ASE 211 Homework 9 Solution

1. Develop a Matlab code which will construct a least squares approximating function of any polynomial order q. The code should consist of several m-files which do the following:

(i) Input the data (x_i, y_i), $i = 1, \ldots, n$ and q.

(ii) Build the matrix A and right hand side $\mathbf{b}$ as discussed in class.

(iii) Solve the linear system $A^T A\mathbf{x} = A^T\mathbf{b}$ for the coefficient vector $\mathbf{x}$.

(iv) Plot the resulting polynomial.

Test your code using the data in problem A9.6 and for $q = 1$, 2, 3. Plot the least squares polynomials for each q.

```
function [alpha]=ls(x,y,n,q)
%
% this function builds the matrix $A$ and the vector $b$ and
% solves A'*A*x = A'*b
%
b=zeros(n,1);
for i=1:n
   b(i)=y(i);
   for j=1:q+1
      A(i,j)=x(i)^(q+1-j);
   end
end
C=A'*A;
d=A'*b;
alpha=C\d;
```

```
function plotls(x,y,n,alpha,q)
dx=(x(n)-x(1))/100;
for j=1:101
   xx(j)=x(1)+dx*(j-1);
   yy(j)=0;
   for k=1:q+1
      yy(j)=yy(j)+alpha(k)*xx(j)^(q+1-k);
   end
end
plot(x,y,'+',xx,yy)
```

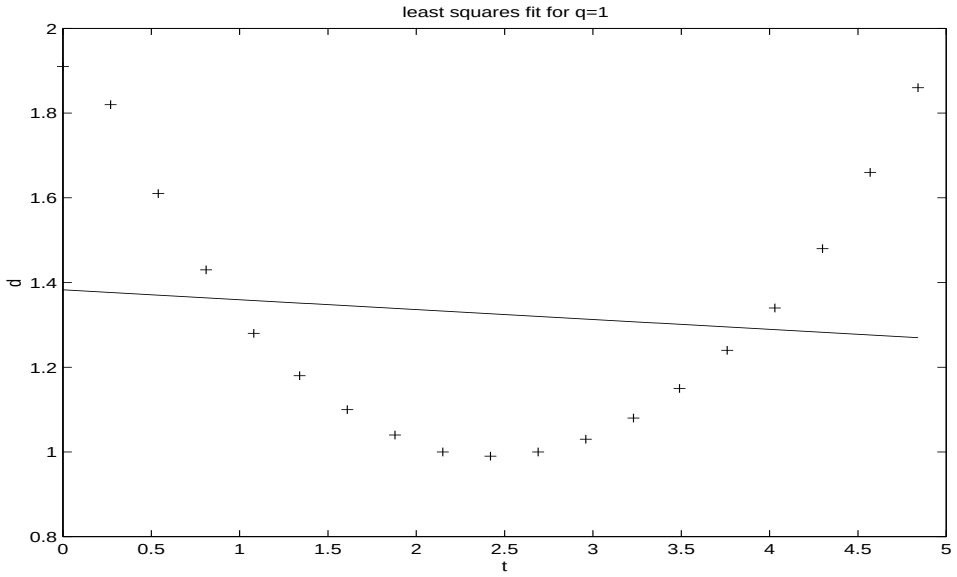

Figure 1: Least squares solution for $q = 1$

Script for problem A9.6:

```
t=[0 .27 .54 .81 1.08 1.34 1.61 1.88 2.15 2.42 2.69 2.96 3.23 3.49 3.76 4.03 4.30 4.5
d=[1.91 1.82 1.61 1.43 1.28 1.18 1.10 1.04 1.00 .99 1.00 1.03 1.08 1.15 1.24 1.34 1.4
n=size(t,2);
q=input('enter q ')
alpha=ls(x,y,n,q);
plotls(x,y,n,alpha,q)
```

2. The data below measures vertical height above a reference height vs. spatial location (e.g., for mapping terrain). Construct least squares polynomials of degree $q = 1, \ldots, 7$ for the data. Plot your least squares polynomials versus the actual data as given in the sample figure below. Also generate and plot a cubic spline which interpolates the data, and compare to your least squares solutions. Which of the functions do you think best represents the data (justify your answer)? Hand in all plots and m-files. You do not have to hand in a diary.

