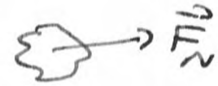
THREE PRINCIPLES



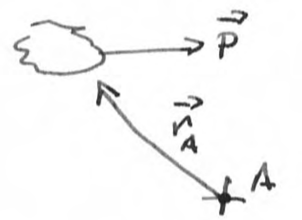
(1) MOMENTUM: $\frac{d\vec{p}}{dt} = \vec{F}_N$




$\vec{p}$ = CONSTANT IF $\vec{F}_N = 0$

(2) ANGULAR MOMENTUM:

$$\frac{d\vec{L}_A}{dt} = \vec{\tau}_{Net,A}$$




$\vec{L}_A = \vec{r}_A \times \vec{p}$

$\vec{\tau}_A = \vec{r}_A \times \vec{F}_N$

NOTE: DEPENDS ON "A"

IF $\vec{\tau}_{net,A} = 0$ $\quad \vec{L}_A$ = CONSTANT

(3) ENERGY: $\Delta E_{sys} = W + Q$

IF NO WORK IS INVOLVED OR HEAT FLOW
($W = 0$, $Q = 0$) ENERGY = CONSTANT

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EXAM PROBLEMS THAT WERE "HARD"

1. PLANET X

GIVEN
$$\begin{cases} M_x = M \quad m_x = m \\ R_x = R \\ g_x = G\frac{M_x}{R_x^2} = G\frac{M}{R^2} \end{cases}$$

WHAT IS $g_y$ in terms of $g_x$?

PLANET Y

$M_y = \cancel{M/2}\ 2M$ $\qquad R_y = R/2$ $\quad \boxed{g_y = ?}$

$m_y = 2m$

$$g_y = \frac{GM_y}{R_y^2} = \frac{2GM}{\frac{1}{4}R^2} = 8\left(\frac{GM}{R^2}\right)$$

$$\boxed{g_y = 8 g_x}$$

NOTE: $\cancel{\ }$ m NOT RELEVANT.

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2. PROTON $\quad m_p = 1.7 \times 10^{-27}$ kg $\qquad$ ($1\text{ ns} = 10^{-9}\text{ s}$)

$\vec{p} = (0, 0, 4.45\times10^{-19})$ kg-m/s $\qquad c = 3\times10^{8}$ m/s

How far does it travel in 2 ns?

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RECALL $\quad \vec{p} = \gamma m_p \vec{v}$ $\qquad \Delta\vec{r} = \vec{v}\Delta t$ $\qquad \vec{v} = v\hat{y}$

FIND $v$.

$$\vec{v} = \frac{\vec{p}}{m_p}\frac{1}{\gamma} \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$v = \frac{p}{m_p}\sqrt{1-v^2/c^2}$ $\qquad \frac{v}{c} = \frac{p}{m_p c}\sqrt{1-v^2/c^2}$ $\qquad$ Let $\beta = v/c$

$\beta = \left(\frac{p}{m_p c}\right)\left(\sqrt{1-\beta^2}\right)$ $\qquad \beta^2 = \frac{p^2}{(m_p c)^2}(1-\beta^2)$ $\qquad$ (p.2)

$$\boxed{\beta^2 = \frac{p^2/m_p^2c^2}{1+\frac{p^2}{m_p^2c^2}}}$$

$$\frac{p}{m_p c} = \frac{4.45\times10^{-19}}{1.7\times10^{-27}\,(3)\times10^{8}}$$

$$= \frac{4.45}{3(1.7)} = 0.8725 \qquad \beta^2 = \frac{(0.8725)^2}{1+(0.8725)^2}$$

$\Delta r = \beta c \Delta t$

$$\beta^2 = \frac{0.7612}{1.7612} = 0.4322$$

$$\boxed{\beta = 0.6574}$$

$= (0.6574)(3\times10^8)(2\times10^{-9})$

$\boxed{\Delta r = 0.395\ m}$

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(3) $\underset{p}{O} \xleftarrow{\;d\;} \xrightarrow{} \underset{p}{O}$ $\qquad$ RATIO OF GRAVITY – ELECTROSTATIC

$$F_g = G\frac{m_p m_p}{d^2} \qquad F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{d^2}$$

$$\frac{F_e}{F_g} = \frac{e^2/4\pi\varepsilon_0}{G\,m_p^2} = 1.2\times10^{36} = a$$

