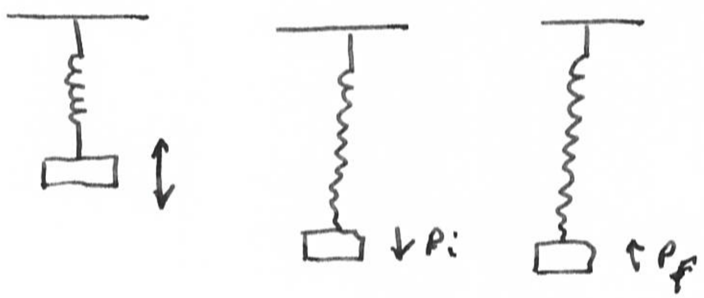
RATE OF CHANGE OF MOMENTUM

$$\vec{F}_N = \frac{d\vec{p}}{dt}$$

We can find an approximation for $\frac{d\vec{p}}{dt}$:

$$\frac{d\vec{p}}{dt} = \underbrace{p(t+\Delta t/2)}_{p_f} - \underbrace{p(t-\Delta t/2)}_{p_i} \,/\, [\Delta t]$$



NEAR THE MAXIMUM DISPLACEMENT $\rightarrow$ $\vec{p}_i \rightarrow \vec{p}_f$ CHANGES SIGN

This is CALLED A TURNING POINT

You might think exactly at this point $\vec{v}=0$ and maybe $\vec{p}=0$ so $\frac{d\vec{p}}{dt}=0$. THAT WOULD BE WRONG!

Suppose $\vec{p}_i = -\vec{p}$ $\quad \vec{p}_f = +\vec{p}$ $\quad$ THEN

$$\vec{F}_{NET} = \frac{\vec{p}-(-\vec{p})}{\Delta t} = 2\frac{\vec{p}}{\Delta t} \neq 0$$

$\downarrow \vec{p}_i \quad \uparrow \vec{p}_f \quad \uparrow \vec{F}_{Net}$

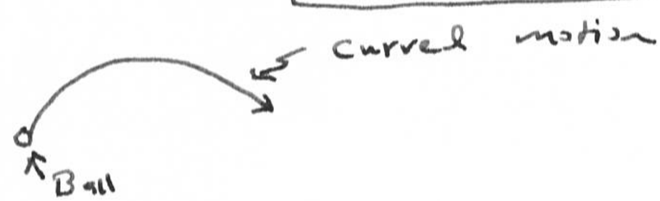
Rules:

EQUILIBRIUM $\rightarrow$ $\frac{d\vec{p}}{dt}=0$ for a LONG TIME. [USUALLY $\vec{v}=0$]

UNIFORM MOTION $\rightarrow$ $\frac{d\vec{p}}{dt}=0$

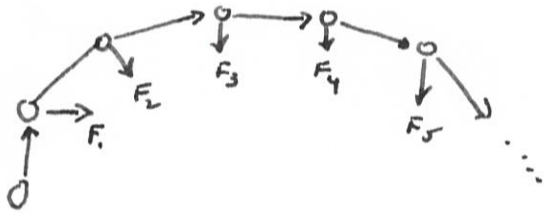
MOMENTARILY AT REST $\Big\}$ $\frac{d\vec{p}}{dt} \neq 0$ Because object does NOT remain at rest.

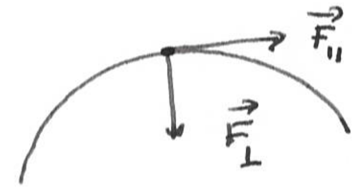
CURVING MOTION
[CIRCULAR]



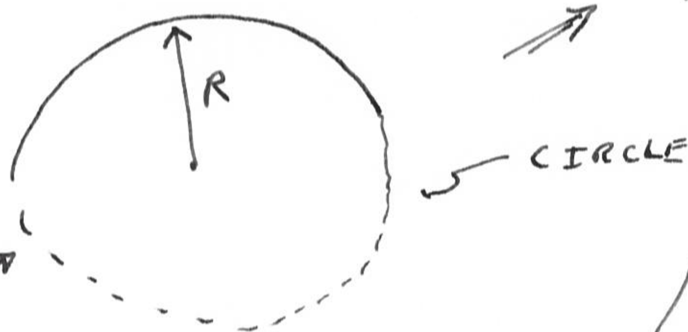
Let's think about how curved occurs →

$\Delta \vec{p}_i = \vec{F}_i \Delta t$ [Did this before]





Suppose this ~~traj~~ trajectory matches motion



$$\left(\frac{d\vec{p}}{dt}\right)_{\parallel} = \vec{F}_{\parallel}$$

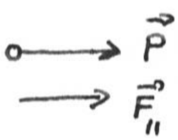
$$\left(\frac{d\vec{p}}{dt}\right)_{\perp} = \vec{F}_{\perp}$$

$\vec{F}_{\parallel}$ TANGENT TO CIRCLE

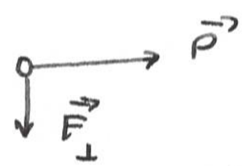
$\vec{F}_{\perp}$ POINTS TO <u>CENTER</u> OF CIRCLE.