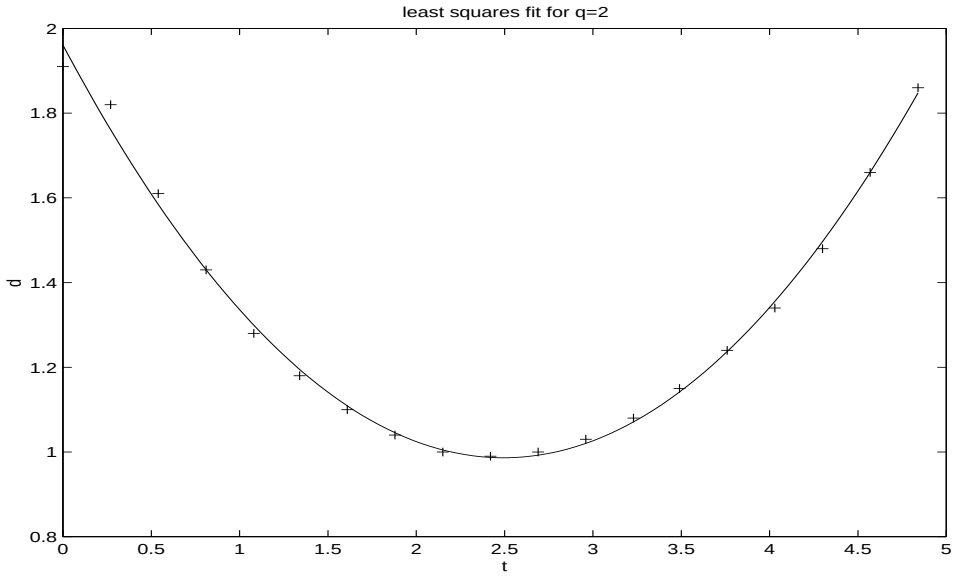

Figure 2: Least squares solution for $q = 2$

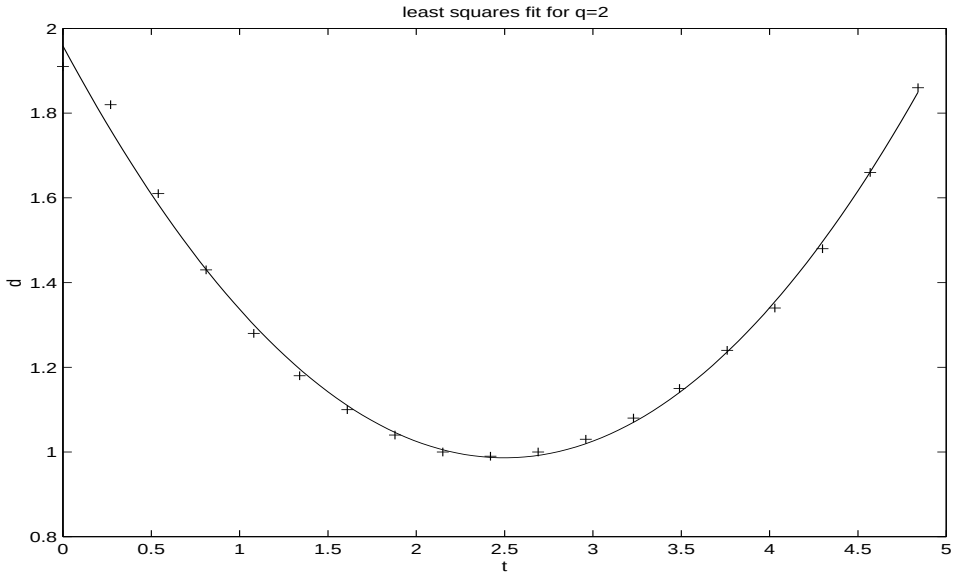

Figure 3: Least squares solution for $q = 3$

Here is the data:

x	y
0	3.3
.5	2.1
1.5	-.5
1.6	1.2
2.8	6.9
3.1	8.1
4.0	7.9
5.5	4.3
6.2	1.1
6.5	-.1
7.0	-3.2
8.2	-4.5
9.6	-2.4
9.8	-2.4
11.1	1.9
12.5	2.6
13.0	4.7

Discussion: The least squares solutions for $q = 6$ and $q = 7$ model the data pretty well without any nonphysical oscillation. These solutions look better than the spline solution, which oscillates quite a bit. The least squares solution is also easier to compute than the spline.

