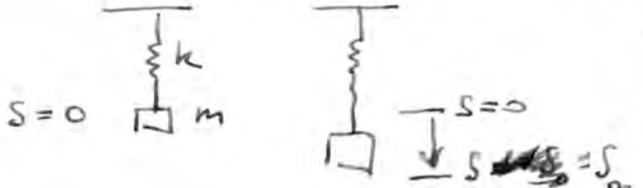
POTENTIAL ENERGY: SPRING



$U_s = \frac{1}{2} k s^2$

$U_g = mgs$

$U = \frac{1}{2} k s^2 + mgs$

(STATIC CASE
$S_{eq} = -\frac{mg}{k}$)

$F = -\frac{du}{dS} = -ks - mg$; $\boxed{S_{eq} = -\frac{mg}{k}}$ $\quad U_{eq} = \frac{1}{2} k \left[\frac{m^2 g^2}{k^2}\right] - \frac{m^2 g^2}{k}$

$\boxed{U_{eq} = -\frac{m^2 g^2}{2k}}$

MOVING !! ⇓

Suppose initially
$v_i = 0$ at $s = 0$
$E_i = 0$

FINAL
$s = s_{eq}$
$v_f = ?$
$E_f = K_f + U_f$

$E_i = 0 = K_f + U_f = E_f$

$K_f = \underbrace{\frac{1}{2} \frac{m^2 g^2}{k}}_{-U_f} = \frac{1}{2} m v_f^2$

$v_f = \sqrt{\frac{m}{k}}\, g$

EQUATION OF MOTION

$F = -ks - mg = m \frac{d^2 s}{dt^2}$

$\boxed{\frac{d^2 s}{dt^2} = -\frac{ks}{m} - g}$

SOLUTION ?

GUESS ⇒ $S = S_0 \cos(\omega t) - S_0$

WHY THIS GUESS? $\quad S(t=0) \quad S(t) = 0 = S_0 - S_0 \quad$ (OK)

$v(t=0) \quad \frac{dS}{dt} = -\omega S_0 \sin(\omega t)$
$= 0$ at $t=0$ (OK)

$\frac{dS}{dt} = -\omega S_0 \sin(\omega t)$

$\frac{d^2 S}{dt^2} = -\omega^2 S_0 \cos(\omega t) \qquad$ But $\cos(\omega t) = \left(\frac{S + S_0}{S_0}\right)$

$\frac{d^2 s}{dt^2} = -\omega^2 S_0 \left[\frac{S + S_0}{S_0}\right] = -\underbrace{\omega^2}_{\frac{k}{m}} S - \underbrace{\omega^2 S_0}_{g}$

$\boxed{S_0 = \frac{g}{\omega^2} = \frac{mg}{k}}$

↗ $= \cancel{\frac{g}{\frac{m}{k}}} = \cancel{\frac{gk}{m}}$

$(S_0 = -S_{eq})$

THE GUESS IS
RIGHT IF ⇒ $\omega = \sqrt{\frac{k}{m}} \qquad \omega^2 S_0 = g$

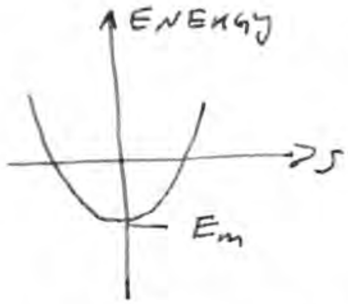
SOLUTION ⇒ $\boxed{S = \frac{mg}{k} \cos(\omega t) - \frac{mg}{k}}$

WHEN $S = -\frac{mg}{k} \quad \cos(\omega t) = 0 \quad \Rightarrow \sin(\omega t) = 1$

$v_f = -\omega S_0 \underset{1}{\underset{\parallel}{[\sin(\omega t)]}} = -\sqrt{\frac{m}{k}}\, g$

A LOT EASIER
USING U !!

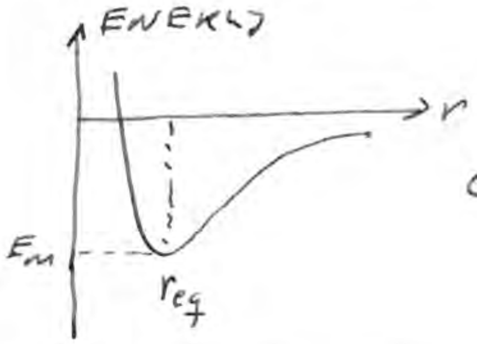
MOLECULE O~~O



↙ OK NEAR $s=0$

NEAR r_{eq} ⇒ MOLECULE LOOKS LIKE "SPRING" HOLDING IT TOGETHER

"REAL" MOLECULE



← r →
O~~O

MORSE POTENTIAL

$$U_M = E_M\left[1-\exp(-\alpha(r-r_{eq}))\right]^2 - E_M$$

$r = r_{eq}$ $\quad U_M(r=r_{eq}) = E_M[1-1]^2 - E_M = -E_M$

$r >> r_{eq}$ $\quad \exp(-\alpha(r-r_{eq})) \approx 0 \quad U_M = E_M(1) - E_M \Rightarrow 0$

$$F_M = -\frac{dU_M}{dr} = -2E_M\left[1-\exp(-\alpha(r-r_{eq}))\right] \times \left[+\alpha\exp(-\alpha(r-r_{eq}))\right] = 0$$

when $r = r_{eq}$ (OK)

Suppose $r \approx r_{eq}$: MATH TRICK!

