REVIEW

Multiparticle ENERGY PRINCIPLE

$$\Delta(E_1 + E_2 + E_3 + \dots) + \Delta(U_{12} + U_{13} + U_{23} + \dots) = W$$

$E_1 = m_1 c^2 + K_1$ — Potential energy between 1 and 2.

SINGLE PARTICLE : IGNORING REST MASS $\Rightarrow$ $\Delta K = W$ [No ΔU because No other particles]

Multiparticle SYSTEM

$\Delta E_{sys} = W$

NOW $\Delta K_1 + \Delta K_2 + \dots \neq W$ BECAUSE WE HAVE POTENTIAL ENERGY

$$\Delta(E_1 + E_2 + \dots) + \cancel{\Delta U}\ \Delta(U_{12} + U_{13} + \dots) = W$$

FOR THIS CASE WE HAVE THE POSSIBILITY OF POTENTIAL ENERGY.

EXAMPLE: Ball & EARTH $\Rightarrow$ SYSTEM BALL

$\Delta K = W_{EARTH}$ No ΔU!

$= -mg\Delta y$

$$\Delta K + \underbrace{mg\Delta y}_{\Delta U} = 0$$

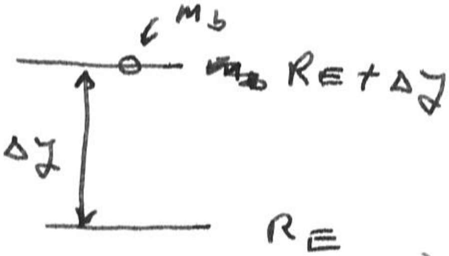
No WORK
$W = 0$

But $\Delta U = mg\Delta y$

IS THIS CORRECT?

EXAMPLE

Let's say
$v_i = 0$
$m_b = 1\,kg$
$\Delta y = 10\,m$



Note: $(\Delta y = +10m)$

INFO
$M_E = 6 \times 10^{24}\,kg$
$R_E = 6.4 \times 10^6\,m$

$M_E >> m_b$
$R_E >> \Delta y$

INITIALLY ?
$K_E = K_b = 0$
FINAL
$K_E = 0 \quad K_b \neq 0$

THINK ABOUT CENTER OF MASS $\underbrace{m_b v_f + M_E v_E}_{p_f} = \underbrace{0}_{p_i}$

$v_E = -\frac{m_b}{M_E} v_b \Rightarrow 0$
So
$K_E = 0$

$$\Delta U = \underbrace{-\frac{G M_E m_b}{R_E}}_{\text{~~INITIAL~~ FINAL}} - \underbrace{\frac{G M_E m_b}{R_E + \Delta y}}_{\text{~~FINAL~~ INITIAL}} = -\frac{G M_E m_b}{R_E}\left[1 - \frac{1}{(1 + \frac{\Delta y}{R_E})}\right]$$

USE

$$\frac{1}{1+x} \cong 1 - x \quad \text{for } x << 1$$

CHECK $x = 0.02$

$$\frac{1}{1+0.02} = \frac{1}{1.02} = 0.9804$$

$$1 - 0.02 = 0.9800$$

$$\Delta U = -\frac{G M_E m_b}{R_E}\left[1 - (1 - \frac{\Delta y}{R_E})\right]$$

$$= -\frac{G M_E m_b}{R_E}\left[\frac{\Delta y}{R_E}\right] = -\underbrace{\left[\frac{G M_E}{R_E^2}\right]}_{g} m_b \Delta y$$

$$\boxed{\Delta U = -m_b g \Delta y} \qquad \boxed{\Delta K + \Delta U = 0} \Rightarrow (\Delta E)_{sys} = 0$$

