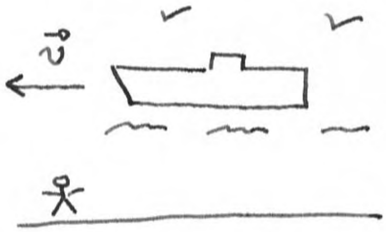
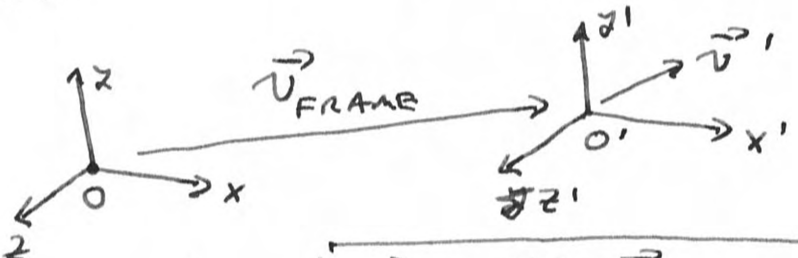
FRAME OF REFERENCE



PERSON ON LAND SEES BOAT MOVE WITH VELOCITY $\vec{v}$.
CAPTAN ON BOAT SEES PERSON ON LAND MOVE $-\vec{v}$. WHO'S RIGHT?
BOTH!

$\vec{v}_{frame}$ = FRAME OF REFERENCE VELOCITY

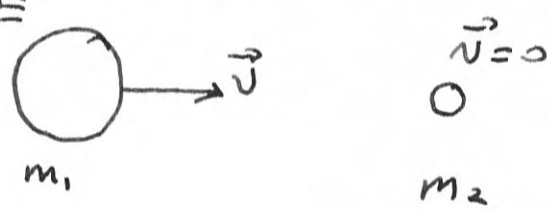


$$\boxed{\vec{v} = \vec{v}' + \vec{v}_{frame}}$$

EXAMPLE: CASE $m_1 >> m_2$ m_1 = Bowling Ball
m_2 = Ping Pong Ball.

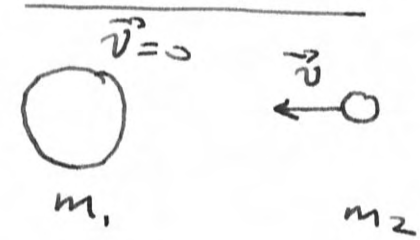
~~OBE~~ OBSERVER A

BEFORE

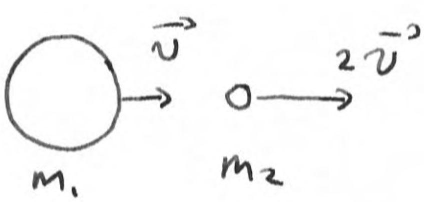


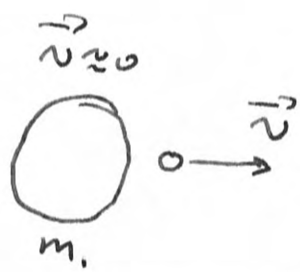
OBSERVER B

~~AB~~ BEFORE



AFTER





$$\vec{v}_B' = \vec{v}_A - \vec{v}_{FRAME}$$

~~BEFROT~~

$\vec{v}_B'$ IS B'S VELOCITY AS SEEN BY ~~A~~ B. (Assume we are A)

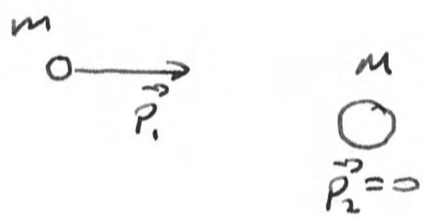
$\vec{v}_{frame} = -\vec{v}$

	BEFORE IN A's WORLD	$\vec{v}_B' = \vec{v}_A - \vec{v}_{FRAM}$
	$m_1 \quad \vec{v}$	$m_1 \quad 0$
	$M_2 \quad 0$	$m_2 \quad -\vec{v}$
AFTER	$m_1 \quad \vec{v}$	$M_1 \quad 0$
	$M_2 \quad 2\vec{v}$	$m_2 \quad \vec{v}$

