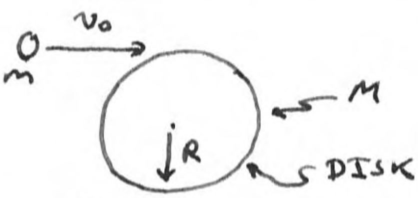
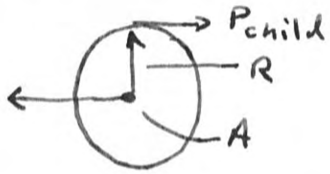
① EXAMPLE PROBLEMS : Exam 4



CHILD Jumps ON "MERRY-GO-ROUND"
How fast does merry-go-round rotate?
[INITIALLY!]

$v_0 = 3\ m/s$ $m = 40\ kg$
$M = 600\ kg$ $R = 2m$

Solution: USE $\Delta \vec{L} = \vec{\tau}_{net}\, \Delta t$
TAKE TORQUE ABOUT AXLE OF RIDE



The forces when the child jumps on ride cancel as they are internal (child + disk). The axle force generates No torque.

$$\Delta L = 0$$

L_i = angular momentum of child
(disk is STATIONARY)

$= R P_{child}$ ← $P \perp R$ right before child jumps on the ride.
$= m v_0 R$
$= (2)(40)(3) = \underline{240\ kg\text{-}m^2/s}$

$$L_f = \underbrace{I_{DISK}\,\omega}_{\text{DISK ROTATION}} + \underbrace{Rmv}_{L_{TRANS}}$$

L_{TRANS} for child (v - velocity after jumping on the ride).

$v = \omega R$
$I_{disk} = \frac{1}{2} M R^2$

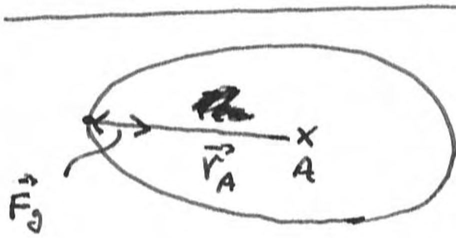
$L_f = \frac{1}{2} M R^2 \omega + R m \omega R$
$= \frac{1}{2} [M + \cancel{m}\ 2m] R^2 \omega$

$$\omega = \frac{L_i}{\frac{1}{2}[M+2m]R^2} = \frac{240}{\frac{1}{2}(\cancel{300}\ 600 + 80)(2)^2} = \cancel{0.176}\ \frac{240}{1360}$$

$\omega = 0.176\ rad/s$

---

② Comets: DOES THE SUN EXERT A TORQUE ON A ~~Comet~~ COMET?



CHOOSE "A" to BE ON THE SUN.

$\vec{\tau}_A = \vec{r}_A \times \vec{F}_g = 0$

BECAUSE $\vec{r}_A \parallel \vec{F}_g$

$\vec{V}_A \parallel \vec{F}_g$ so the cross product is <u>ZERO</u>.

$\vec{L} = \text{CONSTANT}$

Note: L = CONSTANT MEANS

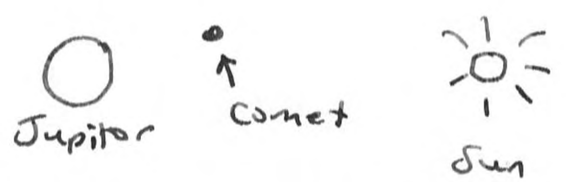
$P_1 r_1 = P_2 r_2$ or $v_1 r_1 = v_2 r_2$

$v_1 = \frac{r_2}{r_1} v_2$ If $r_2 \ll r_1$ then $v_1 \ll v_2$.

Closer the comet is to the sun – the faster it goes!

Other comments: Technically we assumed $M_S \gg m_c$ so that the comets motion does not ~~affect~~ affect the sun's motion. [<u>Good</u> approximation].

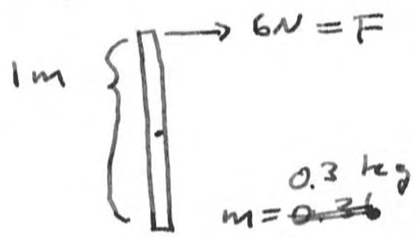
WHAT ABOUT THIS SITUATION →



If a third body is in play – can it change the value of $\vec{L}$? Yes! WHY?

---

(3) Suppose we have a meter stick as shown: ~~What is V_{cm}?~~



→ What is $\frac{d\omega}{dt}$ at this moment?

→ WHAT IS dV_{cm}/dt?