$\exp(-x) \cong 1 - x$ when $x << 1$

$\exp(-\alpha(r-r_{aq})) \cong 1 - \alpha(r-r_{eq})$

$$U_M = E_M\left[1 - \exp(-\alpha(r-r_{eq})\right]^2 - E_M$$

$$\cong E_M\left[1-(1-\alpha(r-r_{eq}))\right]^2 - E_M$$

$$U_M \cong E_M\alpha^2(r-r_{eq})^2 - E_M$$

$s = r - r_{eq}$ $\quad U_M \cong \alpha^2 E_M s^2 - E_M$

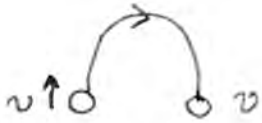
$k_s = 2\alpha^2 E_M$ $\quad \boxed{U_M \cong \frac{1}{2}k_s s^2 - E_M}$

→ GOOD APPROX. ⇒ SPRING NEAR $r \approx r_{eq}$.

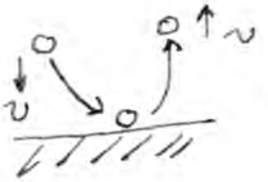
POTENTIAL ENERGY IS NOT PATH DEPENDENT

PATHS ⇒ (1, 2, 3) DO NOT CHANGE

$\Delta U = U_B - U_A$



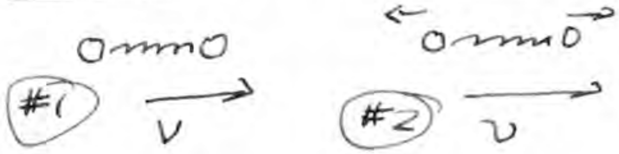
$\Delta u = 0$



$\Delta u = 0$

WHAT IF WE HAD FRICTION? NOT TRUE. FRICTION TAKES OUT ENERGY AS BALL GOES UP AND DOWN

INTERNAL ENERGY



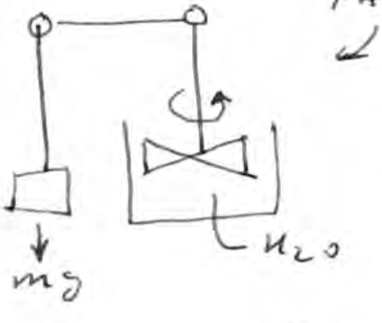
TWO MOLECULES

Kinetic ENERGY of #1 and #2 ARE THE SAME.

BUT ⇒ VIBRATIONAL ENERGY OF #2 IS GREATER.

$$E_{INTERNAL} = E_{vibrational} + E_{ROTATIONAL} + E_{CHEMICAL} + \dots$$

TEMPERATURE = MEASURE OF INTERNAL ENERGY. PUT "ENERGY" IN WATER BY STIRRING PADDLE. PUT 4.2 J ⇒ 1 g of water temp. goes up by 1 K.



HEAT CAPACITY ⇒

$$C = \frac{\Delta E_{THERMAL}}{m\ \Delta T}$$

m in grams
ΔT ⇒ K
ΔE ⇒ J

[CALLED SPECIFIC HEAT CAPACITY]

EXAMPLE: $\Delta E_{sys} = W$ $\boxed{W = 36{,}000\ J}$ $\Delta T = \frac{36000}{(12000)(4.2)}$

$\boxed{m = 12\ kg\ H_2O}$ $\boxed{C(H_2O) = \frac{4.2\ J}{(1g)(1K)} = 4.2\ J/g\text{-}K}$ $\boxed{\Delta T = 0.7\ K}$

TAKES A LOT OF MECHANICAL WORK FOR SMALL TEMPERATURE CHANGE.

ENERGY TRANSFER ⟹ (LAW OF THERMODYNAMICS) THERMAL ENERGY FLOWS FROM A HOT OBJECT TO A COLD OBJECT. ⟹ Q ⇐ SYMBOL FOR THERMAL ENERGY TRANSFER

((ENERGY CONSERVATION))

$$\boxed{\Delta E_{sys} = W + Q}$$

BOOK IS ANNOYING ⟹ It's ok to say "HEAT"

SIGN CONVENTION

↳ □→Q [ENERGY FLOWS OUT OF SYSTEM]

Q → NEGATIVE $Q = -|Q|$

□←Q FLOWS IN $Q = +|Q|$

POWER : ENERGY PER UNIT TIME

~~POF~~ POWER (P) $\equiv \dfrac{\vec{F}\cdot\Delta\vec{r}}{\Delta t}$ Limit $\Delta t \to 0$ $\dfrac{\Delta\vec{r}}{\Delta t} = \vec{v}$

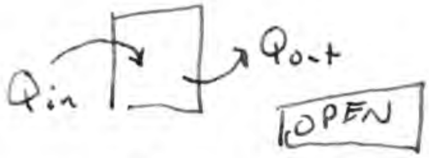
$$\boxed{P \equiv \vec{F}\cdot\vec{v}}$$

POWER - MEASURED IN WATTS

ENERGY TO RUN 100 Watt light bulb for an hr?

$$100\,\frac{J}{s}\cdot 1\,hr\left(\frac{60\,min}{hr}\right)\left(\frac{60\,s}{min}\right) = 3.6\times10^{5}\,J$$

OPEN AND CLOSED SYSTEMS



Q flows in and out ~~but~~ and allowed $\Delta Q \neq 0$

$Q = 0$ Nothing is allowed in or out