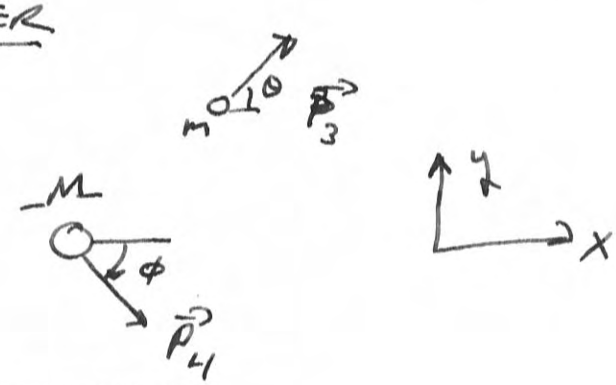
NO PHYSICS CHANGES ⇒ VIEW ~~For~~ FROM A OR B IS EQUALLY VALID

SCATTERING : COLLISIONS

BEFORE



AFTER



NOTE: WE DON'T CARE IF THE PARTICLES COLLIDE BY "CONTACT".

$$\boxed{\vec{P}_i = \vec{P}_f}$$

$$\vec{P}_i = \vec{P}_1 + \underbrace{\vec{P}_2}_{=0} = \vec{P}_1$$

$$\vec{P}_f = \vec{P}_3 + \vec{P}_4$$

X-COMPONENT

$$P_1 = P_3 \cos\theta + P_4 \cos\phi$$

y - component

$$0 = P_3 \sin\theta - P_4 \sin\phi$$

Momentum

WHAT ABOUT KE?

$$\boxed{\frac{P_1^2}{2m} = \frac{P_3^2}{2m} + \frac{P_4^2}{2M}}$$

WE HAVE FOUR UNKNOWNS (P_3, P_4) (ϕ, θ) BUT 3-Equations. NEED TO KNOW $(\phi$ or $\theta)$.

How does this work?

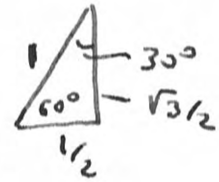
$M = m \quad \theta = 60 \leftarrow$ specified

Momentum Conserved

$\vec{P}_1 = \vec{P}_3 + \vec{P}_4$

$$\boxed{\begin{aligned} P_1 &= \tfrac{1}{2} P_3 + P_4 \cos\phi \qquad (1) \\ 0 &= \tfrac{\sqrt{3}}{2} P_3 - P_4 \sin\phi \qquad (2) \end{aligned}}$$

TRIG REMINDER



$\sin(60) = \sqrt{3}/2$

$\cos(60) = 1/2$

$\sin(30) = 1/2$

$\cos(\cancel{60}\ 30) = \sqrt{3}/2$

KINETIC ENERGY CONSERVED

$$\boxed{P_1^2 = P_3^2 + P_4^2} \quad (3)$$

REWRITE (2)

$$P_3 = \frac{2}{\sqrt{3}} P_4 \sin\phi$$

COMBINE (1) + (2)

$$\left(\frac{P_1}{P_4}\right)^2 = \frac{1}{3}\sin^2\phi + \frac{2}{\sqrt{3}}\sin\phi\cos\phi + \cos^2\phi \qquad (5)$$

COMBINE (2) + (3)

$$\left(\frac{P_1}{P_4}\right)^2 = 1 + \frac{4}{3}\sin^2\phi \qquad (6)$$

EQUATE (5) & (6)

$$\frac{1}{3}\sin^2\phi + \frac{2}{\sqrt{3}}\sin\phi\cos\phi + \cos^2\phi = 1 + \frac{4}{3}\sin^2\phi \qquad (7)$$

REWRITE (7) using $\sin^2\phi + \cos^2\phi = 1 \quad \tan\phi = \dfrac{\sin\phi}{\cos\phi}$

$$1 + \sin^2\phi - \cos^2\phi = \frac{2}{\sqrt{3}}\sin\phi\cos\phi \Rightarrow \tan\phi = \frac{1}{\sqrt{3}}$$

NOTE: $\boxed{\theta + \phi = 90°}$

$$\phi = 30°$$

$$P_3 = \frac{2}{\sqrt{3}} P_4 \underbrace{\sin\phi}_{\frac{1}{2}} = \frac{1}{\sqrt{3}} P_4 \Rightarrow P_1^2 = \frac{1}{3}P_4^2 + P_4^2 = \frac{4}{3}P_4^2$$

$$\boxed{\begin{aligned} P_4 &= \frac{\sqrt{3}}{2} P_1 \\ P_3 &= \frac{1}{2} P_1 \end{aligned}}$$