SOLUTION:

$\frac{dP}{dt} = F_N = m\frac{dV_{cm}}{dt}$ $\quad \frac{dV_{cm}}{dt} = \frac{F_N}{m}$

$= \frac{6}{0.3} = 20 \frac{m}{s}$

$\frac{dL_{rot}}{dt} = \tau$ $\quad I\frac{d\omega}{dt} = F \cdot r \cdot \sin\theta$

$0.025 \frac{d\omega}{dt} = 6(\tfrac{1}{2})(1) = 3$

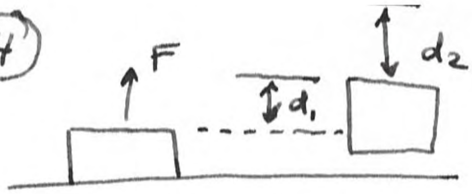
$I = \frac{1}{12} ML^2$

$= \frac{1}{12}(0.3) = 0.025 \ \text{kg m}^2$

$\frac{d\omega}{dt} = \frac{3}{0.025} = 120 \ \text{Rad/sec}$

P3

(4)



You apply a force F to pull string out of a box. The box moves d_1 when d_2 string is pulled out of the box.

WHAT IS THE CHANGE IN THE INTERNAL ENERGY?

POINT PARTICLE PICTURE

$\Delta K_{TRANS} = W_{cm}$ $\qquad$ $\Delta K_{TRANS} = K_{Trans}$ (initially at rest)

$W_{cm} = Fd_1 - \underbrace{md_1 g}_{\text{work by earth.}}$

REAL SYSTEM

$\Delta E_{sys} = \Delta K_{TRANS} + \Delta E_{int}$

$= W_{sys} = F(d_1 + d_2) - mgd_1$

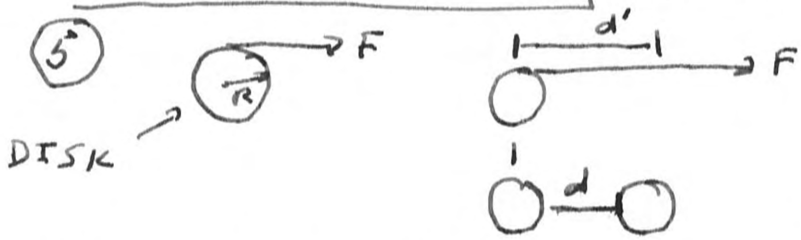
$\Delta K_{TRAN} + \Delta E_{int} = Fd_1 + Fd_2 - mgd_1$

$K_{TRAN} + \Delta E_{int} = Fd_1 - md_1 g + \Delta E_{int} = Fd_1 + Fd_2 - mgd_1$

$\boxed{\Delta E_{int} = Fd_2}$

Another "Point" Particle Problem

(5)



You pull out d' string

DISK MOVES d.

What is d'/d?

POINT PARTICLE

$\Delta K_{TRANS} = K_{TRANS} = W_{cm} = Fd$ $\qquad$ $\boxed{\frac{1}{2}Mv_{cm}^2 = Fd}$

REAL SYSTEM

$\Delta K_{TRANS} + \Delta K_{rot} = (Work)_{sys}$

$K_{TRANS} + K_{rot} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$

$\omega = v/R$

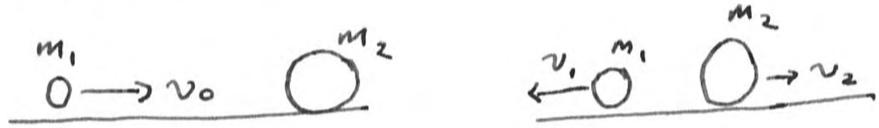
$K_{TRANS} + K_{rot} = Fd'$

$\frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left[\frac{1}{2}MR^2\right]\frac{v_{cm}^2}{R^2} = Fd'$

$\ast \; \frac{3}{4}mv_{cm}^2 = Fd'$

$$\frac{Fd'}{Fd} = \frac{3/4\, Mv_{cm}^2}{1/2\, mv_{cm}^2} = \frac{d'}{d} = \frac{3}{2}$$

(6) Inelastic Collision



Suppose $v_1 = -\frac{1}{3} v_0$, $m_2 = 10\, m_1$, $v_0 = 10\ m/s$, $m_1 = 1\ kg$

WHAT IS v_2? How much energy ~~wh~~ went into ΔE_{int}?

$P_f = P_i$ (ALWAYS)

$+ m_1 v_1 + m_2 v_2 = m_1 v_0$

$+ m_1 \left[-\frac{1}{3} v_0\right] + m_2 v_2 = m_1 v_0$

$m_2 v_2 = m_1 v_0 + \frac{1}{3} m_1 v_0 = \frac{4}{3} m_1 v_0$

$v_2 = \frac{4}{3} \frac{m_1}{m_2} v_0 = \frac{4}{3} \frac{1}{10} 10 = 1.333 \frac{m}{s}$

$\boxed{v_2 = 1.33\ m/s}$

$\Delta E_{sys} = 0$

$\Delta K + \Delta E_{int} = 0$

$\Delta E_{int} = -\Delta K = -\left[K_f - K_i\right]$

$= -\left[\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \frac{m_1}{2} v_0^2\right]$

$= -\left[\frac{1}{2} m_1 \frac{v_0^2}{9} - \frac{m_1}{2} v_0^2 + \frac{1}{2} m_2 \frac{16}{9} \frac{m_1^2}{m_2^2} v_0^2\right]$

