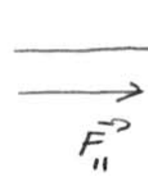
REVIEW

$\Delta \vec{P} = \vec{F}_N \Delta t$

TAKE $\vec{F}_N = \vec{F}_\perp + \vec{F}_\parallel$

$\vec{F}_\parallel$ along $\vec{P}_i$

$\vec{F}_\perp$ perpendicular to $\vec{P}_i$



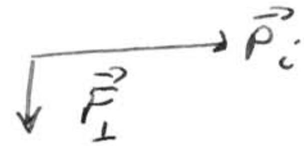
$\vec{P}_i \Rightarrow \vec{F}_\parallel$ changes size of $\vec{P}_i \Rightarrow$ BUT NOT DIRECTION!

$(\vec{P}_f)_\parallel = (\vec{P}_i)_\parallel + \vec{F}_\parallel \Delta t$ $\quad |\vec{P}_{i,\parallel}| \Rightarrow$ CHANGES

$(\vec{P}_{f,\parallel}\ \ \vec{P}_{i,\parallel}\ \ \vec{P}_\parallel)$ All same direction

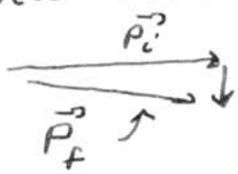
$\vec{F}_\perp$ perpendicular to $\vec{P}_i$

$(\vec{P}_f)_\parallel = (\vec{P}_i)_\parallel + \cancelto{0}{(\vec{F}_\perp \Delta t)_\parallel}$



No change in $(\vec{P}_i)_\parallel$

BUT New direction



$\vec{P}_i$, $\vec{P}_f$, $(\Delta \vec{P}_\perp) = \vec{F}_\perp \Delta t$

For small $\Delta t \rightarrow$ $|\vec{P}_f| \approx |\vec{P}_i|$ But direction CHANGES

SUPPOSE WE WRITE

$\vec{P} = P\hat{P}$ $\quad \hat{P}$ unit vector along $\vec{P}$ direction

$P = |\vec{P}|$

$$\frac{d\vec{P}}{dt} = \underbrace{\frac{dP}{dt}\hat{P}}_{\substack{\text{No change in direction.} \\ \text{CHANGE IN } |\vec{P}|}} + \underbrace{P\frac{d\hat{P}}{dt}}_{\substack{\text{No change in magnitude} \\ \text{CHANGE in Direction}}}$$

$\vec{F}_g = \vec{F}_\perp$

$|\vec{P}| = $ CONSTANT

$\boxed{\text{CIRCULAR ORBIT}}$

WE IDENTIFY

$\vec{F}_\perp = P\dfrac{d\hat{P}}{dt}$ $\quad \vec{F}_\parallel = \dfrac{dP}{dt}\hat{P}$

IN A CIRCULAR ORBIT $\quad \dfrac{dP}{dt} = 0 \quad \dfrac{d\hat{P}}{dt} \neq 0$

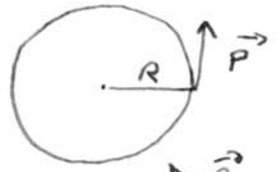
$\left.\vec{F}_\perp = P\dfrac{d\hat{P}}{dt}\right\}$ CIRCULAR ORBIT

$$\boxed{\vec{F}_g = P\frac{d\hat{P}}{dt}}$$

$\vec{F}_\parallel = \dfrac{dP}{dt}\hat{P} = 0$ as $\dfrac{dP}{dt} = 0$

$\rightarrow$ CHANGE IN DIRECTION, NOT in $|\vec{P}| = P$ magnitude.

CIRCULAR MOTION CONTINUED



$$\theta = \frac{S}{R} \qquad S = v\Delta t$$

$$\theta = \frac{v\Delta t}{R}$$ Angle SWEPT OUT in Δt.

$$\theta = \frac{|\Delta\vec{p}|}{|\vec{p}|} = |\Delta\hat{p}|$$

UNIT VECTOR

$$|\vec{P}_f| = |\vec{P}_i| = P$$

MAGNITUDE DOES NOT CHANGE

$$|\Delta\hat{p}| = \frac{v\Delta t}{R} \qquad \frac{|\Delta\hat{p}|}{\Delta t} = \frac{v}{R}$$

