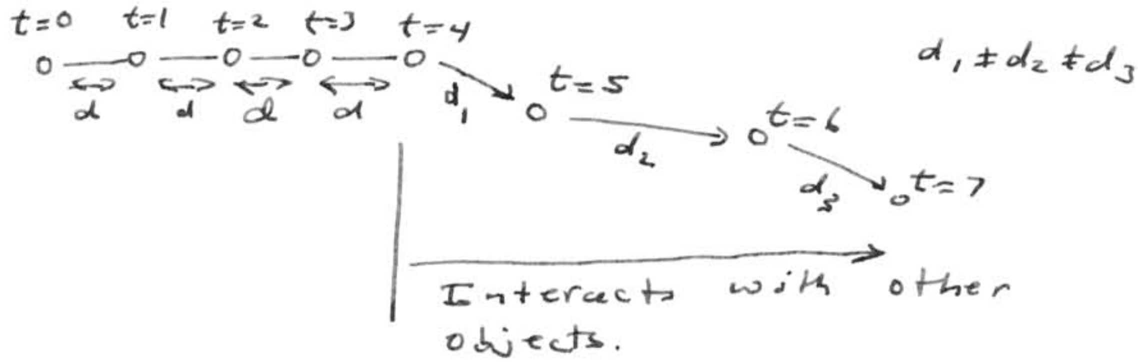
NEWTON'S FIRST LAW (INERTIA)

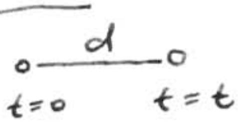
Suppose we see an ~~art~~ object moving in a STRAIGHT LINE at CONSTANT SPEED. It will continue to do so ~~to the~~ except to the extent that it interacts with other objects.



This is why we define a VECTOR. Newton's law can be expressed more readily.

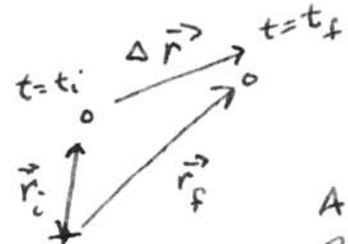
VELOCITY ⇒ Changes in position and direction in a time interval ⇒ characterized by velocity.

SPEED



$$v_{speed} = \frac{d}{t}$$

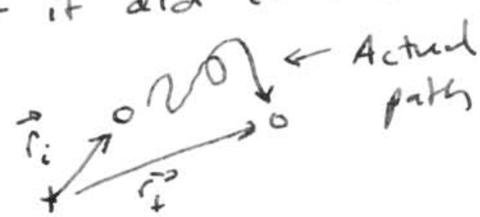
VELOCITY



$$\vec{v}_{AVE} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} = \frac{\Delta\vec{r}}{\Delta t}$$

Why?

Average? We only know its position at $\vec{r}_i$ and $\vec{r}_f$ and not what it did in the interim.



Example : Units in meters – seconds

L2

P. 2

$x_i = 1$ $\quad x_f = 4 \quad \vec{r}_i = \langle 1, 2, -3 \rangle \quad \Delta t = 2$

$y_i = 2 \quad y_f = 0 \quad \vec{r}_f = \langle 4, 0, 5 \rangle \quad \vec{V}_{ave} = \langle \frac{3}{2}, -1, 4 \rangle$

$z_i = -3 \quad z_f = 5 \quad \Delta\vec{r} = \langle 3, -2, 8 \rangle$

$t = 0 \quad t = 2$

Speed $\Rightarrow |\vec{v}_{AVE}| = \sqrt{(\frac{3}{2})^2 + (-1)^2 + 4^2} = \sqrt{19.25} = 4.38\ m/s$

What about direction? Can be characterized by

$$\hat{v}_{ave} = \frac{1}{4.38} \langle \tfrac{3}{2}, -1, +4 \rangle = \langle 0.342, -0.228, +0.913 \rangle$$

What about angles $\quad \hat{v}_{ave} = \langle \cos\theta_x, \cos\theta_y, \cos\theta_z \rangle$

$\cos\theta_x = 0.342 \quad \theta_x = 70.0^\circ$

$\cos\theta_y = -0.228 \quad \theta_y = 103.2^\circ$

$\cos\theta_z = 0.913 \quad \theta_z = 24.1^\circ$

Given $\vec{v}_{ave}$

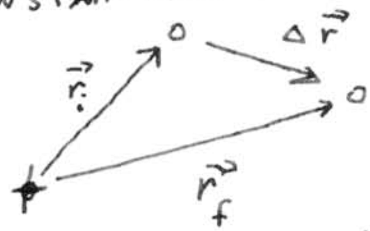
$$\vec{r}_f - \vec{r}_i = \vec{v}_f (t_f - t_i)$$

Often $t_i = 0 \quad \boxed{\vec{r}_f = \vec{r}_i + \vec{v}_{ave} t_f}$

Can also write $\quad \Delta t = t_f - t_i \Rightarrow \boxed{\vec{r}_f = \vec{r}_i + \vec{v}_{ave} \Delta t}$

$$\boxed{\begin{aligned} x_f &= x_i + v_{ave,x} \Delta t \\ y_f &= y_i + v_{ave,y} \Delta t \\ z_f &= z_i + v_{ave,z} \Delta t \end{aligned}}$$