$$\boxed{\Delta K = -\Delta U} \qquad \boxed{K_b = m g \Delta y}$$

$$\frac{1}{2} m v_f^2 = m g \Delta y \qquad v_f^2 = 2 g \Delta y$$

$$v_f^2 = 2(9.8)(10) = 196\ m^2/s^2$$

$$\boxed{v_f = 14\,m/s}$$

ENERGY DIAGRAM

O → ← O

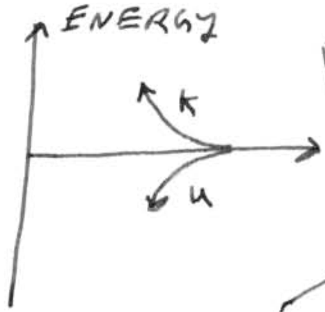
#1 #2

ASTEROIDS

Suppose BOTH AT REST – INITIAL

$E_i = 0$

$[R \to \infty]$



$E_f = K + U = E_i = 0$

$K + U = 0$ so If K increases, U MUST DECREASE

ELECTROSTATIC POTENTIAL ENERGY

#1 #2

F ← (+) (+) → F } TWO PROTONS

$\vec{v}_1$ $\overleftarrow{v_2}$

$$F = + \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

$$U = \frac{q_1 q_2}{4\pi\epsilon_0 r} \Rightarrow F = -\frac{dU}{dr}$$

$q_1 = q_2 = +e$

charge ON PROTON

INITIALLY →

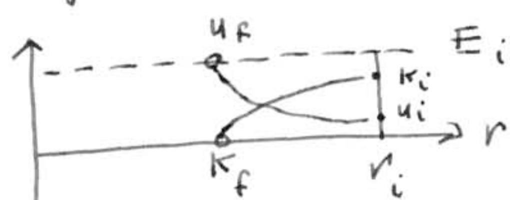
$E_i = K_i + U_i$

$K_i = \frac{1}{2} m_1 v_{1,i}^2 + \frac{1}{2} m_2 v_{2,i}^2$

$U_i = + \frac{e^2}{4\pi\varepsilon_0 R_i}$

FINAL

$$E_f = \underbrace{\frac{1}{2} m v_{1,f}^2 + \frac{1}{2} m v_{2,f}^2}_{K_f} + \underbrace{\frac{e^2}{4\pi\varepsilon_0 r_f}}_{U_f}$$

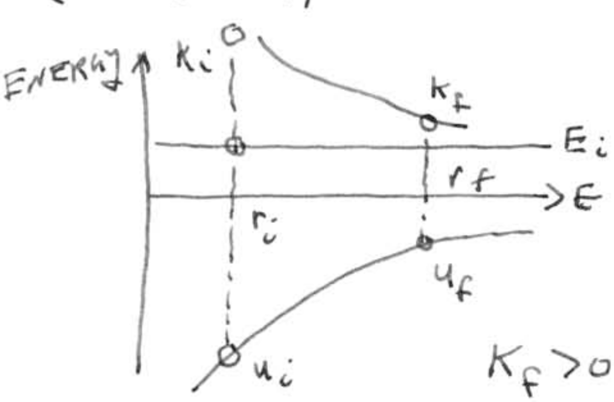


Note: $K_f > 0$ so U_f is ~~first~~ fixed so $E_f = U_f = E_i$

LIMITS ON MOTION

$E_f = E_i$; ~~Also~~

(AND $K_f, K_i > 0$)



AT BEST

Rocket [Coasting – No FUEL!)

Planet

WILL THE ROCKET ESCAPE?

$E_i = K_i + U_i$ $E_f = K_f + U_f$

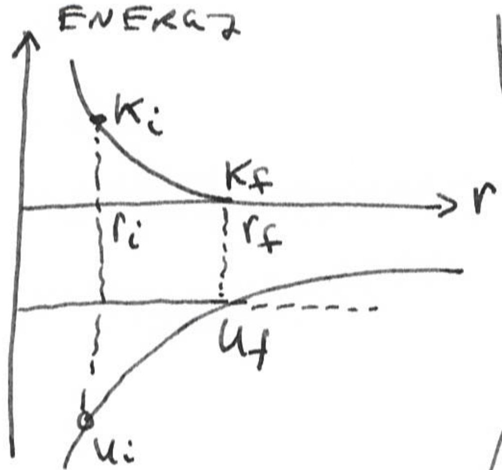
$K_f > 0$ $K_f = K_i + (U_i - U_f)$

TO ESCAPE!

$K_f > 0$ $U_f \to 0 \Rightarrow \boxed{K_i = -U_i}$

(OR $K_i + U_i = 0$)

Suppose $U_i + K_i < 0$ $\qquad K_i + U_i = K_f + U_f$ At r_f



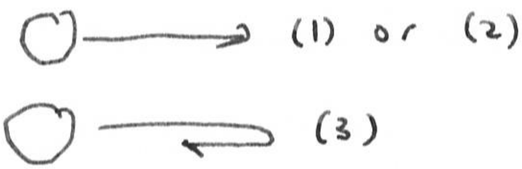
$\boxed{K_f = 0} \Rightarrow$ PARTICLE (OBJECT) CANNOT MOVE PAST $r = r_f$. [L16 p.4]

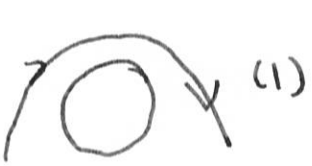
RULES:

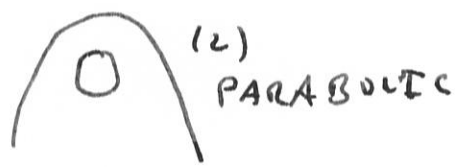
(1) $K_i + U_i > 0$ ROCKET ESCAPES

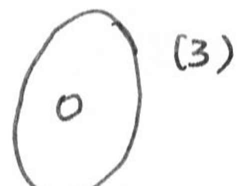
(2) $K_i + U_i = 0$ ROCKET BARELY ESCAPES