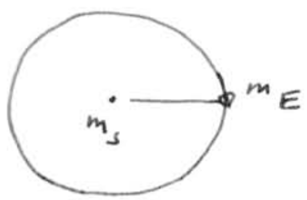
$$\boxed{\frac{|d\hat{p}|}{dt} = \frac{v}{R}}$$

$$\vec{F}_\perp = |\vec{p}|\frac{d\hat{p}}{dt} \qquad |\vec{F}_\perp| = p\left[\frac{v}{R}\right] \qquad \boxed{|\vec{F}_\perp| = \frac{mv^2}{R}}$$

Not relativistic

ORBIT OF EARTH AROUND SUN

$$|F_g| = |\vec{F}_\perp| \qquad F_g = \frac{G m_s m_E}{R_{ES}^2} = \frac{m_E v^2}{R_{ES}}$$



$$\boxed{v = \sqrt{\frac{GM_s}{R_{ES}}}}$$

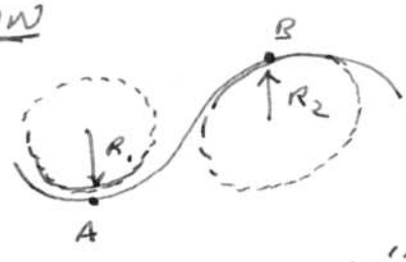
PERIOD

$$T = \frac{2\pi R}{v}$$

$$\boxed{T = \frac{2\pi R_{ES}^{3/2}}{\sqrt{GM_s}}}$$

NOTE: All ~~Curve~~ CURVED MOTION → BROKEN UP INTO "CIRCULAR" TYPE MOTION

$\Longrightarrow$



At "A" particle looks like a circular ~~orbi~~ "orbit" with RADIUS "R_1". "B" → SAME ⟹ RADIUS "R_2"

CALLED RADIUS OF CURVATURE

LR
P. 3

BIG DEAL: ENERGY PRINCIPLE

$$\Delta E_{system} = \underbrace{W_{surrounding}}_{\substack{\text{WORK DONE} \\ \text{ON SYSTEM} \\ \text{BY SURROUNDING}}} + \underbrace{\text{"OTHER ENERGY TRANSFER"}}_{\substack{\text{RADIATION} \\ \text{HEAT}}}$$

RADIATION, HEAT ??? NEXT CHAPTER

WHAT IS WORK? ENERGY?

ENERGY ⇒ CAPACITY TO DO WORK

WORK = FORCE ACTING ON AN OBJECT ⇒ DISPLACES OBJECT (MECHANICAL WORK)

CONSERVATION OF ENERGY:

$$\Delta E_{sys} + \Delta E_{surr} = 0$$

ENERGY CANNOT BE CREATED OR DESTROYED

RELATIONSHIP ~~BETE~~ BETWEEN MASS AND ENERGY

O ← M = MASS

↑ PARTICLE

$$E_{Part} = \gamma mc^2 \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

REST ENERGY: $E_{Part} = mc^2$ $(\gamma = 1)$

$v = 0$

KINETIC ENERGY: $v \neq 0$

$$K = \gamma mc^2 - mc^2 \qquad \text{When } v = 0 \quad \gamma = 1 \quad K = 0$$

$$E_{part} = mc^2 + K$$

SOME MATH:

$$\frac{1}{\sqrt{1-x^2}} = (1 - x^2)^{-1/2} \approx 1 + \frac{1}{2}x^2$$

THIS WORKS IF $x \ll 1$ ⇒

As long as ~~[illegible]~~ $\boxed{x < 0.1}$ we should be ok using

$$\boxed{\frac{1}{\sqrt{1-x^2}} \approx 1 + \frac{1}{2}x^2}$$

EXAMPLE

x	$(1-x^2)^{-1/2}$	$1+\frac{1}{2}x^2$
0	1	1
0.1	1.0050	1.0050
0.5	1.1547	1.1250
0.8	1.6667	1.3200

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = 1 + \frac{1}{2}\frac{v^2}{c^2} \quad \text{for } v \ll c$$

$$E_p = \gamma mc^2 \cong \left[1 + \frac{1}{2}\frac{v^2}{c^2}\right] mc^2 = mc^2 + \frac{1}{2}mv^2 = mc^2 + K$$

$\boxed{K = \frac{1}{2}mv^2}$ KINETIC ENERGY (Non-relativistic)

