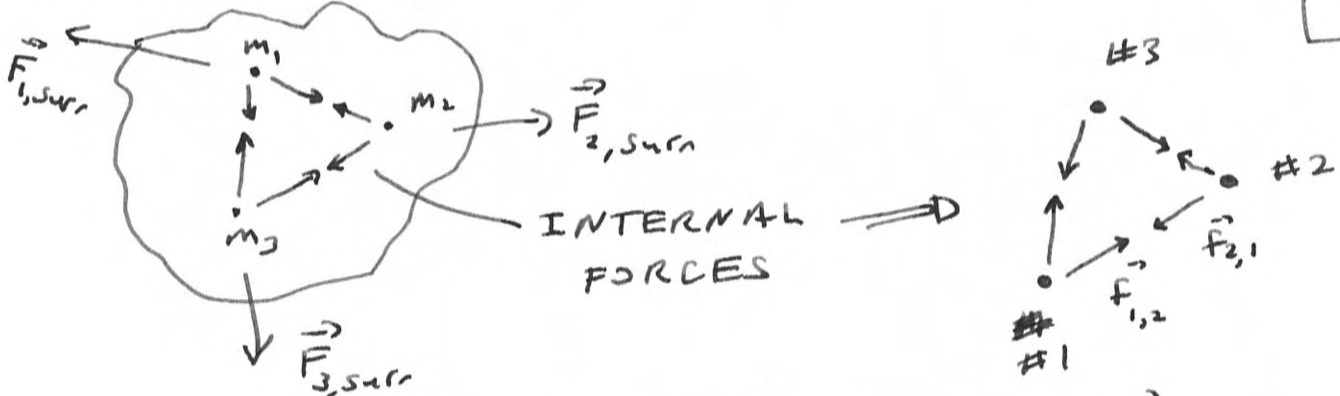
$\vec{\tau}_A = 0 \qquad \frac{d\vec{L}_A}{dt} = 0$

$\vec{L}_A = \text{CONSTANT}$

GENERAL RULE

$$\Delta \vec{L}_{A,sys} + \Delta \vec{L}_{A,surr} = 0$$

MULTIPARTICLE SYSTEM



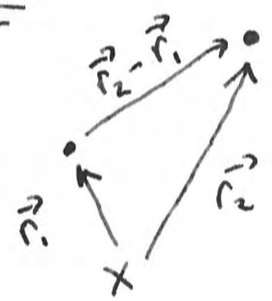
REMEMBER $\Rightarrow$ $\vec{f}_{1,2} = -\vec{f}_{2,1}$; $\vec{f}_{3,1} = -\vec{f}_{1,3}$; ----

$$\left.\begin{aligned}
\frac{d\vec{L}_1}{dt} &= \vec{r}_1 \times (\vec{F}_{1,surr} + \vec{f}_{1,2} + \vec{f}_{1,3}) \\
\frac{d\vec{L}_2}{dt} &= \vec{r}_2 \times (\vec{F}_{2,surr} + \vec{f}_{2,1} + \vec{f}_{3,2}) \\
\frac{d\vec{L}_3}{dt} &= \vec{r}_3 \times (\vec{F}_{3,surr} + \vec{f}_{3,1} + \vec{f}_{3,2})
\end{aligned}\right\} \quad \frac{d\vec{L}}{dt} = \frac{d\vec{L}_1}{dt} + \frac{d\vec{L}_2}{dt} + \frac{d\vec{L}_3}{dt}$$

CONSIDER $\frac{d\vec{L}_1}{dt} + \frac{d\vec{L}_2}{dt}$ $\Rightarrow$ HAS A TERM

$$\boxed{\vec{r}_1 \times \vec{f}_{1,2} + \vec{r}_2 \times \vec{f}_{2,1}} \Rightarrow (\vec{r}_1 - \vec{r}_2) \times \vec{f}_{1,2}$$

AS $\vec{f}_{1,2} = -\vec{f}_{2,1}$ BUT NOTE



$\vec{r}_2 - \vec{r}_1$ IS PARALLEL TO $\vec{f}_{1,2}$ <u>SO</u>

$$\vec{r}_1 \times \vec{f}_{12} + \vec{r}_2 \times \vec{f}_{2,1}$$
$$= (\vec{r}_1 - \vec{r}_2) \times \vec{f}_{1,2} = 0$$

CROSS PRODUCT FOR PARALLEL VECTORS VANISHES!

