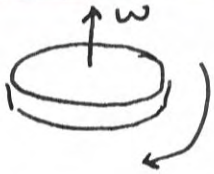
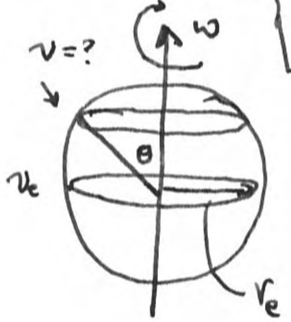
ROTATIONAL KINETIC ENERGY



DISK ROTATES (CONSTANT ANGULAR VELOCITY)
(SPEED)

$$\omega = \frac{2\pi}{T}$$

$S = 2\pi r$ ← Distance Traveled in T → $v = \frac{2\pi r}{T} = \omega r$ $\quad \boxed{v = \omega r}$



EARTH → $r = \cancel{4.6}\,6.4 \times 10^6\,m$

$2\pi r_e = 4.02 \times 10^{7}\,m$

$T = 24\,hrs \Rightarrow = 8.64 \times 10^4\,s$

$$v = \frac{4.02 \times 10^{7}}{8.64 \times 10^4}\,\frac{m}{s} = \underline{460\,m/s}$$

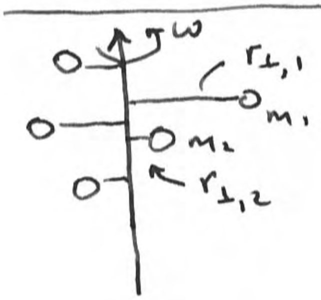
$\sin\theta = \frac{r_\perp}{r_e}$ $\quad \theta = 45$ $\quad r_\perp = \frac{1}{\sqrt{2}} r_e \Rightarrow$

$\boxed{v = \omega r_\perp} \Rightarrow v = 325\,m/s$ at $\theta = 45°$

IN GENERAL ⇒

$$\boxed{v = \omega r_\perp}$$

KINETIC ENERGY

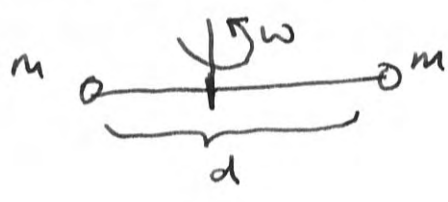


$$K_{rot} = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 + \ldots$$
$$= \tfrac{1}{2} m_1 \omega^2 r_{\perp,1}^2 + \tfrac{1}{2} m_2 \omega^2 r_{\perp,2}^2 + \ldots$$
$$= \tfrac{1}{2} \underbrace{\left[m_1 r_{\perp,1}^2 + m_2 r_{\perp,2}^2 + \ldots \right]}_{\text{MOMENT OF INERTIA } (I)} \omega^2$$

$$I = m_1 r_{\perp,1}^2 + m_2 r_{\perp,2}^2 + \ldots$$

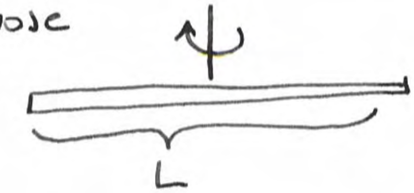
$$\boxed{I = \sum_i m_i r_{\perp,i}^2} \qquad \boxed{K_{rot} = \tfrac{1}{2} I \omega^2}$$

SIMPLE EXAMPLE

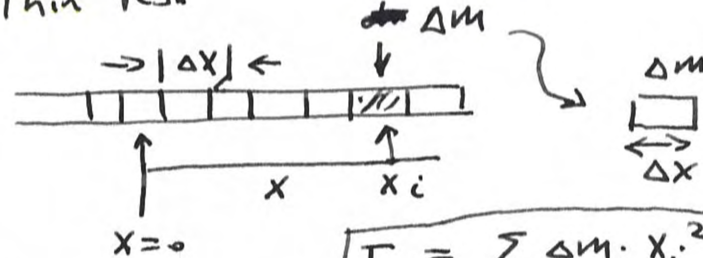


$$I = m\left(\tfrac{d}{2}\right)^2 + m\left(\tfrac{d}{2}\right)^2$$
$$= \tfrac{1}{2} m d^2$$
$$\boxed{K_{rot} = \tfrac{1}{4} m d^2 \omega^2}$$