WHAT ABOUT → $\overset{\alpha}{O} \xleftarrow{\;d\;} \xrightarrow{} \overset{\alpha}{O}$

$\alpha = (2p, 2n)$

Charge $+2e$

$m_\alpha = 4m_p$

$$\frac{F_e}{F_g} = \frac{(2e)^2/4\pi\varepsilon_0}{G\,(4m_p)^2} = \frac{4}{16}\Big[\underbrace{\frac{e^2/4\pi\varepsilon_0}{G\,m_p^2}}_{=a}\Big]$$

$$\boxed{\frac{F_e}{F_g} = \frac{a}{4}}$$

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(4) PARTICLE: WHAT IS VELOCITY OF $\pi^+$ PARTICLE?

$X \longrightarrow \pi^+ + \pi^-$ $\qquad m_X = 496\ MeV/c^2$

$m_\pi = 139\ MeV/c^2$

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$\Delta E = 0$ $\qquad E_i = m_X c^2$ $\qquad \pi^- \xleftarrow{-v} \circ \quad \circ \xrightarrow{v} \pi^+$

$E_f = 2\gamma m_\pi c^2$

$$\boxed{\gamma = \frac{1}{\sqrt{1-v^2/c^2}}}$$

Need to know $\gamma$

$2\gamma m_\pi c^2 = m_x c^2$ $\qquad \gamma = \frac{m_x}{2m_\pi} = \frac{496}{2(139)} = 1.784$

$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ $\qquad 1-\frac{v^2}{c^2} = \frac{1}{\gamma^2}$ $\qquad \frac{v^2}{c^2} = 1-\frac{1}{\gamma^2} = 0.68\cancel{04}$ 57

$\boxed{v = \cancel{0.8309}\,c \quad 0.8281\,c}$

$v = 2.484 \times 10^8$ m/s

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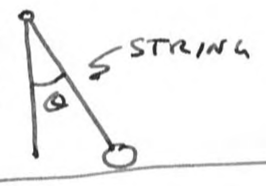
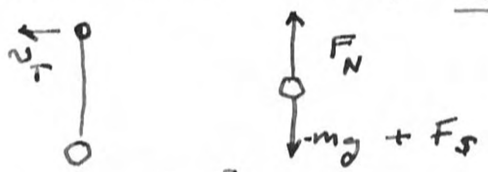
(5) Loop-the-loop: WHAT IS $v$ SO IT GOES OVER THE TOP?

L=0.24m
m=0.5kg




NOTE: THIS IS A STRING.




$\cancel{F_N \neq 0}$ $\quad F_s \neq 0$ OTHERWISE THE STRING GOES LIMP

$F_N = -\frac{mv_T^2}{L} = -mg + F_s$

WHEN $F_s = 0$

$\frac{mv_T^2}{L} = mg$

$\boxed{v_T^{*2} = gL}$

CONSERVE ENERGY $\quad E_i = K_i \quad E_f = K_f + U_f$

$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 + 2mgL$

$v_i^2 = v_f^2 + 4gL = 5gL \qquad v_i = \sqrt{5gL}$

$v_i = \sqrt{5(9.8)\,0.24}$ m/s

$\boxed{v_i = 3.429 \text{ m/s}}$

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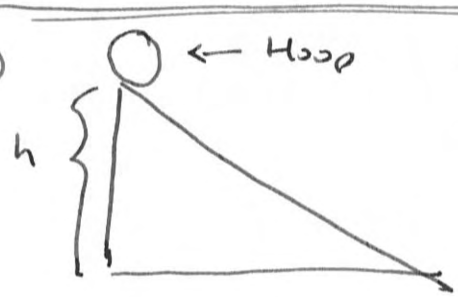
(6)




What is its speed at bottom of incline?

