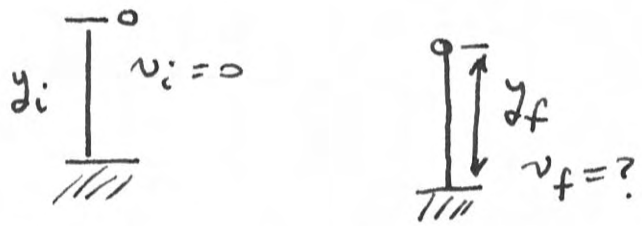
Potential Energy



System: Ball

Force: $F_g = -mg$

GRAVITY DOES WORK ON THE BALL

$W = F_g \Delta y = -mg\Delta y$. Work changes energy →

$\Delta E = \underbrace{\Delta K}_{\text{Kinetic Energy}} = -mg\Delta y$

$m = 0.1$ kg
$y_i = 7$ m
$y_f = 4$ m

$\Delta K = -mg\Delta y = -0.1(9.8)(-3)$
$= 2.94$ J

ASIDE: USE ΔK to find v_f

$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W$ $\quad$ $\frac{1}{2}mv_f^2 = W$

$v_f^2 = \frac{2W}{m} = \frac{2(2.94)}{0.1}\ m^2/s^2$
$= 58.8\ m^2/s^2$

$v_f = \pm 7.67$ m/s
We take "−" sign.

$\boxed{v_f = -7.67\ m/s}$

SIGN: $\pm v$ leads to Same K, but we known it must be "−" sign.

We can check this

$v_f = v_i + \frac{F}{m}\Delta t$ $\quad$ $v_f = -g\Delta t$

$y_f = y_i + v_i\Delta t + \cancel{\frac{F}{m}}\frac{F}{2m}\Delta t^2$

$y_f = 7 - \frac{1}{2}g\Delta t^2$

$y_f = 4$ $\quad$ $\Delta t^2 = \left(\frac{6}{9.8}\right)s^2$
$\Delta t = 0.782$ sec

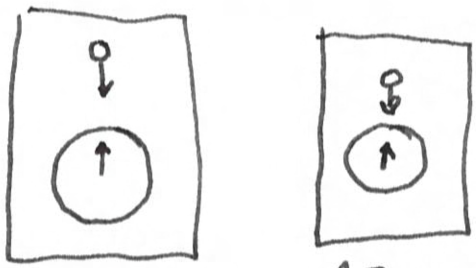
$v_f = -9.8(0.782)s$
$= -7.67$ m/s

ENERGY OR FORCES YIELD SAME RESULT!

Did we create energy?

L15 P2

Suppose $M_E \gg m_{ball}$. BALL GOT ENERGY!



BEFORE AFTER

BALL + EARTH AS SYSTEM

ΔU = CHANGE IN POTENTIAL ENERGY

$\Delta E_{system} = 0$

DID WE MISS AN ENERGY TERM?

$\Delta K_{BALL} = W_{By\ EARTH}$

$\Delta K_{BALL} - W_{By\ EARTH} = 0$

WE CAN THINK OF $W_{By\ EARTH}$ AS A "POTENTIAL ENERGY"

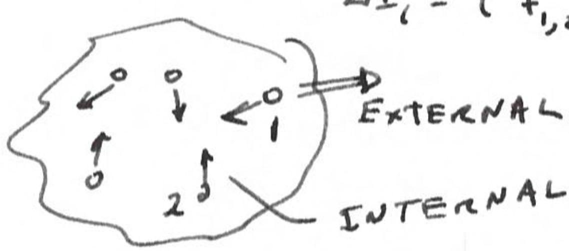
$$\boxed{\Delta K + \Delta U = \Delta E_{sys} = 0}$$

$\Delta U[\text{GRAVITY}] = \Delta[mgy] = mg\Delta y$

MAKES SENSE $\rightarrow$ ENERGY CAN EXTRACTED USING GRAVITY.

BUNCH OF PARTICLES

$\Delta E_1 = (\vec{f}_{1,2} + \vec{f}_{1,3} + \vec{f}_{1,4} + \dots + \vec{F}_{1,E}) \cdot \Delta \vec{r}_1$



$\Delta E_2 = (\vec{f}_{2,1} + \vec{f}_{2,3} + \vec{f}_{2,4} + \dots \vec{F}_{2,E}) \cdot \Delta \vec{r}_2$

DIVIDE UP ENERGY

$\Delta E = \Delta E_1 + \Delta E_2 + \dots$

$\Delta E_1 = W_{1,int} + W_{1,E}$

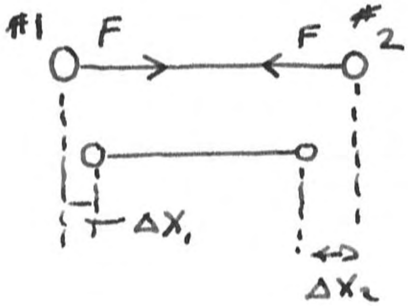
$\Delta E = W_{int} + W_{EXT}$

$\underbrace{\Delta E - W_{int}}_{\Delta U} = W_{EXT}$ OR SURROUNDINGS!