Suppose



UNIFORM Thin Rod



$\boxed{\text{WHAT IS } I\,?}$

$$\boxed{I = \sum_i \Delta m_i\, x_i^2}$$

But Rod Is UNIFORM

So $\Delta m_i = \Delta m$

$$\rho = \text{DENSITY} = \frac{M}{L} \begin{matrix} \leftarrow \text{Mass Rod} \\ \leftarrow \text{Length} \end{matrix}$$

$$\rho = \frac{M}{L} \Rightarrow \boxed{\Delta m = \rho\, \Delta x}$$

$$\boxed{\Delta m = \frac{M}{L} \Delta x}$$

$$I = \frac{M}{L} \sum_i x_i^2 \Delta x$$

Let $\Delta x \to 0$

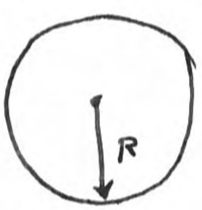
$$\boxed{I = \frac{M}{L} \int_{-L/2}^{+L/2} x^2\, dx}$$

$$I = \frac{M}{L} \left.\frac{x^3}{3}\right]_{-L/2}^{L/2}$$

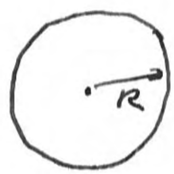
$$= \frac{M}{L}\left[\frac{L^3}{24} - \frac{-L^3}{24}\right]$$

$$\boxed{I = \frac{1}{12} M L^2}$$

ANOTHER EXAMPLE



← DISK



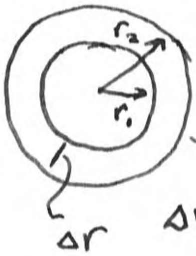
← RING

ALL MASS IS AT R

$$I = \sum_i \Delta m_i R_i^2 = R^2 \underbrace{\sum_i \Delta m_i}_{M} = MR^2$$

SAME

$$\boxed{I = MR^2}$$



MASS = M

AREA $= \pi r_2^2 - \pi r_1^2$

$\Delta r = r_2 - r_1$ $\quad = \pi\left[(r_1 + \Delta r)^2 - r_1^2\right] = \pi\left[2\Delta r\, r_1 + \Delta r^2\right]$

Let $\Delta r \to 0$ $\quad$ AREA $= 2\pi r \Delta r$ $\quad$ Mass $\Rightarrow \Delta m = \rho \Delta r$

$$\rho = \frac{M}{\pi R^2}$$

$$\Delta m = \rho\, [2\pi r \Delta r]$$

$$I = \sum_i \Delta m\, r_i^2$$

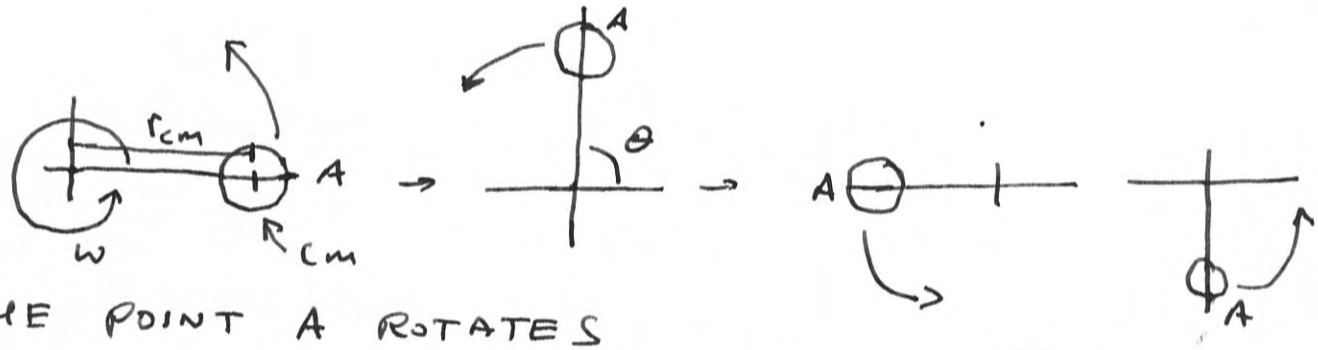
$$= \cancel{2\pi r \rho} \sum_i \overbrace{[\rho 2\pi r_i \Delta r]}^{\Delta m} r_i^2$$

$$= 2\pi\rho \sum_i r_i^3 \Delta r_i = \frac{2\pi M}{\pi R^2} \int_0^R r^3\, dr$$

