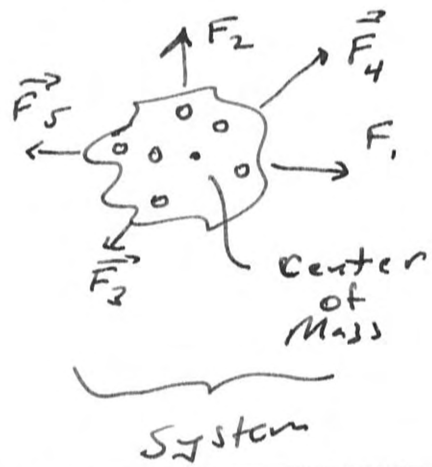
POINT PARTICLE PICTURE

RECALL CENTER OF MASS $\rightarrow$ $\vec{P}_{sys} = M\vec{v}_{cm}$

M: TOTAL MASS

$$\frac{d\vec{P}_{sys}}{dt} = \vec{F}_{Net}$$



$$\vec{F}_{Net} = \sum_i \vec{F}_i$$

Center of Mass – Looks like a <u>point</u> <u>particle</u> moves under influence of $\vec{F}_{Net}$.

RECALL FOR A POINT PARTICLE

$K = K_{TRANSLATIONAL}$

and

$\Delta K_{TRANS} = W_p \leftarrow$ WORK DONE ON POINT PARTICLE

$$\boxed{W_p = \int_i^f \vec{F}_{Net} \cdot d\vec{r}_{cm}}$$

SO $\boxed{\Delta K_{TRANS} = \int_i^f \vec{F}_{Net} \cdot d\vec{r}}$

REAL SYSTEM

{ CONSIDER ALL PARTICLES: INFORMATION ON CHANGE OF TOTAL ENERGY OF SYSTEM }

$$\Delta E_{sys} = W + Q$$

Most of the time we will take $Q = 0$

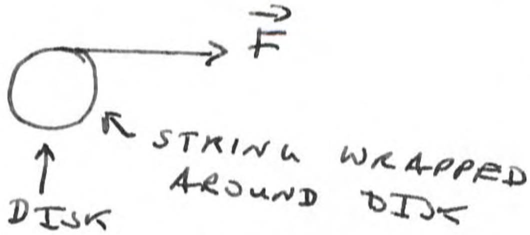
$$\boxed{\Delta E_{sys} = W}$$

WHEN APPLIED TO TO THE <u>SYSTEM AT LARGE</u>

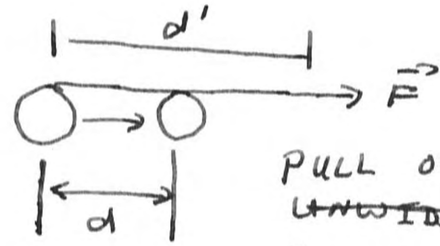
$$\Delta E_{sys} = \Delta K_{cm} + \underbrace{\Delta K_{rel}} = W_{real}$$

$$\Delta K_{rel} = \Delta K_{rot} + \Delta K_{vib} + \cdots$$

SOME EXAMPLES!



[THINK HOCKEY PUCK]



PULL ON STRING.
~~UNWIDS~~ UNWINDS
SO STRING EXTENDS
d', but puck moves d.

QUESTION:
WHAT IS d?
[RELATIVE TO d']

CENTER OF MASS MOVES
FROM INITIAL POSITION BY d
So $\Delta K_{Trans} = Fd$

WORK on CM →
$F \cdot d = W_{point}$

REAL SYSTEM

$\Delta K_{TOTAL} = Fd'$ → TOTAL WORK DONE ON SYSTEM

$\Delta K = \Delta K_{Trans} + \Delta K_{rot}$ ← DISK CAN "ROTATE" as pulled.

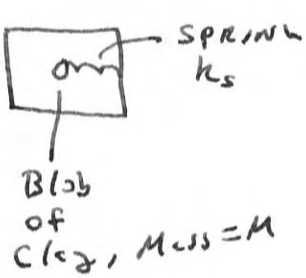
$\Delta K_{TRANS} = \frac{1}{2} m v_{cm}^2$
$\Delta K_{rot} = \frac{1}{2} I \omega^2$
} INITIALLY NO KINETIC ENERGY ⇒ $\Delta K_{Trans} = K_{trans}$, $\Delta K_{rot} = K_{rot}$

