FRICTION: Control of frictional forces important!

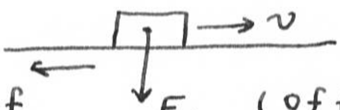
Sliding friction

$f_{friction} \cong \mu_k F_N$

$F_N \Rightarrow$ Normal force

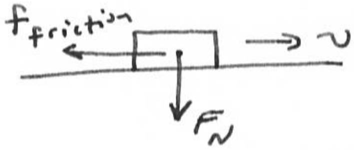
μ_k = Coefficient of sliding friction

Direction $\Rightarrow$ ~~oppositiv~~ opposite to motion



(often $F_N = -mg$)

$f_{friction} \Rightarrow$ INPENDENT of v.



STATIC FRICTION

$f_{friction} \leq \mu_s F_N$

STATIC FRICTION $\Rightarrow$ OBJECT NOT moving

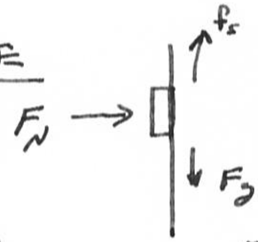
MAXIMUM $f_{friction}$ [STATIC] is $\mu_s F_N$.

EXAMPLE

$\mu_s = 0.6$

MASS = 3kg

WILL THIS SLIDE?



$F_N = 40N$

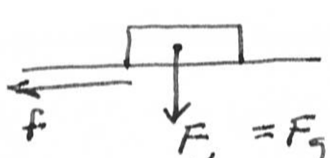
$F_g = -mg = -(3)(9.8) = -29.4\ N$

$f^s = \mu_s F_N = (0.6)(40) = 24N$

$F_g > f^s\ (max) = 24N$ SO OBJECT WILL SLIDE!

EXAMPLE

~~μ_s~~ $\mu_k = 0.4$
$m = 3kg$
$v_i = 6\ m/s$



How far will block slide?

$F_N = mg = 29.4\ N$

$f_s = \mu_s F_N = 11.8\ N$

No NET FORCE IN y-direction

$F_N + F_g = 0$

ALL FORCES HERE ARE CONSTANT

$$\boxed{x_f = x_i + v_i \Delta t + \frac{1}{2} \frac{f^k}{m} \Delta t^2}$$

WE SHOWED THIS! LETS USE IT.

WHEN DOES BLOCK STOP? HOW FAR WILL IT TRAVEL?

$$x_f = \underset{0}{\underset{\|}{x_i}} + 6\,\Delta t - \frac{11.8}{2(3)}\Delta t^2$$

$$\boxed{x_f = 6\Delta t - 1.97\,\Delta t^2} \Rightarrow \boxed{v_f = \frac{dx_f}{dt} = 6 - 3.94\,\Delta t}$$

$$v_f = 0 \qquad 6 - 3.94\,\Delta t = 0 \qquad \boxed{\Delta t = 1.52\,s}$$

$$x_f = 6(1.52) - 1.97(1.52)^2 = 9.14 - 4.55 = \underline{\underline{4.6\,m}}$$

READ SECTION 4.9 (BEST handled with computer model).

FORCE from <u>momentum principle</u>

$$\Delta\vec{p} = \vec{F}_{Net}\,\Delta t \qquad \vec{p}_f = \vec{p}_i + \vec{F}_{Net}\,\Delta t$$

$$\boxed{\frac{\Delta\vec{p}}{\Delta t} = \vec{F}_{NET}}$$

Calculus

$$\lim_{\Delta t\to 0}\frac{\Delta\vec{p}}{\Delta t} = \frac{d\vec{p}}{dt} = \vec{F}_{NET}$$

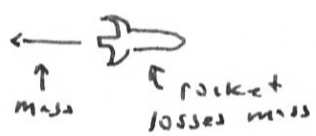
$$\boxed{\vec{F}_{NET} = \frac{d\vec{p}}{dt}}$$

Assume (Non-relativistic) momentum

$\hookrightarrow \vec{p} = m\vec{v} \quad (v \ll c)$

$$\frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt}$$

Note: $\frac{dm}{dt} = 0$ if mass is a constant. EXAMPLE WHERE $\frac{dm}{dt} \neq 0$ WATER ROCKET



$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} \quad \text{if} \quad \frac{dm}{dt} = 0$$

$$\vec{F}_N = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} \qquad \boxed{\vec{F}_N = m\vec{a}} \qquad \boxed{\vec{a} \equiv \frac{d\vec{v}}{dt}}$$

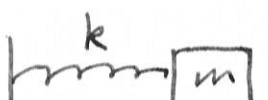
Non-relativistic form

$\vec{a}$ = acceleration

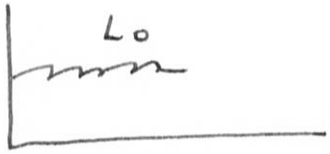
SPRING SOLUTION

We want

$x(t) \to$ EQUATION OF MOTION



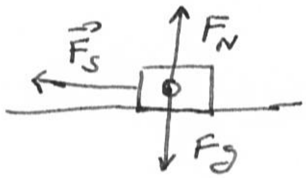
<u>No</u> friction.