$\Delta E + \Delta U = W_{surr}$

$$\boxed{\Delta U \equiv -W_{int}}$$

GRAVITATIONAL POTENTIAN ENERGY



$W_1 = F\Delta X_1 \quad W_2 = F\Delta X_2$

$\Delta U = -(W_1 + W_2) = -F(\Delta X_1 + \Delta X_2)$

$\Delta X_1 + \Delta X_2 =$ Total DISPLACEMENT $= \Delta X$

$\Delta U = -F\,\Delta X \Rightarrow F_x = -\dfrac{\Delta U}{\Delta X}$

NOW CONSIDER ~~LIMIT~~ LIMIT WHERE $\Delta X \to 0$

$$\boxed{F_x = -\frac{dU}{dx}}$$

IN GENERAL

$o \xrightarrow{\vec{r}} o$

$$\boxed{F_r = -\frac{dU}{dr}}$$

(FORCE ALONG $\vec{r}$)

SOME CALCULUS

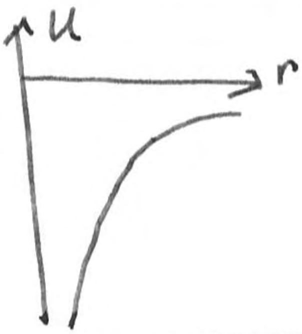
$$F_r = -G\frac{m_1 m_2}{r^2} \rightarrow -\frac{dU}{dr} = -\frac{Gm_1 m_2}{r^2}$$

$$U = -\frac{Gm_1 m_2}{r} + \text{CONSTANT}$$

CONSIDER $r \to \infty \quad U \to 0$

then CONSTANT $\to 0$

$$\boxed{U = -\frac{Gm_1 m_2}{r}}$$



GENERAL PROPERTIES OF POTENTIAL ENERGY [GRAVITY] *

- DEPENDS ONLY ON ~~SEPARA~~ SEPARATION OF PARTICLES
- APPROACHES ZERO AS $r \to \infty$

(QUESTION: CAN WE DO A POTENTIAL ENERGY FOR FRICTION? (NO!))

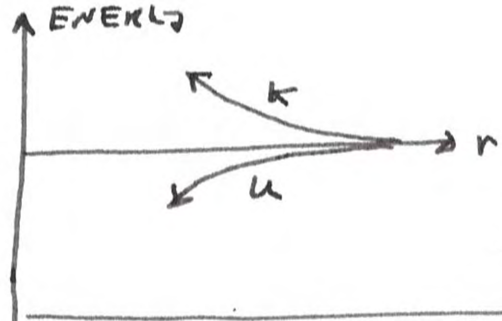
* ALSO TRUE FOR ELECTROSTATIC

ENERGY DIAGRAM

O → ← O

STAR #1 STAR #2

INITIALLY BOTH AT REST $E = 0$



$K + U =$ CONSTANT $= 0$

AS STARS INTERACT K increases as U decreases so $E = K + U = 0$

ELECTROSTATIC POTENTIAL ENERGY

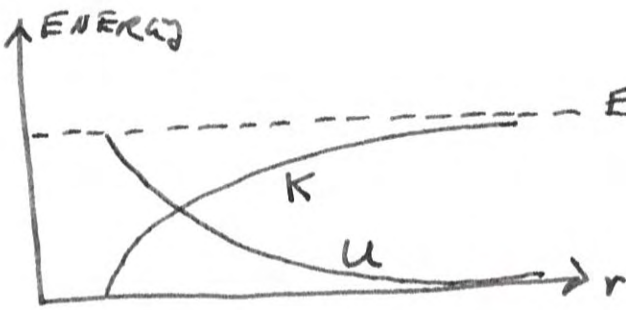
$\overleftarrow{F}$ ⊕ $\rightarrow v_1$ $\quad$ $\overleftarrow{v_2}$ ⊕ $\rightarrow F$

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r} \qquad F = -\frac{dU}{dr} = +\frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

INITIALLY $K = \frac{m_1 v_{1,i}^2}{2} + \frac{m_2 v_{2,i}^2}{2}$ $\quad U = 0$ $\quad [r \to \infty]$

$E = E_0$

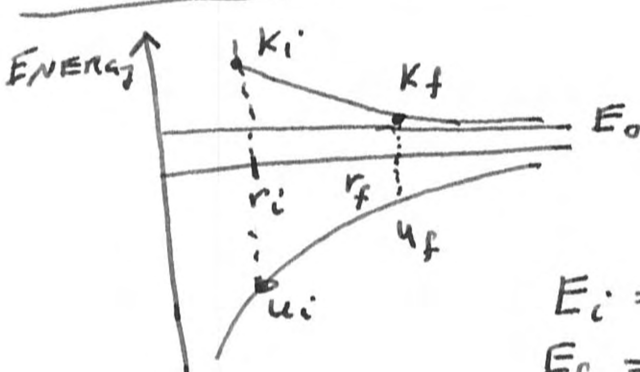
$v_{1,i}; v_{2,i} =$ INITIAL VELOCITY



HERE $U \to +\infty$ as $r \to 0$

LIMITS ON MOTION

~~CONSERA~~ CONSERVATION OF ENERGY SETS BOUNDS



INITIAL VELOCITY v_i, INITIAL SEPARATION, r_i.

