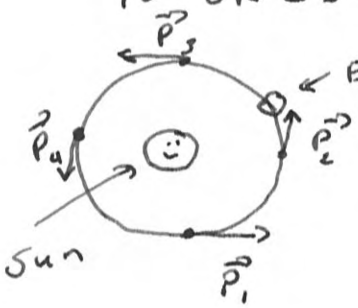
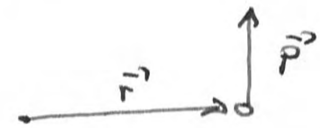
CHAPTER 11 (LAST ONE!)

ANGULAR MOMENTUM → RELATED TO ORBITAL MOTION



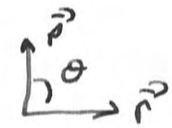
Momentum of the earth changes in a CONSTANT FASHION.

DEFINE: $L = rp\sin\theta$



Note: $\vec{r} \perp \vec{p}$ (always) so

$L = pr = $ CONSTANT



$\theta = 90°$

L ⇒ ANGULAR MOMENTUM

VEL. of earth $= \dfrac{2\pi r}{T} = \dfrac{2(3.14)(1.5\times10^{11})}{86{,}400 \text{ s} \times 365}$

↙ r

↖ T (86,400 sec per day × 365 days)

$v = 3\times10^{4}$ m/s

$m_E = 6\times10^{24}$ kg

$p = 1.8\times10^{29}$ kg·m/s

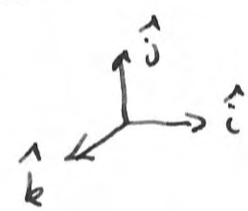
$L = rp = (1.5\times10^{11})(1.8\times10^{29}) = 2.7\times10^{40}$ kg·m²/s

(UNITS OF L ⇒ kg-m²/s)

L as a VECTOR ⇒ $\boxed{\vec{L} \equiv \vec{r}\times\vec{p}}$ CROSS PRODUCT

DEFINITION

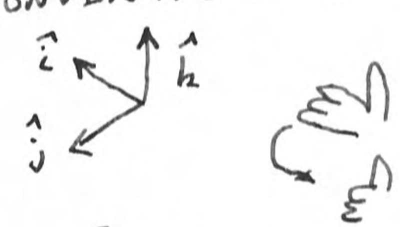
$\vec{C} = \vec{A}\times\vec{B}$ $C \equiv AB\sin\theta$



Note: $\hat{i}\times\hat{i} = 0$ BECAUSE $\theta = 0$ and $\sin\theta = 0$.

$\hat{i}\times\hat{j} = \hat{k}$ $k = ij\sin\theta = (1)(1)\sin 90 = 1$

BY DEFINITION $\hat{k}$ is perpendicular to both $\hat{i}$ and $\hat{j}$. SIGN CONVENTION GIVEN BY RIGHT HAND RULE



(SEE p. 418)

Rules: $\hat{i}\times\hat{i}=0$, $\hat{i}\times\hat{j}=\hat{k}$, $\hat{j}\times\hat{i}=-\hat{k}$ } CAN FIGURE ALL FROM THIS.

[TRICK
ijkijkijk
$\rightarrow$ (+) e.g. $jk\rightarrow +$ $ij\rightarrow +$
$ki\rightarrow +$]

EXAMPLE:

$$\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\qquad \vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$$

$$\vec{A}\times\vec{B}=(A_x\hat{i}+A_y\hat{j}+A_z\hat{k})\times(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$$

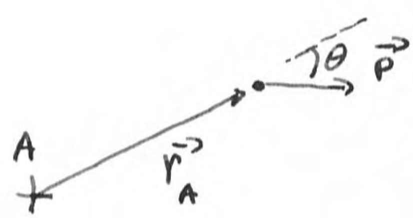
$$=A_xB_y\hat{k}-A_xB_z\hat{j}+(-A_yB_x\hat{k})+A_yB_z\hat{i}$$
$$+A_zB_x\hat{j}-A_zB_y\hat{i}$$

$$\boxed{\vec{A}\times\vec{B}=(A_yB_z-A_zB_y)\hat{i}+(A_zB_x-A_xB_z)\hat{j}+(A_xB_y-A_yB_x)\hat{k}}$$