Suppose $\vec{p} = |\vec{p}|\,\hat{p}$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$



Speeds up particle But DOES NOT CHANGE IT'S DIRECTION

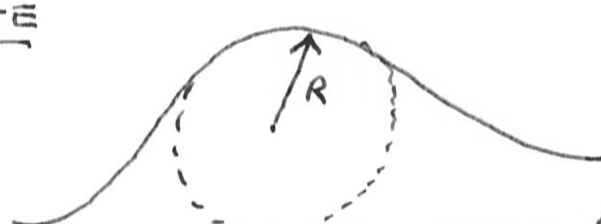


CHANGES ITS DIRECTION BUT INITIALLY NOT ITS SPEED

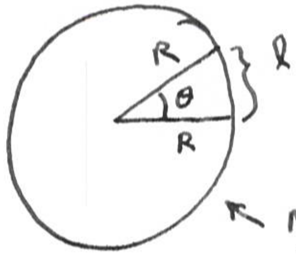
$$\boxed{\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p}}$$

$$\boxed{\vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt}}$$

<u>NOTE</u>

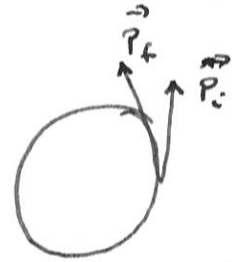


MOTION MAY LOOK LIKE IT IS CIRCULAR AT SOME POINT IN THE TRAJECTORY!

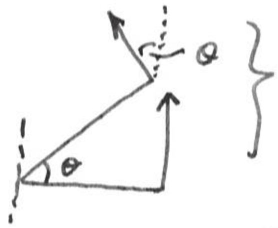


$$\boxed{\theta = \frac{l}{R} = \frac{v \Delta t}{R}}$$

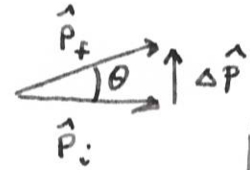
Moves at constant speed around a circle



$|\vec{p}_f| = |\vec{p}_i| = p$ CONSTANT SPEED



ANGLE IS ALSO θ



$$\boxed{\theta = \frac{|\Delta\vec{p}|}{p}}$$

$|\Delta\vec{p}| = p\,|\Delta\hat{p}|$

$\frac{|\Delta\vec{p}|}{p} = |\Delta\hat{p}|$

$|\Delta\hat{p}| = \frac{|\vec{v}|\Delta t}{R}$ $\quad \left|\frac{\Delta\hat{p}}{\Delta t}\right| = \frac{|\vec{v}|}{R}$

Limit $\Delta t \to 0$

$$\boxed{\left|\frac{d\hat{p}}{dt}\right| = \frac{|\vec{v}|}{R}}$$

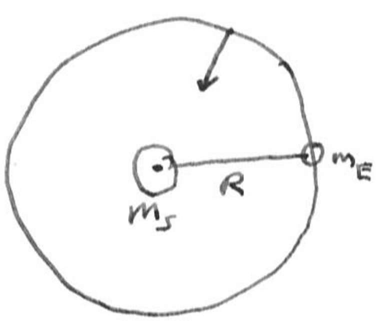
RECALL: $\vec{F}_\perp = |\vec{p}|\,\frac{d\hat{p}}{dt}$ $\quad |\vec{F}_\perp| = |\vec{p}|\,\left|\frac{d\hat{p}}{dt}\right|$

No relativity ($\gamma = 1$) $\quad |\vec{p}| = m|\vec{v}|$

$|\vec{F}_\perp| = m|\vec{v}|\left[\frac{|\vec{v}|}{R}\right] = m\frac{|\vec{v}|^2}{R} \Rightarrow |\vec{F}_\perp| = m|\vec{a}_\perp|$

$$\boxed{|\vec{a}_\perp| = \frac{|\vec{v}|^2}{R}}$$

EXAMPLE: EARTH MOVES IN A CIRCULAR ORBIT



$\vec{F}_g$ is always perpendicular to orbit

$\vec{F}_\perp = \vec{F}_g$ $\quad \vec{F}_\parallel = 0$

$\vec{F}_\parallel = 0$ Means $\frac{d|\vec{p}|}{dt} = 0$ $\Rightarrow$ EARTH MOVES AT CONSTANT SPEED.