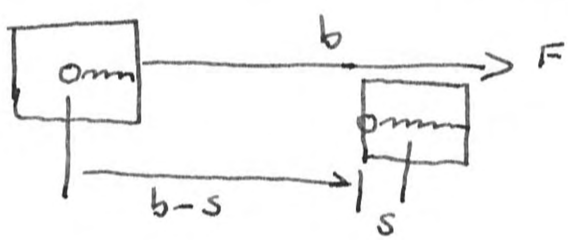
$\omega = v_{cm} \cdot R$ $\quad \Delta K_{rot} = \frac{1}{2}\left[\frac{1}{2} M R^2\right] \frac{v_{cm}^2}{R^2} = \frac{1}{4} M v_{cm}^2$

$$\frac{Fd'}{Fd} = \frac{K_{rot} + K_{cm}}{K_{cm}} = \frac{\frac{1}{4} M v_{cm}^2 + \frac{1}{2} M v_{cm}^2}{\frac{1}{2} M v_{cm}^2}$$

$$\boxed{\frac{d'}{d} = \frac{3}{2}} \quad \text{or} \quad \boxed{d = \frac{2}{3} d'}$$

ANOTHER EXAMPLE: BOX WITH CLAY





Clay STICKS TO BOX → BOX has "No Mass"

NO GRAVITY!

QUESTION: WHEN BOX TRAVELS A DISTANCE b, clay smacks and sticks ~~box~~ back end of box. What's the velocity of box?

$$\Delta K_{TRANS} = \frac{1}{2} m v_{cm}^2 = F(b-s)$$

Note: Center of mass moves less than b.

$$\boxed{v_{cm} = \sqrt{\frac{2F(b-s)}{m}}}$$

Did clay have its internal energy increase?

$\Delta K = W_{sys}$ $\qquad \Delta K = \Delta K_{trans.} + U_{spring} + \Delta E_{int}$

$W_{sys} = Fb \Rightarrow \Delta K = F(b-s) + \frac{1}{2} k_s s^2 + \Delta E_{int} = Fb$

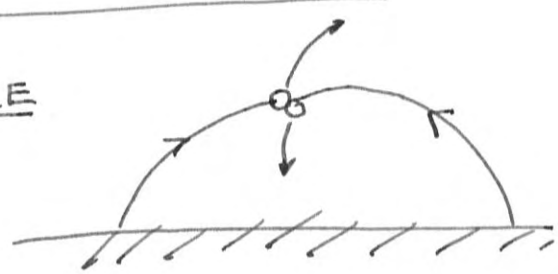
$$\boxed{\Delta E_{int} = Fs - \frac{1}{2} k_s s^2}$$

Note: $Fs > \frac{1}{2} k_s s^2$ as $\Delta E_{int} > 0$

END CHAPTER 9

CHAPTER 10: COLLISIONS

EXAMPLE



TWO BALLS HIT

A COLLISION INVOLVES STRONGLY INTERACTING (LARGE FORCES) ACTING FOR A SHORT TIME

ELASTIC AND INELASTIC COLLISIONS

ELASTIC → NO CHANGE IN THE INTERNAL ENERGY $\boxed{\Delta E_{int} = 0}$

$\Delta K = 0 \qquad \boxed{K_f = K_i}$

} EXAMPLE BILLIARD BALLS

INELASTIC → $\Delta E_{int} \neq 0$, $K_f \neq K_i$ } "REAL" CALCULATIONS

All collisions are "inelastic" as $\Delta E_{int} = 0$ can ~~ever~~ never really happen - limiting cases $\Delta E_{int} \to 0$ close to "ELASTIC"

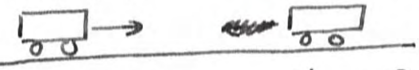
"MAXIMALLY" INELASTIC – PARTICLES
<u>STICK</u> TOGETHER

$\boxed{K_f \neq K_i}$

STILL $\boxed{\vec{P}_i = \vec{P}_f}$

EXAMPLE ELASTIC COLLISION

~~ENDS~~ IDENTICAL CARTS



$v_{i,1} = v_0$ $\quad v_{i,2} = 0$

WHAT HAPPENS AFTER COLLISION? $\quad \vec{P}_i = \vec{P}_f \quad K_f = K_i$

$$m v_0 = m v_1 + m v_2 \qquad \tfrac{1}{2} m v_0^2 = \tfrac{1}{2} m v_1^2 + \tfrac{1}{2} m v_2^2$$

Note: mass all cancel out $\quad v_0 = v_1 + v_2 \quad v_0^2 = v_1^2 + v_2^2$

$$v_0^2 = v_1^2 + 2 v_1 v_2 + v_0^2 = v_1^2 + v_2^2$$

Solution: $v_1 v_2 = 0$

MEANS EITHER v_1 or $v_2 = 0$

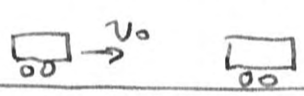
[Both cannot be zero unless $v_0 = 0$!]