EXAMPLE: $\vec{A}=\langle 2,3,4\rangle$ $\vec{B}=\langle 5,6,7\rangle$

$$\vec{A}\times\vec{B}=\langle 3\cdot7-4\cdot6,\ 4\cdot5-2\cdot7,\ 2\cdot6-3\cdot5\rangle$$
$$=\langle -3,6,-3\rangle$$

TRANSLATIONAL (ORBITAL) ANGULAR MOMENTUM $\Rightarrow$ TRANSLATION Center of mass moving

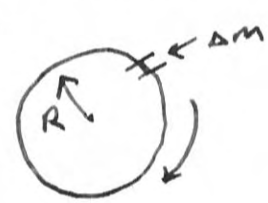


$$\vec{L}_A=\vec{r}_A\times\vec{p}$$

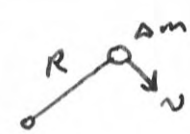
$$L_A=r_Ap\sin\theta$$

$$\vec{L}_A=\langle r_yp_z-r_zp_y,\ r_zp_x-r_xp_z,\ r_xp_y-r_yp_x\rangle$$

WHAT ABOUT MOMENTUM associated WITH ROTATION? CENTER OF MASS FIXED.



WHEEL



$v=\omega R$

$v\perp R$ $\quad L(\Delta m)=R\Delta p$

$$=R\Delta m v=\Delta m R^2\omega$$

$$L=\sum L(\Delta m)=\left(\sum\Delta m R^2\right)\omega$$
$$\sum\Delta m R^2 = I=MR^2$$

$$\boxed{L_{ROT}=I\omega}$$

NEW DEFINITION: $\vec{\omega}$ ⟹ ω as a vector.

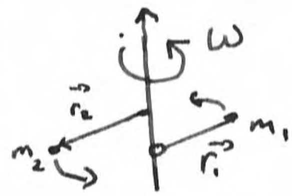
DEFINITION
$\vec{\omega}$ defined
by RH Rule

$$\boxed{\vec{L}_{ROT} = I\vec{\omega}}$$

DERIVATION

$$\boxed{v = r_\perp \omega}$$

~~This implies~~



$$|\vec{r}\times\vec{p}| = r_\perp m v$$
$$= r_\perp m\, r_\perp \omega$$
$$= (m r_\perp^2)\,\omega$$

FOR ~~ALL~~ A NUMBER OF PARTICLES ⟶

$$L_{rot} = \left[m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \dots\right]\omega = I\omega \Rightarrow \vec{L}_{rot} = I\vec{\omega}$$

So if we know I we can find L_{rot}

ROTATIONAL KINETIC ENERGY ⟹

$$K_{ROT} = \frac{1}{2} I\omega^2 = \frac{1}{2}\frac{(I\omega)^2}{I} = \frac{L_{rot}^2}{2I}$$

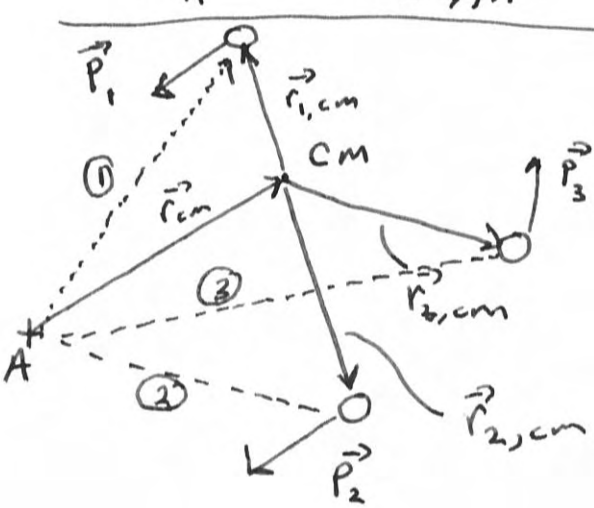
$$K_{TOT} = K_{TRAN} + \underbrace{K_{REL}}_{\text{(RELATIVE TO CENTER OF MASS)}} = K_{TRANS} + K_{rot} + K_{vib}$$

SIMILAR ISSUES FOR $\vec{L}$ → Can we BREAK UP $\vec{L}$?

