Summary on unit 4(update: 11/21/10)

Sec 10.1-7 Collisions: Interaction over a short time. $\Delta \mathbf{P} = \mathbf{F}\Delta t \approx 0$, $\Delta E = \Delta(K + E_{int}) = W + Q \approx 0$.

3 types: Elastic: ΔK=0. Inelastic: $\Delta K < 0$. Maximally inelastic: stuck together, or $K_{rel} = 0$.

Change of internal energy: $E = K + E_{int}$. $\Delta E = 0$ implies $\Delta E_{int} = -\Delta K = -\Delta K_{rel}$ (Verify last step)

Changing reference frames. Notations: $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2$, where $\mathbf{p}'_1 \equiv \mathbf{p}_{1f}$ and $\mathbf{p}'_2 \equiv \mathbf{p}_{2f}$.

- Bowling ball (Bb) hits pingpong ball(Pb) at rest. In lab frame: $Mv_1 + mv_2(0) = Mv'_1 + mv'_2$.
- Bb-frame is where Bb at rest. Velocities in Bb-frame and in lab-frame are related by $V^* = V - v_{Bb}$, Pb in Bb-frame $v{*}_2 = v_2 - v_{Bb} = -v_{Bb}$.
- After elastic bounce, the Bb-frame velocity is: $v_2'^* = -v_2^* = v_{Bb}$.
- In lab frame: Since in general $V = V^* + v_{Bb}$, final lab-frame Pb velocity is $v'_2 = v_2'^* + v_{Bb} = 2v_{Bb}$.

What is the sign of v'_1 in elastic collisions where v_2=0 in 3 cases: $m_1 < m_2$, $m_1 = m_2$, and $m_1 > m_2$?

Elastic headon collisions for arbitray m_1, m_2, v_1 v_2: $v'_1 = 2v_{cm} - v_1$ and $v'_2 = 2v_{cm} - v_2$. Derive.

Ballistic Pendulum 1d collision: (i) Initial; (A) after collision m+M stuck together m+M; (B) block reached final height. Steps to determine v_1. Geometry: From L, θ to h. From h to K_A. From K$_A$ to v_1.

(i) to (A): $\Delta P = P_A - P_i$, $P_i = mv_1$, $P_A = (m+M)v_A$. (A) to (B): $\Delta E = \Delta K + \Delta U = 0$. $\Delta E_{int} = 0$, why?

Beyond 1d collisions:

Impact parameter b: b decreases, scattering angle increases.

Rutherford scattering: It led to the discovery of nuclei in atoms.

Sec 11.1-9 Angular momentum in rotational motion

<u>Translational rotation</u>: $\mathbf{L} \equiv \mathbf{L}_O = \mathbf{r} \times \mathbf{p} = rpsin\theta\, \hat{\mathbf{n}} = r_\perp p\, \hat{\mathbf{n}}$

Angular momentum Principle (AMP) about O (subscript suppressed): $d\mathbf{L}/dt = \vec{\tau}$, $\vec{\tau} = d(\mathbf{r} \times \mathbf{p})/dt = r_\perp F \hat{\mathbf{n}}$.

Planetary motion: $d\mathbf{L}_O/dt = \mathbf{r} \times \mathbf{F} = 0$, since $\mathbf{F} = -GMm\hat{\mathbf{r}}/r^2$. This implies that $\mathbf{L}_O = const$.

Kepler's 2nd law: $\frac{\Delta A}{\Delta t} = const$. $\Delta A = \frac{1}{2} r_\perp v \Delta t$. $\frac{\Delta A}{\Delta t} = \frac{r_\perp p}{2m} = \frac{L}{2m}$.

<u>Angular momentum of multiparticle system</u>: $\mathbf{L}_{tot} = \mathbf{L}_{trans} + \mathbf{L}_{rot}$.

Example: A dumbbell has masses $m_1 = m_2 = m$, separated by 2a. Rotational angular velocity: $\omega_1 \hat{\mathbf{n}}$.

Assume its cm rotates along $\hat{\mathbf{n}}$, with a uniform velocity along a ciculr arc, where $v_{cm} = r_{cm}\omega_2$.

The two angular momenta vectors are $L_{rot} = 2ma^2\omega_1\hat{\mathbf{n}}$ and $\mathbf{L}_{cm} = 2mr_{cm}^2\omega_2\hat{\mathbf{n}}$.

<u>Angular momentum of rotation of a rigid body</u>. $\mathbf{L}_{tot,A}\hat{\mathbf{n}} = (\mathbf{L}_{cm,A} + \mathbf{L}_{rot})\hat{\mathbf{n}}$.

AMP for rigid body rotation about A. $\frac{d\mathbf{L}_A}{dt} = \frac{d\mathbf{L}_{trans,A}}{dt} + \frac{d\mathbf{L}_{rot}}{dt} = \mathbf{r}_{cm,A} \times \mathbf{F}_{cm} + \tau_{rot}\hat{\mathbf{n}}$.

Boy+MGR: Torque, negligible: $\frac{\Delta L}{\Delta t} \approx 0$. $L_i = amv_i$, $L_f = ma^2\omega_f + I_{MGR} \cdot \omega_f$.

Meteorite hits a satellite. Collisions: $\Delta \mathbf{P} = 0$, $\Delta \mathbf{L} = \Delta L\hat{\mathbf{n}} = 0$. Use $\Delta E = 0$ to find ΔE_{int}.

<u>Prediction on θ</u>. Linear to rotation: $v = \frac{ds}{dt}$ to $\omega = \frac{d\theta}{dt}$, $p = mv$ to $L = I\omega$, F to τ.

AMP for Rigid body rotation: $\Delta L = \tau \Delta t$ leads to $L_f - L_i = \tau \Delta t$, or $\omega_f - \omega_i = \frac{\tau \Delta t}{I}$.

Position update: $\theta_f = \theta_i + \omega_{avg}\Delta t$, where for constant F, $\omega_{avg} = \frac{\omega_f + \omega_i}{2}$.