$E_i = K_i + U_i$

$E_f = K_f + U_f$ $\qquad K_i + U_i = K_f + U_f$

$$\boxed{E_i = E_f = E_0}$$

$\cancel{K_i = K_f + (U_f - U_i)}$

$$K_f = K_i + (U_i - U_f) \leftarrow \text{REWRITE. SERVES AS BOUND}$$

Suppose $r_f \to \infty$ $U_f \to \infty$

$$\boxed{K_f (r \to \infty) = K_i + U_i}$$

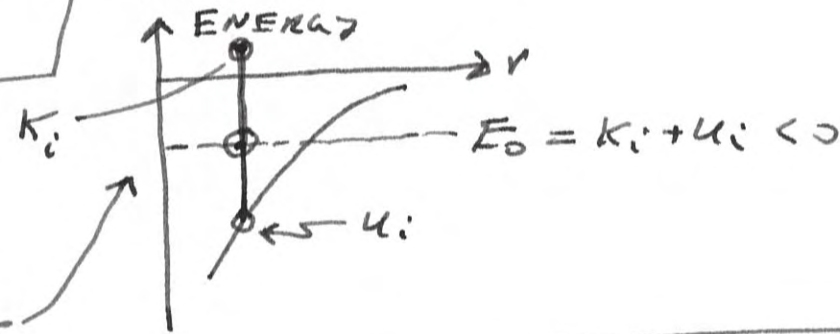
Suppose FURTHER

$$\boxed{K_i + U_i < 0}$$

WE HAVE A PROBLEM.

$K_f < 0$ IS NOT ALLOWED!

EXAMPLE



ROCKET CANNOT ESCAPE!

TO ESCAPE $K_f > 0$ But

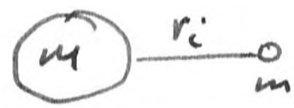
$K_f = K_i + U_i < 0$

MINIMAL CONDITION FOR ESCAPE

$$K + U = 0$$

RULES:

$$K_i = \frac{1}{2} m v_i^2 \quad U_i = \frac{-GMm}{r_i}$$



(1) $K_i + U_i > 0$ ROCKET ESCAPES

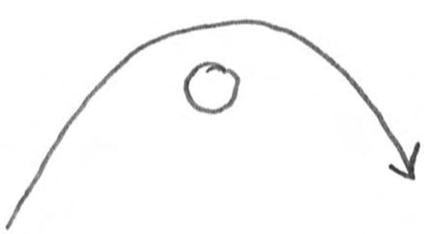
(2) $K_i + U_i = 0$ BARELY ESCAPES

} UNBOUND STATE

(3) $K_i + U_i < 0$ NO ESCAPE (BOUND STATE)

(1) or (2) STRAIGHT LINE

(3) CIRCULAR OR ELLIPTICAL ORBIT



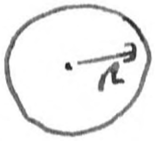
(1) HYPERBOLIC

PARABOLIC

EXAMPLE PROBLEMS

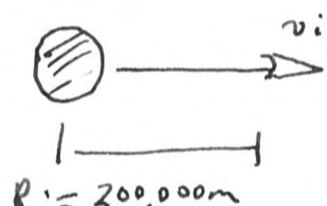
ESCAPE FROM EARTH!

$$E(R) = \frac{1}{2} m v_i^2 - \frac{GmM}{R} = 0$$

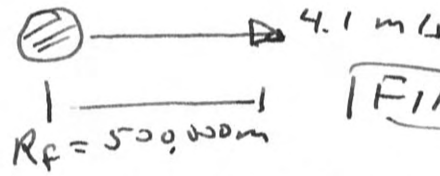


$$v_i = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.7\times10^{-11})(6\times10^{24})}{6.4\times10^{6}}} = 1.12\times10^{4}\ \frac{m}{s}$$

MASS of ASTEROID



INITIAL



FINAL

Mass of ROCKET = 1500kg

WHAT IS THE MASS OF THE ASTEROID?

$E_{initial} = K_i + U_i$

$E_{FINAL} = K_f + U_f$

$$\frac{1}{2} m v_i^2 + \left[-\frac{GMm}{R_i}\right] = \frac{1}{2} m v_f^2 + \left[-\frac{GMm}{R_f}\right]$$

$$G\, Mm \left[\frac{1}{R_i} - \frac{1}{R_f}\right] = \frac{1}{2} m \left[v_i^2 - v_f^2\right]$$

NOTE: MASS OF ROCKET CANCELS OUT

$$M = \frac{\frac{1}{2}\left[v_i^2 - v_f^2\right]}{G\left(\frac{1}{R_i} - \frac{1}{R_f}\right)} = 2\times10^{16}\ kg$$