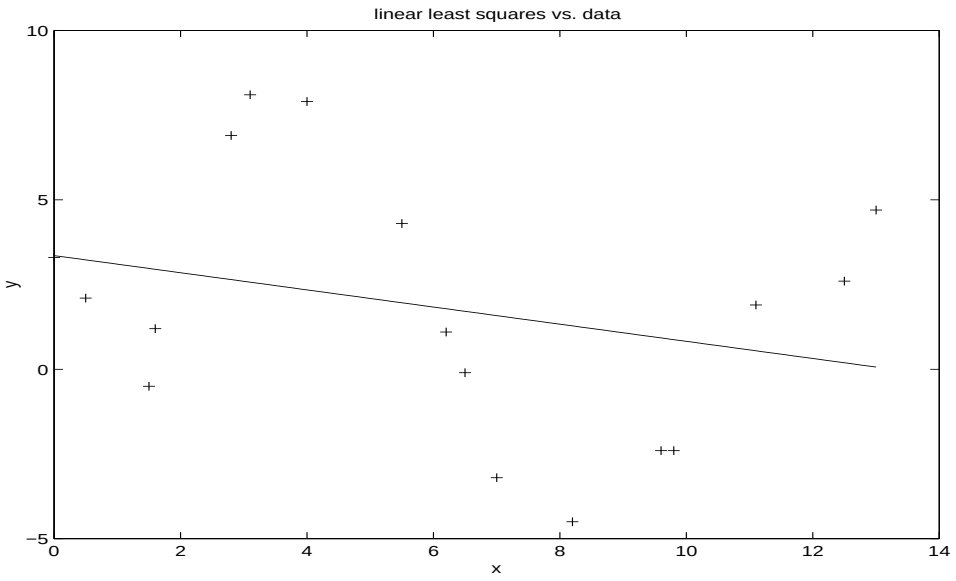

Figure 4: Least squares solution for $q = 1$

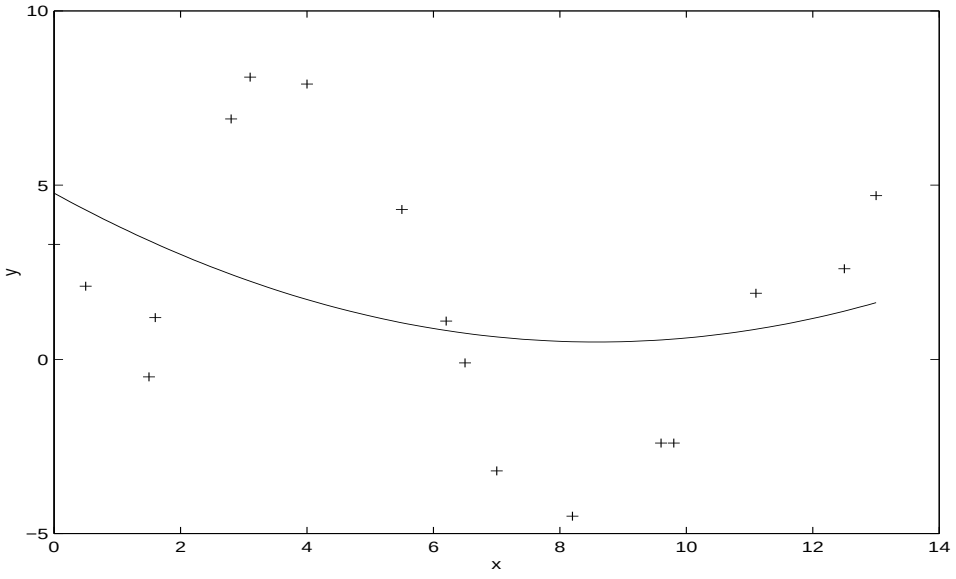

Figure 5: Least squares solution for $q = 2$

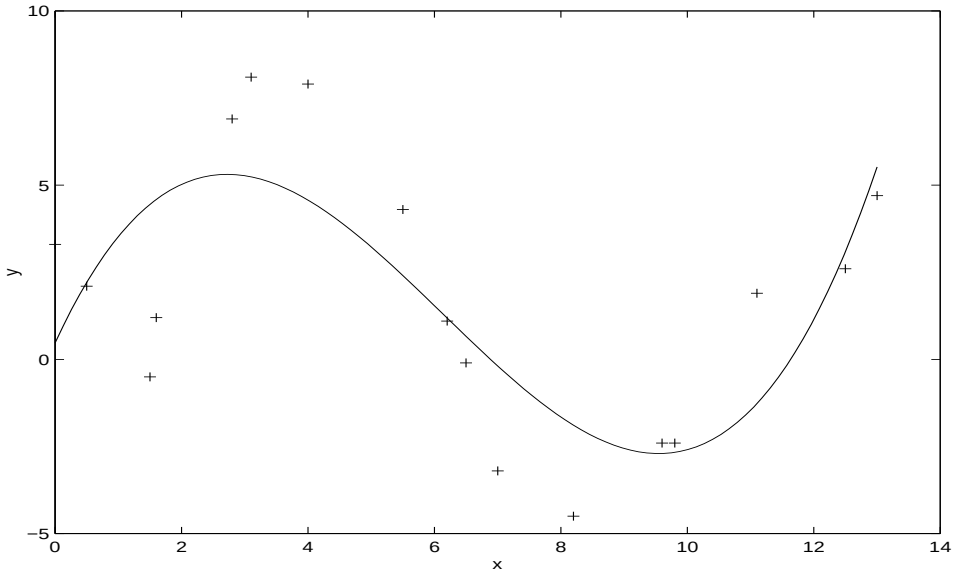