THIS MEANS →

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{L}_1 + \vec{L}_2 + \vec{L}_3)$$

$$= \underbrace{\vec{r}_1 \times \vec{F}_{1,surr} + \vec{r}_2 \times \vec{F}_{2,surr} + \vec{r}_3 \times \vec{F}_{3,surr}}_{\text{NET TORQUE ON SYSTEM}}$$

$\cancel{\frac{d\vec{L}_{TOTA}}{dt} = \vec{\tau}}$

$$\boxed{\frac{d\vec{L}_A}{dt} = \vec{\tau}_{NET,A}}$$

FOR MULTI-PARTICLE SYSTEMS

SUPPOSE A IS MEASURED FROM <u>CENTER OF MASS</u>. $\vec{L}_A = \vec{L}_{TRANS,A} + \vec{L}_{ROT}$

$\vec{L}_{TRANS,A} = 0$ AND $\boxed{\frac{d\vec{L}_{ROT}}{dt} = \vec{\tau}_{NET,CM}}$

(INTERNAL FORCES ~~DO NOT~~ DO NOT CHANGE $\vec{L}_A$)

SUMMARY:
THREE PRINCIPLES

(1) $\vec{F}_N = \frac{d\vec{p}}{dt}$ MOMENTUM

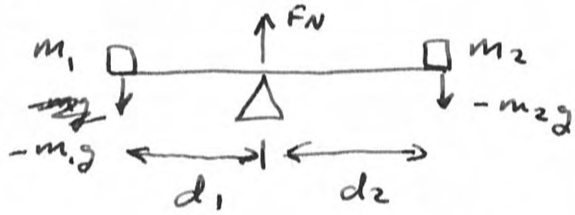
(2) $\Delta E_{sys} = W + Q$ ENERGY CONSERVATION

(3) $\frac{d\vec{L}_A}{dt} = \vec{\tau}_{Net,A}$ ANGULAR MOMENTUM

EXAMPLES: IF SYSTEM IS STATIC!

$$\frac{d\vec{L}_A}{dt} = 0$$ and there can be NO NET TORQUE.

$\vec{\tau}_A = 0$

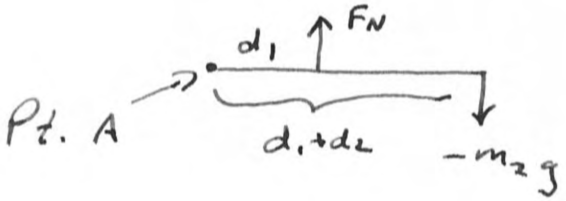


$m_1 = 90$ kg
$m_2 = 40$ kg
$d_1 = 1.2$ m $d_2 = ?$

NET TORQUE ABOUT A (BALANCE POINT) : $\Sigma\tau_A \Rightarrow$

$$+m_1 g d_1 - m_2 d_2 = 0 \qquad d_2 = \frac{m_1}{m_2} d_1 = \frac{90}{40}(1.2) = \underline{\underline{2.7\text{ m}}}$$

SUPPOSE WE TOOK A TO BE AT THE END OF THE SEESAW $\Rightarrow$



$$+F_N d_1 - (d_1 + d_2) m_2 g = 0$$

BUT $\boxed{F_N = +m_1 g + m_2 g}$

$$(m_1 g + m_2 g) d_1 - (d_1 + d_2) m_2 g$$

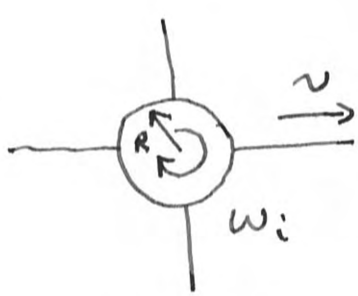
$$= m_1 g d_1 + \cancel{m_2 g d_1} - \cancel{m_2 g d_1} - m_2 g d_2 = 0$$