SIMPLE (NOT MESSY) RULE

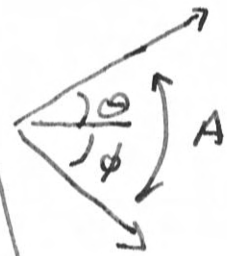
$\vec{P}_1 = \vec{P}_3 + \vec{P}_4$ CONSERVATION OF MOMENTUM

$P_1^2 = P_3^2 + P_4^2$ CONSERVATION OF ENERGY

$$\vec{P}_1 \cdot \vec{P}_1 = P_1^2 = (\vec{P}_3 + \vec{P}_4) \cdot (\vec{P}_3 + \vec{P}_4) = P_3^2 + P_4^2 + 2\vec{P}_3 \cdot \vec{P}_4$$

$$P_1^2 = P_3^2 + P_4^2 \qquad P_1^2 = P_3^2 + P_4^2 + 2\vec{P}_3 \cdot \vec{P}_4$$

$$\boxed{\vec{P}_3 \cdot \vec{P}_4 = 0}$$



$$\vec{P}_3 \cdot \vec{P}_4 = P_3 P_4 \cos A$$

$\cos(A) = 0$ MEANS

$$\boxed{\theta + \phi = A = 90°}$$

END
CHPT 10

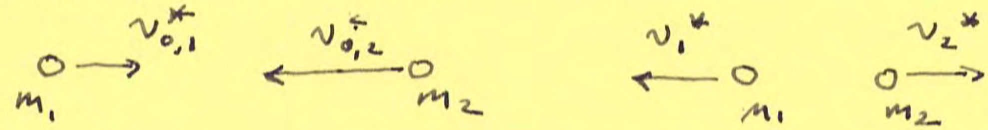
CENTER OF MASS : SOLUTION

(E1)

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} v_0 \qquad v_2 = \frac{2m_1}{m_1 + m_2} v_0 \qquad v_{cm} = \frac{m_1 v_0}{m_1 + m_2}$$

LAB FRAME

CENTER OF MASS FRAME



NOTE $\left.\begin{bmatrix} v_1^* = -v^*_{0,1} \\ v_2^* = -v^*_{0,2} \end{bmatrix}\right\}$ PROOF $\rightarrow$ $\boxed{m_1 v^*_{0,1} + m_2 v^*_{0,2} = 0}$

$$+ m_1 v_1^* + m_2 v_2^*$$
$$= -(m_1 v^*_{0,1} + m_2 v^*_{0,2})$$
$$= 0$$

So $\boxed{m_1 v_1^* + m_2 v_2^* = 0}$

$$\underbrace{\tfrac{1}{2} m v^*_{0,1} + \tfrac{1}{2} m v^*_{0,2}}_{K_i} = \tfrac{1}{2} m_1 (-v^*_{0,1})^2 + \tfrac{1}{2} m_2 (-v^*_{0,2})^2$$
$$= \tfrac{1}{2} m_1 v_1^* + \tfrac{1}{2} m_2 v_2^* = K_f$$

So $\boxed{K_i = K_f}$ and $\boxed{P_f = P_i = 0}$ THIS PROVES $v_1^* = -v^*_{0,1}$ and $v_2^* = -v^*_{0,2}$

CROSS CHECK $\quad v^* = v_{Lab} - v_{cm}$

$$v^*_{0,1} = v_0 - v_{cm} \qquad v^*_{0,2} = -v_{cm}$$

$$v_1^* = -v^*_{0,1} \qquad v_2^* = +v_{cm}$$

$$v_1^* = v_1 - v_{cm} \qquad v_2^* = v_2 - v_{cm}$$

$$[v_{cm} - v_0] = v_1 - v_{cm} \qquad v_2 = v_{cm} + v_2^* = 2v_{cm}$$

$$\boxed{v_1 = 2v_{cm} - v_0} \qquad \boxed{v_2 = 2v_{cm}}$$

CHECK $\quad v_1 = \dfrac{2m_1 v_0}{m_1 + m_2} - v_0 \qquad \boxed{v_2 = \dfrac{2m_1 v_0}{m_1 + m_2}}$

$$\boxed{v_1 = \frac{m_1 - m_2}{m_1 + m_2} v_0}$$