WE CAN WRITE

$$\vec{L}_A = \vec{L}_{TRANS,A} + \vec{L}_{ROT.}$$

EXAMPLE



VECTORS:

1. $\vec{r}_{cm} + \vec{r}_{1,cm}$
2. $\vec{r}_{cm} + \vec{r}_{2,cm}$
3. $\vec{r}_{cm} + \vec{r}_{3,cm}$

$\vec{L} \Rightarrow$ Add up contributions

$\vec{L}_1 = (\vec{r}_{cm} + \vec{r}_{1,cm}) \times \vec{p}_1 \quad \vec{L}_2 = (\vec{r}_{cm} + \vec{r}_{2,cm}) \times \vec{p}_2 \quad \vec{L}_3 = (\vec{r}_{cm} + \vec{r}_{3,cm}) \times \vec{p}_3$

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 = \underbrace{\vec{r}_{cm} \times (\underbrace{\vec{p}_1 + \vec{p}_2 + \vec{p}_3}_{\vec{p}_T})}_{\text{TRANSLATIONAL}} + \underbrace{[\vec{r}_{1,cm} \times \vec{p}_1 + \vec{r}_{2,cm} \times \vec{p}_2 + \vec{r}_{3,cm} \times \vec{p}_3]}_{\text{ROTATONAL}}$$

$$\boxed{\vec{L}_{A,TRANS} = \vec{r}_{cm} \times \vec{P}_{TOTAL}}$$

Depends on A

$$\boxed{\vec{L}_A = \vec{L}_{TRANS,A} + \vec{L}_{ROT}}$$

$$\boxed{\vec{L}_{ROT} = [\vec{r}_{1,cm} \times \vec{p}_1 + \vec{r}_{2,cm} \times \vec{p}_2 + \vec{r}_{3,cm} \times \vec{p}_3]}$$

DOES NOT DEPEND ON A.

ANGULAR MOMENTUM PRINCIPLE

CONSIDER $\vec{r}_A \times \vec{p}$

$$\frac{d(\vec{r}_A \times \vec{p})}{dt} = \underbrace{\frac{d\vec{r}_A}{dt} \times \vec{p}}_{\vec{v} \times \vec{p} = m[\vec{v} \times \vec{v}] = 0} + \vec{r}_A \times \frac{d\vec{p}}{dt} = \vec{r}_A \times \vec{F}_N$$

RECALL $\frac{d\vec{P}}{dt} = \vec{F}_N$

(TRUE IF $\vec{p} = \gamma m \vec{v}$ also)

DEFINE: $\boxed{\vec{\tau}_A \equiv \vec{r}_A \times \vec{F}}$ CALLED TORQUE

SINCE $\vec{L}_A = \vec{r}_A \times \vec{P} \Rightarrow \boxed{\frac{d\vec{L}_A}{dt} = \vec{\tau}_A}$ FOR A PARTICLE

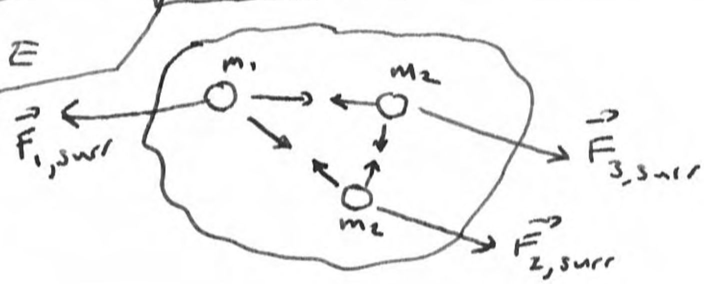
$\vec{\tau}_A = \vec{r}_A \times \vec{F}$