Figure 6: Least squares solution for $q = 3$

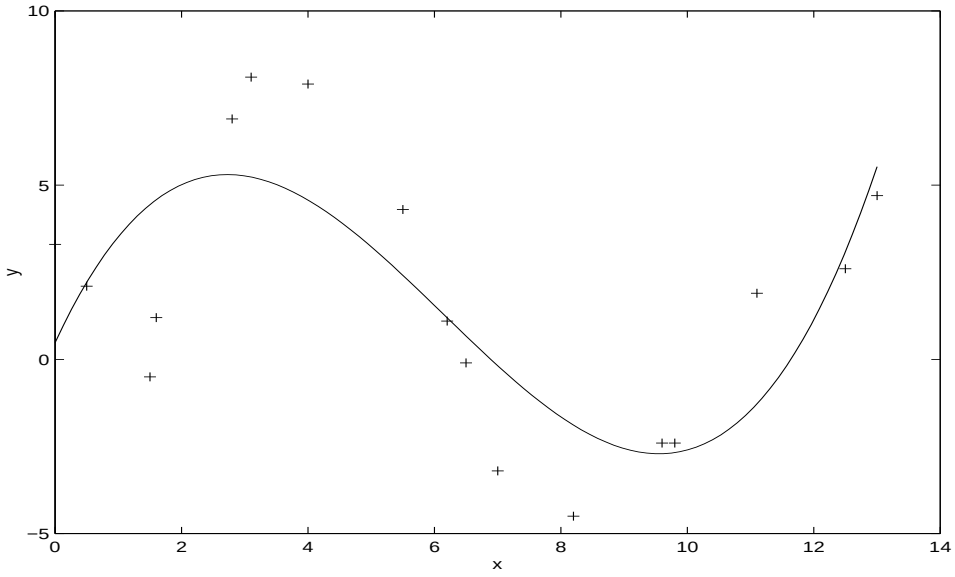

Figure 7: Least squares solution for $q = 4$

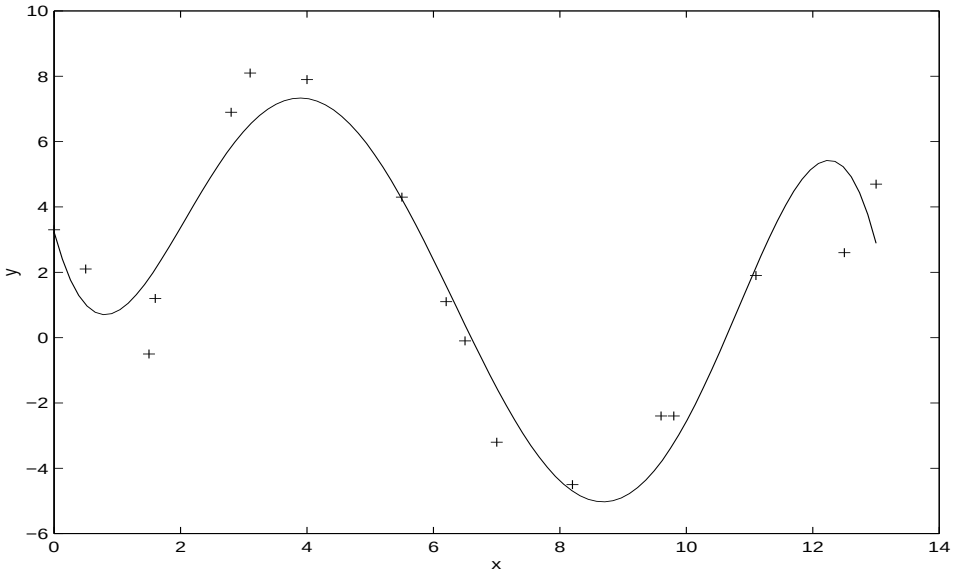