(3) $K_i + U_i < 0$ ROCKET NEVER ESCAPES









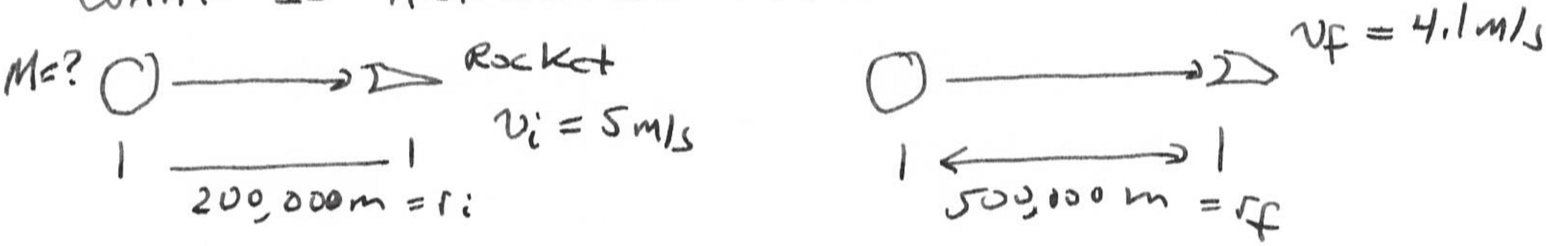
CLOSED ORBIT CIRCLE OR ELLIPSE

EXAMPES: ESCAPE FROM EARTH

$$K_i + U_i = \frac{1}{2} m v_i^2 - \frac{GMm}{R_E} = 0$$

$$v_i = \sqrt{\frac{2GM}{R_E}} = \sqrt{\frac{2(6.7 \times 10^{-11})(6 \times 10^{24})}{6.4 \times 10^4}} = 11.2 \text{ km/s}$$

WHAT IS ASTEROID MASS



$$E_i = K_i + U_i = E_f = K_f + U_f$$

$$\left.\frac{1}{2} m v_i^2 - \frac{GMm}{r_i} = \frac{1}{2} m v_f^2 - \frac{GMm}{r_f}\right\} \quad \frac{GMm}{r_f} - \frac{GMm}{r_i} = \frac{1}{2} m [v_f^2 - v_i^2]$$

$$M = \frac{\frac{1}{2}[v_f^2 - v_i^2]}{G\left[\frac{1}{r_f} - \frac{1}{r_i}\right]} = 2 \times 10^{16} \text{ kg}$$

$\boxed{\text{ENERGY AND MASS}}$

BEFORE

$e^+ + e^- \rightarrow 2\gamma$

MASS
POSITRON
ELECTRON

AFTER
TWO GAMMA RAYS
MASS?

$$\boxed{E = \gamma m c^2} \qquad m = \frac{E}{\gamma c^2}$$ } Define mass

ENERGY IS MASS!

(AT High Velocity
$m >> (m)_{REST}$)

$E = mc^2 + K = \gamma m c^2$

DEFINES KINETIC ENERGY

Multiparticle Systems

$$M = \frac{E_{sys}}{c^2} = \frac{m_1c^2 + K_1 + U_1 + m_2c^2 + K_2 + U_2 + \cdots}{c^2}$$

$$M = m_1 + m_2 + m_3 + \ldots + \left(\frac{K_1 + K_2 + K_3 + U}{c^2}\right)$$

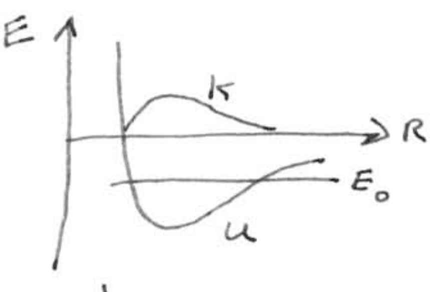
Sum of $U_1 + U_2 + U_3$

ORDINARY EXPERIENCE ($v_1, v_2, \ldots << c$ — AND $U/c^2 \rightarrow 0$)

$\{M = m_1 + m_2 + \cdots\}$ Non-relativistic Limit

EXAMPLE: O_2 MOLECULE

$O \overset{R}{\longleftrightarrow} O$
#1 #2



E_0 IS CALLED BINDING ENERGY

$$M = \frac{E_{sys}}{c^2} = 2m + \frac{K+U}{c^2}$$

$$= 2m + E_0/c^2$$

$E_0 = -5\,eV$

$1\,eV = 1.6 \times 10^{-19}\,J$

$E_0 = -8 \times 10^{-19}\,J$

$$\frac{E_0}{c^2} = \frac{-8 \times 10^{-19}\,J}{9 \times 10^{16}\,m^2/s^2} = -8.8 \times 10^{-36}\,kg$$

$O_2 \Rightarrow (16 \times m_p) \times 2 = 2m$

$(32)(1.7 \times 10^{-27})\,kg \doteq 5.4 \times 10^{-26}\,kg$

CHANGE IN MASS!
ONE PART IN 10^{10}

READ ABOUT NUCLEAR APPLICATIONS