OUT OF PLANE OF PAPER

CONSERVATION OF ANGULAR MOMENTUM

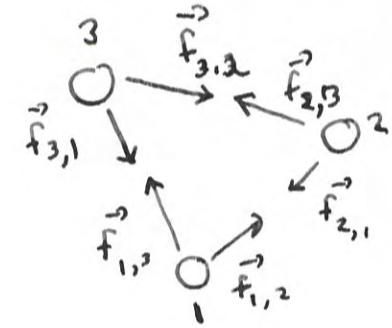
$$\Delta \vec{L}_{A,system} + \Delta \vec{L}_{A,surroundings} = 0$$

MULTIPARTICLE SYSTEMS



SIMPLE SYSTEM

L25
P5

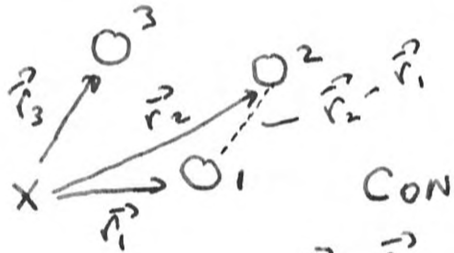


INTERNAL
Forces : REMEMBER $\left\{ \begin{array}{l} \vec{f}_{2,1} = -\vec{f}_{1,2}, \\ \vec{f}_{3,1} = -\vec{f}_{3,1} \\ \vec{f}_{2,3} = -\vec{f}_{3,2} \end{array} \right.$

$$\frac{d\vec{L}_1}{dt} = \vec{r}_1 \times (\vec{F}_{1,surr} + \vec{f}_{1,2} + \vec{f}_{1,3})$$

$$\frac{d\vec{L}_2}{dt} = \vec{r}_2 \times (\vec{F}_{2,surr} + \vec{f}_{2,1} + \vec{f}_{2,3})$$

$$\frac{d\vec{L}_3}{dt} = \vec{r}_3 \times (\vec{F}_{3,surr} + \vec{f}_{3,1} + \vec{f}_{3,2})$$



CONSIDER $\vec{r}_1 \times \vec{f}_{1,2} + \vec{r}_2 \times \vec{f}_{21}$

$(\vec{r}_1 - \vec{r}_2) \times \vec{f}_{21} = 0$

BUT $\vec{r}_1 - \vec{r}_2$ IS ALONG A LINE FROM 1-2. ~~BA~~ SO $\vec{r}_1 - \vec{r}_2 \parallel \vec{f}_{12}$

CROSS PRODUCT OF $(\vec{r}_1 - \vec{r}_2) \times \vec{f}_{12} = 0$!

ONLY EXTERNAL OR ~~RE~~ SURROUNDING FORCES SURVIVE

$$\boxed{\frac{d\vec{L}_{TOT,A}}{dt} = \vec{\tau}_{net,A}}$$

$$\boxed{\Delta \vec{L}_{TOT,A} = \vec{\tau}_{net,A}\, \Delta t}$$

DIFFERENCE EQUATION

Suppose A is CENTER OF MASS? $\rightarrow \frac{d\vec{L}_{cm}}{dt} = \frac{d\vec{L}_{rot}}{dt}$

~~NOT~~ NO TRANSLATIONAL $\vec{L}$

SUMMARY

THREE PRINCIPLES:

(1) $\frac{d\vec{p}}{dt} = \vec{F}_{NET}$ Momentum

(2) $\frac{d\vec{L}_A}{dt} = \vec{\tau}_{net,A}$ Angular Momentum

(3) $\Delta E = W + Q$ ENERGY