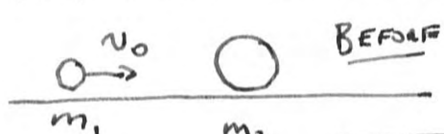
$= -\left[-\frac{8}{9}\left(\frac{1}{2} m_1 v_0^2\right) + \frac{16}{9} \frac{m_1}{m_2} \frac{m_1}{2} v_0^2\right] = -\left[-\frac{8}{9} + \frac{16}{9} \frac{1}{10}\right] \frac{1}{2} m v_0^2$

$= -\left[\frac{-80 + 16}{90}\right] \frac{1}{2} m v_0^2 = \frac{64}{90}\left[\frac{1}{2} m v_0^2\right] = \frac{32}{45}\left[\frac{1}{2} m_1 v_0^2\right]$

$\Delta E_i = \frac{32}{45}\left[\frac{1}{2}(1)(100)\right] = \frac{32(50)}{45} = \boxed{35.55\ J}$

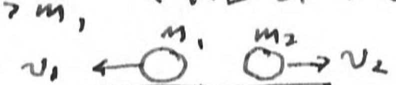
---

(7) ELASTIC COLLISION

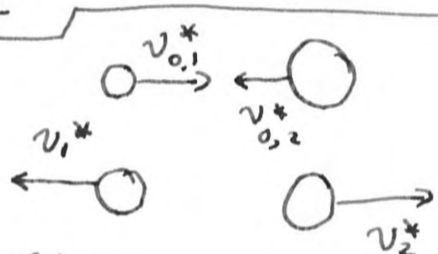


Suppose $m_2 >> m_1$

(What is final KE of Each?)



Center of Mass



REMEMBER

$v_1^* = -v_{0,1}^*$

$v_2^* = -v_{0,2}^*$

$v^* = v_{Lab} - v_{cm}$

$\uparrow$

C of M

$v_{cm} = \frac{m_1 v_0}{m_1 + m_2}$

$v_1^* = -v_{0,1}^*$

$\downarrow$

$v_1 - v_{cm} = -\left[v_0 - v_{cm}\right]$

$$v_1 = 2v_{cm} - v_0 = 2\left[\frac{m_1 v_0}{m_1+m_2}\right] - v_0 = \left(\frac{m_1-m_2}{m_1+m_2}\right) v_0$$

$m_1 >> m_2$ $\boxed{v_1 = -v_0}$ $\quad v_2^* = v_2 - v_{cm} = -[0 - v_{cm}]$

$m_1 >> m_2$ $\boxed{v_2 = 0}$ $\quad v_2 = 2v_{cm} \quad v_2 = \frac{2m_1 v_0}{m_1+m_2}$

Issue: if $v_1 = -v_0$ and $v_2 = 0$ <u>IS MOMENTUM CONSERVED</u>?

$$m_1 v_0 = m_1 v_1 + m_2 v_2 \Rightarrow m_1 v_0 = -m v_0 + 0 \quad ? \text{ No?!}$$

$\boxed{\text{WHAT WENT WRONG?}}$ LOOK at $m_2 v_2 \Rightarrow$

$m_2 \to \infty$ $v_2 \to 0$ WE NEED TO BECAREFUL

If WE HAVE "∞" times "ZERO"!

~~AND~~ ANSWER: BECAREFUL IN DISCARDING TERMS

$$v_1 = \left(\frac{m_1-m_2}{m_1+m_2}\right) v_0 \quad \text{and} \quad v_2 = \frac{2m_1 v_0}{m_1+m_2}$$

KEEP (m_1, m_2)

$$m_1 v_0 = m_1\left(\frac{m_1-m_2}{m_1+m_2}\right) v_0 + \frac{2m_1 m_2}{m_1+m_2} v_0$$

$$= \frac{m_1^2 - m_1 m_2 + 2m_1 m_2}{m_1+m_2} = \frac{m_1(m_1+m_2)}{m_1+m_2} v_0$$

$$= m_1 v_0$$

PROBABLY BEST TO WRITE

$v_1 \cong -v_0 \quad v_2 \cong 0 \Leftarrow$ "APPROXIMATE"

<u>ADVANCED COMMENT</u>: If we use $\frac{1}{1+x} \cong 1 - x$

then

$$v_1 \cong -v_0 + \left(\frac{m_1}{m_2}\right)^2 v_0$$

$$v_2 \cong \frac{2m_1}{m_2} v_0$$

$$m_1 v_1 = \cancel{v_0} - m v_0 + (m_1 v_0)\left(\frac{m_1}{m_2}\right)^2$$

$$m_2 v_2 = +2m_1 v_0$$

$$m_1 v_1 + m_2 v_2 = m_1 v_0 + m_1 v_0 \left(\frac{m_1}{m_2}\right)^2$$

$\boxed{\text{Now take limit } m_2 \to \infty}$ and $m_1 v_1 + m_2 v_2 = m_1 v_0$

as REQUIRED!