$x=0$

$L = L_o + x$

$s = L - L_o = x$



$\boxed{F_N = F_g}$

x-direction

$$\frac{dp}{dt} = F_s = -k_s x$$

$p = m v_x$ $\quad m\frac{dv_x}{dt} = -k_s x \quad m\frac{d^2x}{dt^2} = -k_s x \quad \boxed{\frac{d^2x}{dt^2} = -\frac{k_s}{m}x}$

$\left(v_x = \frac{dx}{dt}\right)$

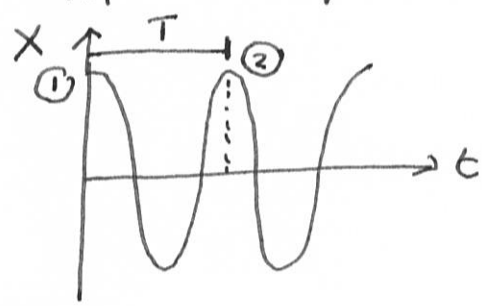
Guess $x = A\cos(\omega t)$

$$\frac{dx}{dt} = -A\omega \, \cancel{\cos}\ \sin(\omega t) \qquad \frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t)$$
$$= -\omega^2 [A\cos(\omega t)]$$
$$= -\omega^2 x$$

Compare:

$$\left.\begin{aligned}\frac{d^2x}{dt^2} &= -\omega^2 x\\ \frac{d^2x}{dt^2} &= -\frac{k_s}{m}x\end{aligned}\right\}$$

$x = A\cos(\omega t)$ is a solution provided $\omega^2 = \frac{k_s}{m}$ $\quad \omega = \sqrt{k_s/m}$



$\omega \Rightarrow$ Angular frequency

① $x = A$ $\quad t = 0$

② $x = A$ $\quad \omega t = 2\pi$

$t = \frac{2\pi}{\omega}$ = PERIOD

$T \equiv \frac{2\pi}{\omega}$ ← ω is in RADIANS/s

Nomenclature

PERIOD $\quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k_s}}$ $\quad$ (s)

FREQUENCY $\quad f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_s}{m}}$ $\quad$ (Hz)

↳ cycles per second

EXAMPLE

$m = 0.05$ kg

$L_o = 25$ cm $= 0.25$ m

$k_s = 4$ N/m $\quad L = 0.30$ m

(a) $A = ?$ $\quad x(0) = A\cos(0) = A = 5$ cm

(b) PERIOD $\quad \omega = \sqrt{k_s/m} = \sqrt{4/0.05} = 8.94$ rad/sec

$T = \frac{2\pi}{\omega} = \frac{2\pi}{8.94} = 0.7$ s

Issues: REAL SPRINGS HAVE MASS
AND ARE NOT HARMONIC

HARMONIC: $F_s = -k_s x$

ANHARMONIC: $F_s = -k_s x + \beta x^2$

often ~~Books~~ β can be neglected.

$\boxed{\text{SPEED OF SOUND}}$

$v_s = \omega d$ [No proof]

Copper

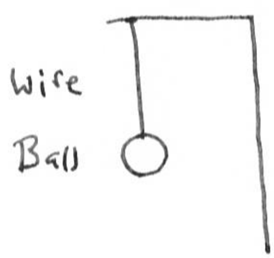
$$\omega = \sqrt{\frac{k_a}{m_a}}$$

$$v_s = \sqrt{\frac{k_a}{m_a}}\, d = \sqrt{\frac{30}{1.0\times10^{-25}}}\,(2.2\times10^{-10})\ \text{m/s} = 3800\ \frac{\text{m}}{\text{s}}$$

Expt. = 3600 m/s

AIR: $v_s = 340$ m/s

CHAPTER 5 | RATE OF CHANGE OF MOMENTUM



WHAT IF FORCE OF WIRE ON Ball?

$$\Delta\vec{P} = \vec{F}_N \Delta t \Rightarrow \vec{F}_N = \frac{d\vec{p}}{dt}$$

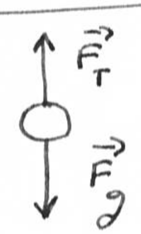
IF NOTHING IS MOVING $\Rightarrow \dfrac{d\vec{p}}{dt} = 0$