$p = mv$

$\boxed{K = \frac{p^2}{2m}}$

WHAT IF $v \sim c$? RELATIVISIC

$$E_p = \gamma mc^2 \Rightarrow E_p^2 = \gamma^2 (mc^2)^2 \qquad \gamma^2 = \frac{1}{1-\frac{v^2}{c^2}}$$

$$E_p^2\left(1 - \frac{v^2}{c^2}\right) = (mc^2)^2 \qquad E_p^2 - E_p^2\frac{v^2}{c^2} = (mc^2)^2$$

$$E_p^2 \Rightarrow \gamma^2 (mc^2)^2$$

$$\left(\frac{1}{1-\frac{v^2}{c^2}}\right)\frac{v^2}{c^2}\cdot m^2c^4 = \left(\frac{m^2v^2}{1-\frac{v^2}{c^2}}\right)c^2$$

$\boxed{p = \gamma m v}$

REMEMBER $\rightarrow$ $= \gamma^2 m^2 v^2 c^2 = p^2c^2$

$$\boxed{E_p^2 = (mc^2)^2 + (pc)^2}$$

MECHANICAL WORK

WORK = FORCE X DISTANCE → MORE PRECISELY

$= F_x \Delta x + F_y \Delta y + F_z \Delta z$

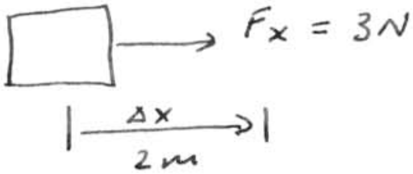
MATH: $\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$ ← DOT PRODUCT

$W = \vec{F}\cdot\Delta\vec{r} \qquad \Delta\vec{r} = \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}$

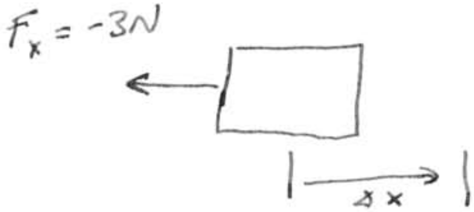
← SYSTEM $\qquad \Delta E_{sys} = W_{surroundings}$

SIGN ~~CONE~~ CONVENTION

$\boxed{1\text{ Joule} = 1\ N\cdot m}$

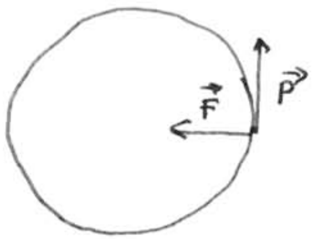


$W_{surr} = (+3)(+2)\ N\cdot m$
$= 6\ J$
$\boxed{\Delta E = +6J}$

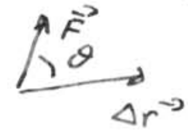


$W_{surr} = (-3)(2)\ N\cdot m$
$= -6J$
$\boxed{\Delta E = -6J}$

CIRCULAR ORBIT :



$$\vec{F} \cdot \Delta\vec{r} = |\vec{F}|\,|\Delta\vec{r}|\cos\theta$$

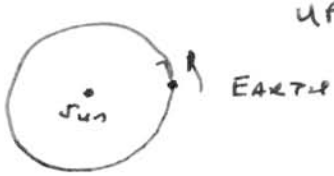


Suppose $\Delta\vec{r} \perp$ to $\vec{F}$

$\cos(\theta = 90^\circ) = 0$

$W = \vec{F} \cdot \Delta\vec{r} = 0$

EARTH $\Rightarrow$ NOT SPEEDING UP $\}$ $\Delta E = 0$ so $W = 0$ and $\vec{F} \Rightarrow \Delta\vec{r}$ are Perpendicular.



RECALL $\vec{F}_{\parallel} = 0$ $\quad \vec{F}_{\perp} \to \vec{F}_g = \vec{F}_{\perp}$ $\quad \vec{F}_g \perp$ to $\Delta\vec{P}$

Recall $\Delta\vec{r} = \dfrac{\vec{P}}{m}\Delta t$ so $\Delta\vec{r} \perp \vec{F}_g$

CONSIDER $\vec{F} = \vec{F}_{\parallel} + \vec{F}_{\perp}$

$$W = (\vec{F}_{\parallel} + \vec{F}_{\perp}) \cdot \Delta r = |\vec{F}_{\parallel}|\,\Delta r \qquad \boxed{W = F_{\parallel}\,\Delta r}$$

** $\vec{F}_{\perp}$ DOES NO WORK $\Rightarrow$ BUT CHANGES THE DIRECTION OF MOMENTUM **