$v = \omega R$

$E_f = E_i$

$E_i = mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}MR^2\frac{v^2}{R^2}$

$gh = \frac{1}{2}v^2 + \frac{1}{2}v^2 \qquad \boxed{v = \sqrt{gh}}$

Suppose hoop


$$E_i = (M+m)gh \qquad E_f = \tfrac{1}{2}(m+M)v^2 + \tfrac{1}{2}I\omega^2$$

$I = \frac{1}{2}MR^2$ unchanged!

$$(M+m)gh = \tfrac{1}{2}(M+m)v^2 + \tfrac{1}{2}MR^2\frac{v^2}{R^2}$$

$$v^2 = \frac{(M+m)gh}{\frac{1}{2}(M+m) + \frac{1}{2}M} = \left(\frac{M+m}{M+m/2}\right)gh$$

$$\boxed{v = \sqrt{2gh\left(\frac{M+m}{2M+m}\right)}}$$

If $M >> m$ $\cancel{2M+m}$

$$\boxed{v = \sqrt{gh}}$$

SPEEDS UP!

If $M << m$

$$\boxed{v = \sqrt{2gh}}$$

(7)


Lowest Point $\frac{mv^2}{L} = mg$

True or false?

(a) At $\theta = 0$ change in momentum is zero.

(b) At $\theta_{max}$ the speed is zero and tangential acceleration is a maximum

(c) $F_T = 2mg$

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(a) False. Momentum is changing.

(b) True.

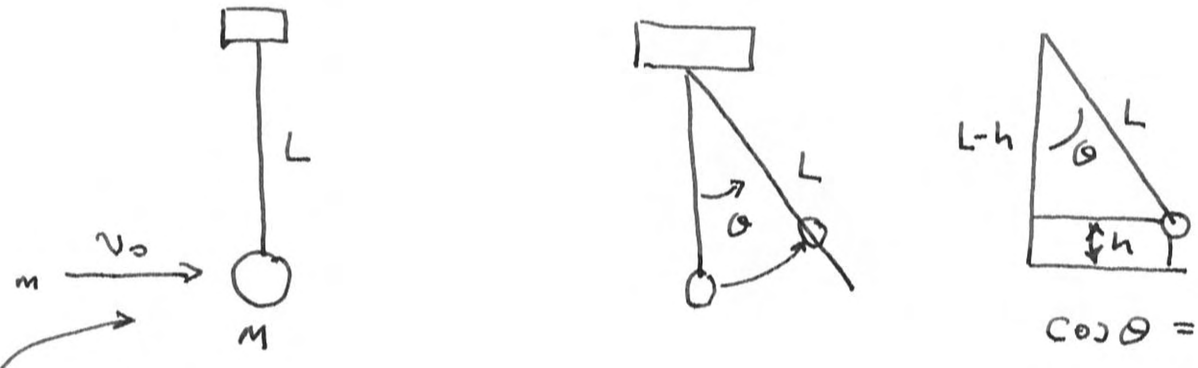
(c) $\underbrace{F_N = F_T - mg = \frac{mv^2}{L}}_{\text{TRUE at } \theta = 0} \Rightarrow \underline{\text{IF}}\ \frac{mv^2}{L} = mg \Rightarrow$

$F_T - mg = mg$

$F_T = 2mg$

True.

# (8) Ballistic Pendulum




$$\cos\theta = \frac{L-h}{L}$$

$$h = L\,(1-\cos\theta)$$

Momentum Conservation

$$\begin{bmatrix} m = 9.7\ g & M = 4\ kg \\ L = 1.6\ m & \theta = 36^\circ \end{bmatrix}$$

$$P_i = m v_o$$

$$P_f = (m+M)\,v_f$$

$$v_o = \left(\frac{M+m}{m}\right) v_f \cong \left(\frac{M}{m}\right) v_f$$

WHAT IS $v_f$?

$$\tfrac{1}{2} M v_f^2 = mgh \qquad v_f^* = \sqrt{2gL(1-\cos\theta)}$$

$$v_o = \left(\frac{M}{m}\right)\sqrt{2gL(1-\cos\theta)}$$

$$= \frac{4}{0.0097}\sqrt{2(9.8)(1.6)(1-\cos(36))} = 1009.2\ \frac{m}{s}$$