Surrounding: WIRE & GRAVITY

← SYSTEM

GRAVITY → "DISTANCE" FORCE [EARTH]

WIRE → CONTACT FORCE.



$$\vec{F}_T + \vec{F}_g = 0$$

$$\vec{F}_g = -mg\,\langle 0,1,0\rangle$$

y - COMPONENTS

$F_T = +mg$

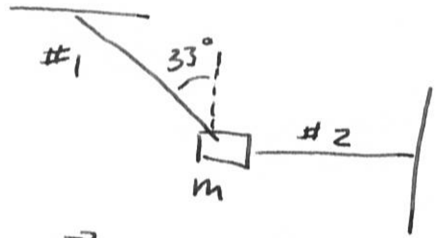
Suppose $m_{ball} = 1$ kg

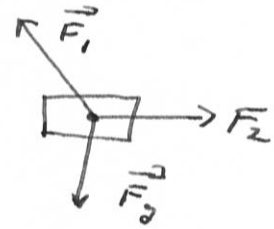
$F_T = 9.8$ N

Rules: (a) Choose System
(b) Show force at a distance (gravity or electrostatic)
(c) Show CONTACT FORCES

EXAMPLE

Two strings hold block.
WHAT ARE THE FORCES OF TENSION?





$\frac{d\vec{P}}{dt} = 0 \quad \vec{F}_N = 0$ z-component Not present

x-components $\quad F_{1,x} + F_{2,x} = 0$

$F_{\cancel{y},1} \cos(123) + F_{2,x} = 0$

y-component

$F_{1,y} + F_{\cancel{y}g} = 0$

$F_1 \cos(33) + F_g = 0$

$F_{2,y} = F_2$

$\cos(33) = 0.839 \qquad \cos(123) = -0.545$

$\cancel{0.839 F_1 + F_2 = 0}$

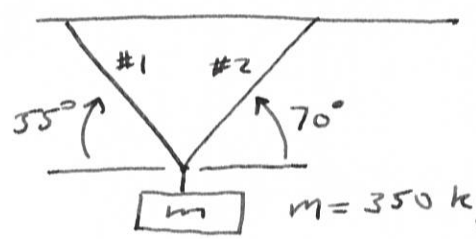
$-0.545 F_1 + F_2 = 0$

$0.839 F_1 - mg = 0$

$F_1 = \frac{(3)(9.8)}{0.839} = \underline{35.0 N}$

$F_2 = 0.545 F_1 = \underline{19.1 N}$

EXAMPLE



m = 350 kg

~~Copper~~ Aluminum wires

Radius 1.2 mm

$Y = 6.9 \times 10^{10}\ N/m^2$

Find F_1, F_2 and <u>STRAIN</u> in each wire

Find forces:

$\vec{F}_1 = F_1 \langle \cos(180-55), \cos(90-55), 0 \rangle$

$\vec{F}_2 = F_2 \langle \cos(70), \cos(90-70), 0 \rangle$

$\vec{F}_g = -mg \langle 0, 1, 0 \rangle \qquad mg = 3430 N$

z-component useless

X- Component

$F_1 \cos(125) + F_2 (70) = 0 \qquad -0.574 F_1 + 0.340 F_2 = 0$

$F_1 = 0.60 F_2$

Y- Component

$F_1 \cos(35) + F_2 \cos(20) - 3430 = 0$

$0.819 F_1 + 0.940 F_2 = 3430 \qquad 0.491 F_2 + 0.94 F_2 = 3430$

$\boxed{F_2 = 2400 N}$

$\boxed{F_1 = 1438 N}$

L10
p. 6

STRAIN $= F/A$ $\quad A = \pi r^2 = (3.14)(1.2 \times 10^{-3})^2\ m$

$A = 4.5 \times 10^{-6}\ m^2$

$(\text{STRAIN})_1 = F_1/A = 5.3 \times 10^8\ N/m^2$ $\quad \frac{\Delta L}{L} = Y^{-1}\ F/A$

$(\text{STRAIN})_2 = F_2/A = 3.2 \times 10^8\ N/m^2$

For #2

$$\left(\frac{\Delta L}{L}\right)_2 = Y^{-1}(F/A)_2 = [6.9 \times 10^{10}]^{-1}[5.3 \times 10^8] = 0.8 \times 10^{-2}$$

$$= \underline{0.8\ \%}$$

For #1

$$\left(\frac{\Delta L}{L}\right)_1 = Y^{-1}(F/A)_1 = [6.9 \times 10^{10}]^{-1}[3.2 \times 10^8]$$

$$= 0.5 \times 10^{-2} \text{ or } \underline{0.5\ \%}$$