$m_1 d_1 = m_2 d_2$

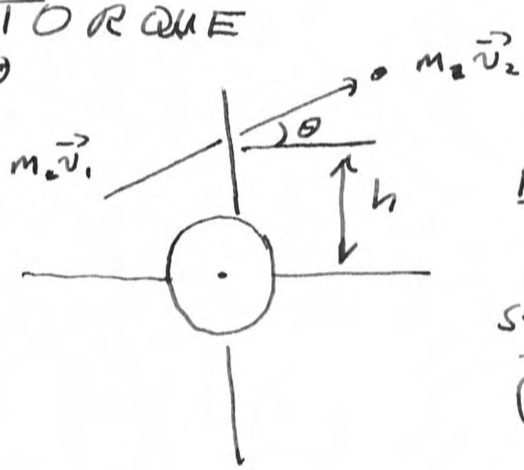
$$\boxed{d_2 = \frac{m_1}{m_2} d_1}$$

AS BEFORE! CHOICE OF POINT A CANNOT CHANGE PHYSICAL OUTCOME.

SYSTEM WITH ~~ZERO~~ TORQUE
~~(NON ZERO)~~
ZERO



METEOR HITS SOLAR PANEL



NO NET FORCE FROM SURROUNDINGS

$(\vec{\tau}_A = 0)$

SATELLITE SPINNING IN ORBIT: MASS $\underline{\underline{M}}$
VELOCITY
v in x-direction

[FIND (v_x, v_y) for SATELLITE AND ω_f]

RULES!

CONSERVE MOMENTUM : $\vec{P}_f = \vec{P}_i$

ORIGINAL VELOCITY $v \rightarrow$ x-direction

X-components

$$M v_x + m v_2 \cos\theta = M v + m v_1 \cos\theta$$

$$\boxed{v_x = v + \frac{m}{M}(v_1 - v_2)\cos\theta}$$

Y-COMPONENTS

$$M v_y + m v_2 \sin\theta = m v_1 \sin\theta$$

$$\boxed{v_y = \frac{m}{M}(v_1 - v_2)\sin\theta}$$

CONSERVE ANGULAR MOMENTUM (z-comp. out of paper)

$L_i = L_f$

$$\underbrace{I\omega_i + h v_1 m \sin(90-\theta)}_{\text{BEFORE COLLISION}} = \underbrace{I\omega_f + h m v_2 \sin(90-\theta)}_{\text{After Collision}}$$

$$\boxed{\omega_f = \omega_i + \frac{hm}{\left(\frac{2}{5}MR^2\right)}(v_1 - v_2)\cos\theta}$$

SATELLITE SPHERE

$I = \frac{2}{5} MR^2$

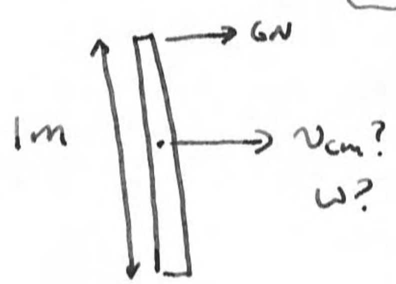
Note: THIS CAN BE WRITTEN

$$\omega_f = \omega_i + \frac{hm\left[\frac{M}{m}(v_x - v)\right]}{I} = \omega_i + \frac{hM(v_x - v)}{I}$$

$$\omega_f = \omega_i + \frac{\Delta L_z}{I} \Bigg\} \quad \Delta L_z = I\Delta\omega \quad \text{(MAKES SENSE)}$$

~~ΔLz~~

SYSTEMS WITH NON-ZERO TORQUE



$m = 0.3\,kg$

SUPPOSE WE HAVE a meter STICK $\Rightarrow$ 6N FORCE APPLIED TO END OF STICK.

WHAT IS $\frac{dv_{cm}}{dt}$? $\omega_i = ?$

CENTER OF MASS PICTURE

$F_{Net} = 6\ N$ $\quad \frac{dp}{dt} = m\frac{dv_{cm}}{dt} = F_N \quad a_{cm} = \frac{dv_{cm}}{dt} = \frac{F_N}{m}$

$a_{cm} = 20\ m/s^2$ $\quad = 6/0.3$

ANGULAR MOMENTUM

$\frac{dL_{rot}}{dt} = \tau \qquad I\frac{d\omega}{dt} = |\vec{r}_{cm} \times \vec{F}| = rF\sin\theta$

$= \frac{1}{2} 6 (1) = 3\ N\cdot m$

$I = \frac{1}{12}ML^2 = \frac{1}{12}(0.3)(1)^2$

$= 0.025\ kg/m^2$

$\frac{d\omega}{dt} = \frac{3}{0.025} = 120\ rad/s^2$

END COURSE !