INSTANTANEOUS VELOCITY



Suppose we consider $\Delta t \to 0$

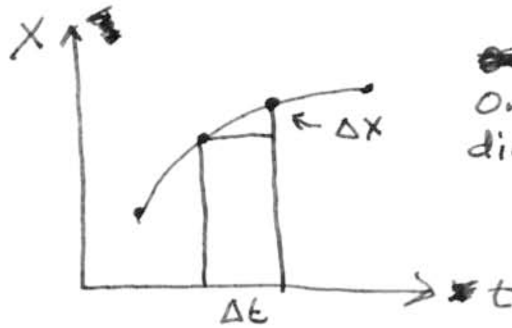
$$\lim_{\Delta t \to 0} \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} = \lim \frac{\Delta\vec{r}}{\Delta t}$$

what is this? Ask NEWTON!

$$\boxed{\vec{v} = \frac{d\vec{r}}{dt}}$$

This is why CALCULUS was invented.

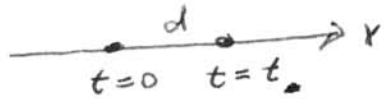
$$v_x = \frac{\Delta x}{\Delta t}\Big|_{t \to 0} = \frac{dx}{dt}$$



One dimension

NEWTON'S LAW ⇒ CONSTANT SPEED + STRAIGHT LINE

One dimension



slope v_x = constant

$v_x = \frac{d}{t}$ = constant

ACCELERATION : CHANGE IN VELOCITY ⇒

$\vec{a} = \frac{d\vec{v}}{dt}$ $\vec{a}_{ave} = \frac{\Delta\vec{v}}{\Delta t}$

[If v_x = CONSTANT $a_x = \frac{dv_x}{dt} = 0$]

Another way of stating NEWTONS FIRST LAW:
An object will undergo acceleration if another object interacts with it.

STILL NOT HELPFUL. At this point we don't have the means to predict how the acceleration occurs. ~~Wee~~ We need to KNOW MORE!

Mass ⇒ amount of matter ⇐ is an important factor. Suppose we want to accelerate an object ⇒ e.g. STOP a boat. It takes much more effort to stop the TITANIC (e.g. an iceberg) than stopping a row boat!

MOMENTUM : Definition

$\vec{p} = m\vec{v}$ (or $\vec{p} = \gamma m \vec{v}$, but $\gamma = 1$ for "ordinary" experience. At high velocities ⇒

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

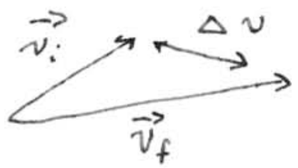
Momentum is proportional to velocity. The proportionality factor is the mass.

Momentum like velocity but scaled by mass $\Rightarrow \vec{p} = m\vec{v}$ ← velocity

↑ Scalar

For same velocity $\vec{p}$ bigger for bigger mass!

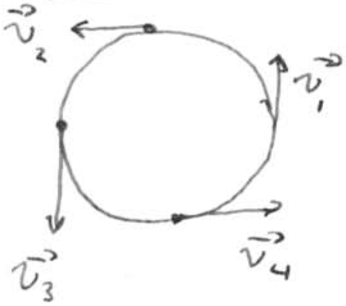
Find $\Delta\vec{p}$ by finding $\Delta\vec{v}$ and "scale" by mass (m).



$$\Delta\vec{v} + \vec{v}_i = \vec{v}_f \Rightarrow \Delta\vec{v} = \vec{v}_f - \vec{v}_i$$

$$\boxed{\Delta\vec{p} = m\Delta\vec{v}}$$

Interesting case: CIRCULAR MOTION



If object moves at CONSTANT speed →

$$|\vec{v}_1| = |\vec{v}_2| = |\vec{v}_3| = |\vec{v}_4| = v$$

$$m|\vec{v}_1| = m|\vec{v}_2| = m|\vec{v}_3| = m|\vec{v}_4| = mv$$

$|\vec{p}|$ = CONSTANT, but $\vec{p}$ changes direction!

EXAMPLE OF WHY WE TAKE $\gamma = 1$

Supersonic jet travels at 1000 mph → 1600 km/hr

$$1600\,\frac{km}{hr} \times \frac{1\,hr}{60\,min} \times \frac{1\,min}{60\,sec} \times \frac{1000\,m}{1\,km} = 444\ m/s$$

Speed of light $3.0 \times 10^8\ \frac{m}{s}$

$$\frac{v}{c} = \frac{4.44}{3.0} \times \frac{10^2}{10^8} \simeq 1.5 \times 10^{-6} \qquad \left(\frac{v}{c}\right)^2 = 2 \times 10^{-12}$$

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}} = \frac{1}{\sqrt{1 - 2 \times 10^{-12}}} = 1$$

↑ My calculator can't tell difference from 1.

So why introduce this now?