$$|\vec{F}_\perp| = G\frac{m_E m_S}{R^2} = m_E\frac{v^2}{R} = |\vec{p}|\,\left|\frac{d\hat{p}}{dt}\right|$$

$v^2 = \frac{GM_s}{R}$ $\boxed{v = \sqrt{\frac{GM_s}{R}}}$ $v = \frac{2\pi R}{T} = \sqrt{\frac{GM_s}{R}}$

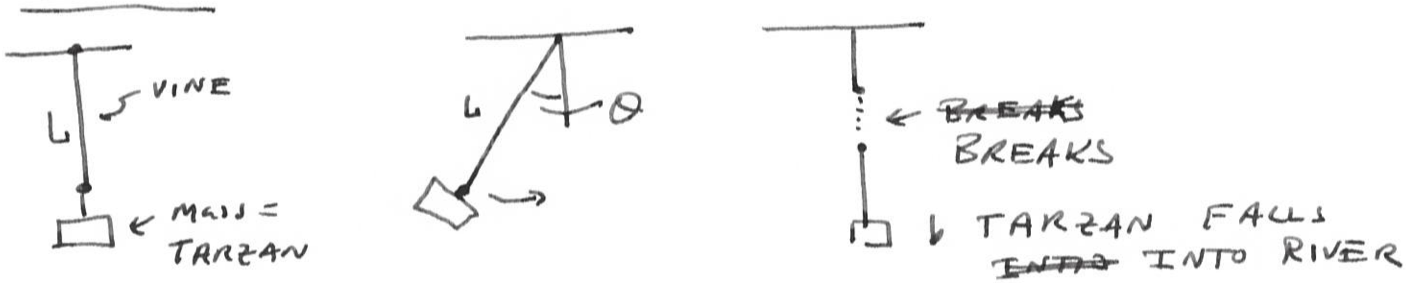
$$\boxed{T = \frac{2\pi R^{3/2}}{\sqrt{GM_s}}}$$

CHECK $M_s = 2 \times 10^{30}$ kg
$G = 6.7 \times 10^{-11}$ N-m²/kg²
$R = 1.5 \times 10^{11}$ m

$$T = \frac{2(3.14)[1.5 \times 10^{11}]^{3/2}}{\sqrt{(6.7 \times 10^{-11})(2 \times 10^{30})}} = \frac{3.65 \times 10^{17}}{1.16 \times 10^{10}} \text{ s} = 3.15 \times 10^7 \text{ s}$$

$$3.17 \times 10^7 \text{ s} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ day}}{24 \text{ hr}} = 365 \text{ days } (\ddot\smile)$$

EXAMPLE: TARZAN OF THE APES



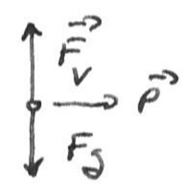
VINE SUPPORTS TARZAN

SURROUNDING: VINE (CONTACT) and EARTH (DISTANCE)

SYSTEM: TARZAN

$\boxed{\frac{d\vec{p}}{dt} = \vec{F}_N}$ USE AT $\theta \approx 0$ AT $\theta = 0$

$\vec{F}_{\parallel} = 0$ as No component from $\vec{F}_v$ or $\vec{F}_g$



$$\left|\frac{d\vec{p}}{dt}\right| = |\vec{F}_{\perp}| = |F_v - mg| = \frac{mv^2}{R} = \frac{mv^2}{L}$$

Suppose $m = 90$ kg
$L = 8$ m
$v = 12$ m/s

Find F_v at $\theta \approx 0$

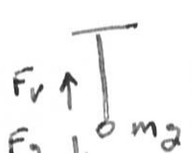
$F_v = mg + \frac{mv^2}{L}$

why $|F_v| > |F_g|$

If vine did not break he would SWING UP!

$$F_v = (90)(9.8) + \frac{(90)(12)^2}{(8)^2} = \underline{\underline{2{,}500 \text{ N}}}$$

IF vine ~~has~~ hung limp $\Rightarrow$



$F_v = F_g = \underline{\underline{882 \text{ N}}}$

FORCE ON VINE MUCH LARGER WHEN HE SWINGS!