Suppose $\boxed{v_1 = 0}$ then $v_0 = \cancel{v_1} + v_2$ $\quad \boxed{v_2 = v_0}$

Can $v_2 = 0$? Then $\Rightarrow v_1 = v_0$ and $v_2 = 0$ THIS SOLUTION IS "UNPHYSICAL" AS IT WOULD SUGGEST CART #1 "PASSES" RIGHT THROUGH CART #2

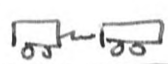
INELASTIC COLLISION

BEFORE



AFTER

$v = ?$



STILL $P_f = P_i$

$m v_0 = 2 m v_f$ $\quad \boxed{V_f = \tfrac{1}{2} v_0}$

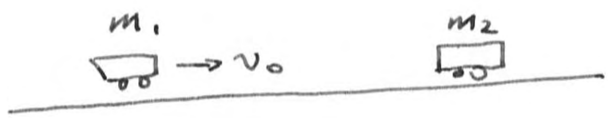
$$K_i = \tfrac{1}{2} m v_0^2$$

$$K_f = \tfrac{1}{2} (2m) v_f^2 = m \left[\tfrac{1}{4} v_0^2 \right] = \tfrac{1}{4} m v_0^2$$

$$K_f - K_i = -\Delta E_{int} = \tfrac{-1}{4} m v_0^2$$

} WENT INTO INCREASING INTERNAL ENERGY

$$\boxed{\Delta E_{int} = \tfrac{1}{4} m v_0^2}$$



Suppose ELASTIC COLLISION WITH DIFFERENT MASSES

INITIALLY

$$P_i = m_1 v_0$$
$$K_i = \tfrac{1}{2} m_1 v_0^2$$

FINAL

$$P_f = m_1 v_1 + m_2 v_2$$
$$K_f = \tfrac{1}{2} m_2 v_1^2 + \tfrac{1}{2} m_2 v_2^2$$

$$\boxed{\begin{aligned} m_1 v_0 &= m_1 v_1 + m_2 v_2 \\ m_1 v_0^2 &= m_1 v_1^2 + m_2 v_2^2 \end{aligned}}$$

$$v_0 - v_1 = \cancel{\frac{m_2}{m_1} v_2} \; \frac{m_2}{m_1} v_2$$
$$v_0^2 - v_1^2 = \frac{m_2}{m_1} v_2^2$$

$$v_0^2 - v_1^2 = (v_0 + v_1)(v_0 - v_1)$$

$$(v_0 + v_1)\underbrace{(v_0 - v_1)}_{\frac{m_2}{m_1} v_2} = \frac{m_2}{m_1} v_2^2$$

$$(v_0 + v_1) = v_2$$

$$\boxed{v_2 - v_1 = v_0} \quad (1)$$

$$\boxed{\frac{m_2}{m_1} v_2 + v_1 = v_0} \quad (2)$$

Add (1) + (2)

$$\left(1 + \frac{m_2}{m_1}\right) v_2 = 2 v_0$$

$$\boxed{v_2 = \frac{2 m_1 v_0}{m_1 + m_2}}$$

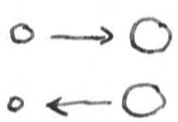
$$v_1 = v_2 - v_0$$
$$= \left[\frac{2 m_1 v_0}{m_1 + m_2}\right] - v_0$$

$$v_1 = \left[\frac{2 m_1 - (m_1 + m_2)}{m_1 + m_2}\right] v_0$$

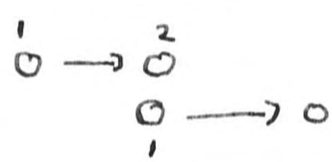
$$\boxed{v_1 = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_0}$$

Limiting cases:

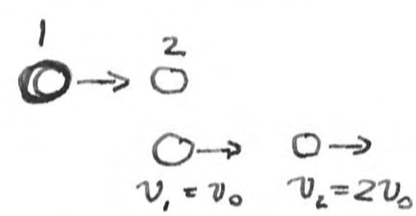
$$\boxed{\begin{aligned} & m_2 \gg m_1 \\ & v_2 \approx 0 \\ & v_1 = -v_0 \end{aligned}} \qquad \boxed{\begin{aligned} & m_1 = m_2 \\ & v_2 = v_0 \\ & v_1 = 0 \end{aligned}} \qquad \boxed{\begin{aligned} & m_1 \gg m_2 \\ & v_2 = 2 v_0 \\ & v_1 = v_0 \end{aligned}}$$



PING-PONG Ball collides with Bowling Ball



AS BEFORE



NOTE:
$m_2 \rightarrow 2 v_0$!