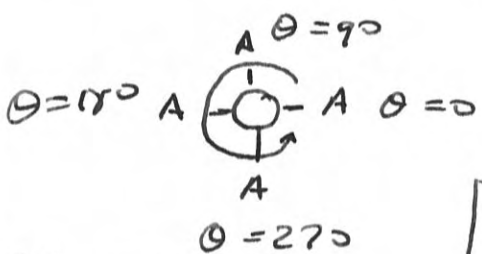
$$= \frac{2\pi M}{\pi R^2} \left.\frac{r^4}{4}\right]_0^R = \frac{1}{2} M R^2$$

NOTE: THICKNESS OF DISK DOES NOT MATTER!

RIGID MOTION ABOUT POINT THAT IS NOT CENTER OF MASS.



THE POINT A ROTATES



* LOOKS like OBJECT ROTATED WITH "ω"

*(BODY MUST BE RIGID)

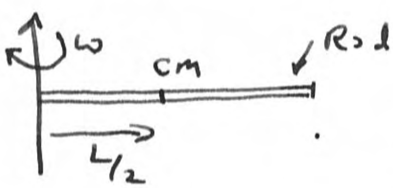
$$\boxed{I_{AXIS} = M r_{cm}^2 + I_{cm}}$$

$$K_{Tot} = K_{TRAN} + K_{rot}$$

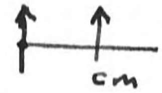
$$= \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

$$= \frac{1}{2} M [r_{cm}^2 \omega^2] + \frac{1}{2} I_{cm} \omega^2$$

$$= \frac{1}{2} [M r_{cm}^2 + I_{cm}] \omega^2$$



$$I = M\frac{L^2}{4} + \frac{1}{12} M L^2 = \frac{1}{3} M L^2$$

NOTE : ROTATIONAL AXIS MUST BE PARALLEL. ()

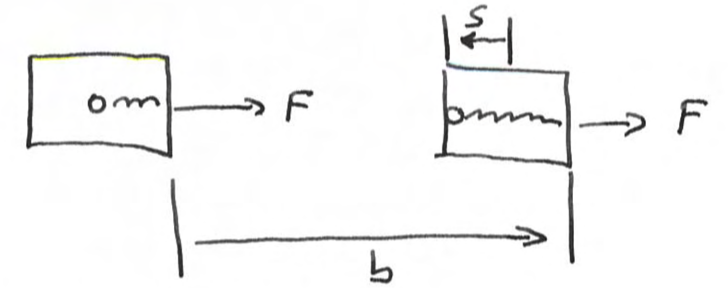
POINT PARTICLE : WE CAN FOLLOW AN OBJECT BY

$$\frac{d \vec{P}_{sys}}{dt} = \vec{F}_{net}$$

Point PARTICLE

$\Delta K_{trans} = W \leftarrow$ work

$$\Delta K_{trans} = \Delta \left[\frac{1}{2} M v_{cm}^2 \right] = \int_i^f \vec{F}_{NET} \cdot d\vec{r}_{cm}$$



WORK DONE:

$F \Delta X_{cm}$ CAREFUL!

$X_{cm} \to$ CHANGES

$\Delta X_{cm} = b - s$

$\Delta K = W = \frac{1}{2} M v_{cm}^2 = F(b-s)$ $\quad v_{cm} = \sqrt{\frac{2F(b-s)}{M}}$

Note: Ball sticks to box (No mass)

So net $v = v_{cm} = \sqrt{\frac{2F(b-s)}{M}}$

END MATERIAL CHAPTER 9