WORK (REVIEW)

WORK = FORCE × DISPLACEMENT

$W = F_x \Delta x$ IN GENERAL →

$W = F_x \Delta x + F_y \Delta y + F_z \Delta z$ ↗ NOTE: $\vec{F}$ is CONSTANT!

DEFINITION: Dot product

$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$

$W = \vec{F}\cdot\Delta\vec{r} = (F_x \hat{i} + F_y \hat{j} + F_z \hat{k})\cdot(\Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k})$

$\left.\begin{aligned} \hat{i}\cdot\hat{i} &= 1 \\ \hat{i}\cdot\hat{j} &= 0 \\ \hat{i}\cdot\hat{k} &= 0 \end{aligned}\right\}$ Similar for j, k

NOTE: UNITS OF ENERGY (or Work)

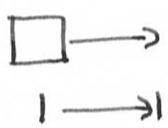
⟹ J = N·m ← Meter

↑ Joule ↑ Newton

WORK CAN BE NEGATIVE OR ~~POSITE~~ POSITIVE }

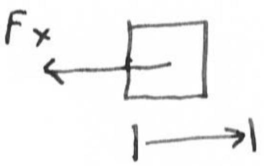
NEGATIVE work → System DOES work ON surrounding

POSITIVE work → System has work done on it by Surrounding



$F_x = 2N$ $\Delta x = 2m$

$\Delta x = 2m$ $W = +4\ N\cdot m = 4\ J$



$F_x = -2N$ $\Delta x = 2m$

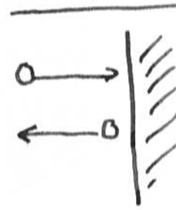
$W = -4\ J$ } FORCE opposite to DISPLACEMENT

2m

$\overleftarrow{F_x}$, $\overleftarrow{\Delta x}$ or $\overrightarrow{F_x}$, $\overrightarrow{\Delta x}$ } Positive work

$\overrightarrow{F_x}$, $\overleftarrow{\Delta x}$ or $F_x \leftarrow$, $\overrightarrow{\Delta x}$ } NEGATIVE work

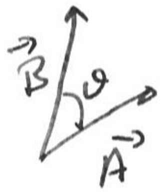
ZERO WORK



Ball exerts force on wall, but wall does not move.

MORE ON DOT PRODUCT

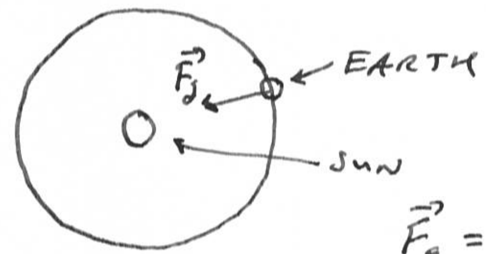
$$W = \vec{F} \cdot \Delta\vec{r} = |\vec{F}||\Delta\vec{r}| \cos\theta$$



$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$

* NOTE: $\vec{F} \perp \Delta\vec{r}$

$\cos(\theta) = 0$ No work done!



$$\vec{F}_g = \vec{F}_\perp$$

$$(\vec{F}_\parallel = 0)$$

$$\Delta E_{EARTH} \Rightarrow \Delta K = W$$

ΔK: Kinetic energy CHANGE

IF EARTH MOVES AT A CONSTANT SPEED

$\Delta K = 0$ Kinetic energy is a CONSTANT

$\Delta E_{EARTH} = 0$ MEANS $W = 0$

W = WORK DONE ON EARTH BY $\vec{F}_g$.

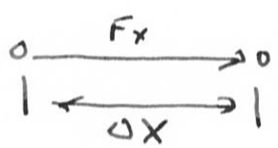
But $\vec{F}_g \perp \Delta\vec{r}$ so

$$W = \vec{F}_g \cdot \Delta\vec{r} = \vec{F}_\perp \cdot \Delta\vec{r} = 0 \ !$$

No work done.

(Limit $\Delta\vec{r} \to 0$ so $\vec{F}_g$ is a CONSTANT)

RELATIONSHIP BETWEEN ENERGY AND MOMENUM.



$W = F_x \Delta X$ (No other source of energy transfer → e.g. heat).

$W = \Delta E_{sys}$

Assume $(\Delta X, \Delta t)$ small and F_x CONSTANT OVER ΔX.

$$\Delta E = F_x \Delta X = \left(\frac{\Delta P_x}{\Delta t}\right) \Delta X$$

$$\frac{\Delta E}{\Delta X} = \frac{\Delta P_x}{\Delta t}$$

So $F_x = \dfrac{\Delta P_x}{\Delta t}$

Now take limit $\Delta t \to 0$

$$\boxed{\frac{dE}{dx} = \frac{dP_x}{dt}}$$

LET'S CHECK THIS

$E = \gamma m c^2$ SUPPOSE $v \ll c$

$$E \cong mc^2 + \frac{1}{2} m v_x^2$$

$$\frac{dE}{dx} \cong m v_x \frac{dv_x}{dx} = m \left[\frac{dx}{dt}\right] \frac{dv_x}{dX} = m \frac{dv_x}{dt} = \frac{dP_x}{dt}$$

SO $\frac{dE}{dx} = \frac{dP_x}{dt}$ WORKS FOR $E = \gamma m c^2$ FOR $v \ll c$

IF $v \approx c$, $\left\{ \frac{dE}{dx} = \frac{dP_x}{dt} \right\}$ STILL WORKS (SEE BOOK).

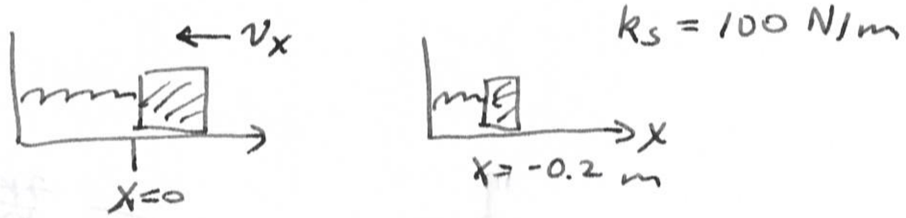
SUPPOSE WORK RESULTS ~~AS THE~~ FROM A NONCONSTANT FORCE: BREAK UP PROBLEM

$$W = \vec{F}_1 \cdot \Delta \vec{r}_1 + \vec{F}_2 \cdot \Delta \vec{r}_2 + \vec{F}_3 \cdot \Delta \vec{r}_3 + \ldots$$

$$\boxed{W = \sum_i \vec{F}_i \cdot \Delta \vec{r}_i} \Longleftrightarrow \boxed{W = \int_i^f \vec{F} \cdot d\vec{r}}$$

NEED TO FIGURE OUT INTEGRAL OR ~~DO~~ DO THIS NUMERICALLY.

EXAMPLE



How much work involved in compressing spring?

$$W = \int F_x \, dx = -100 \int_0^{-0.2} x \, dx = -100 \left. \frac{x^2}{2} \right]_0^{-0.2} = -2\,J$$

$\Delta E = -2J$

KINETIC ENERGY OF BOX REDUCED BY 2J.