Figure 8: Least squares solution for $q = 5$

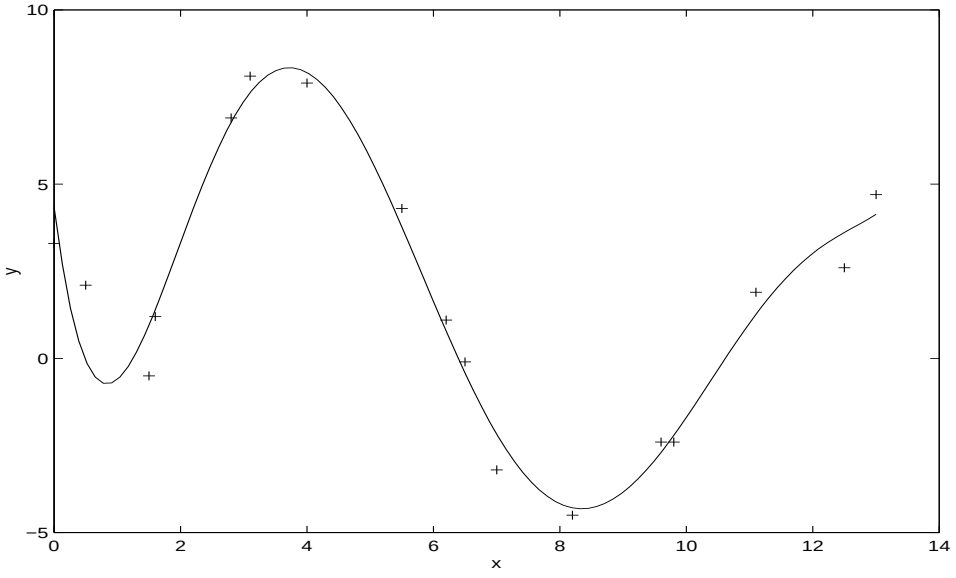

Figure 9: Least squares solution for $q = 6$

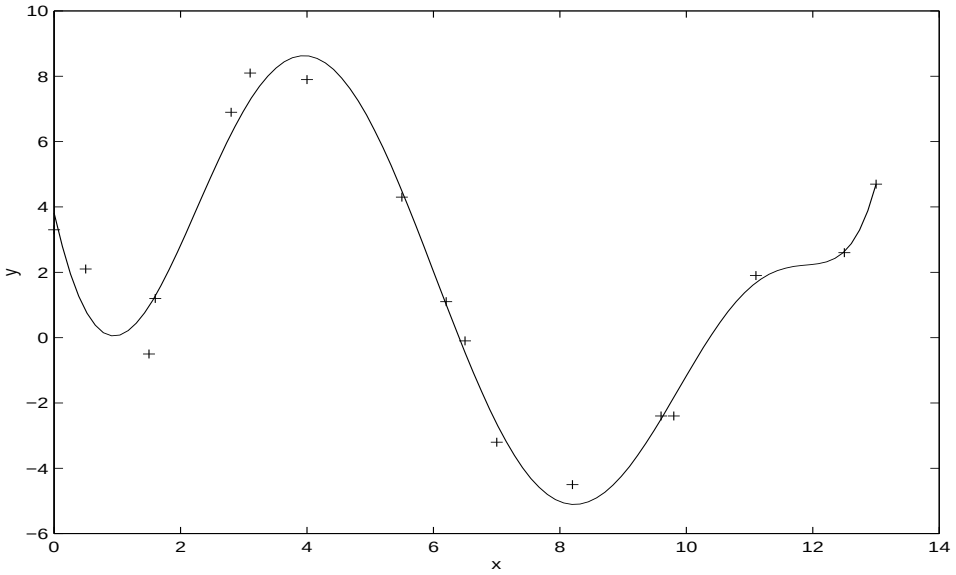

Figure 10: Least squares solution for $q = 7$

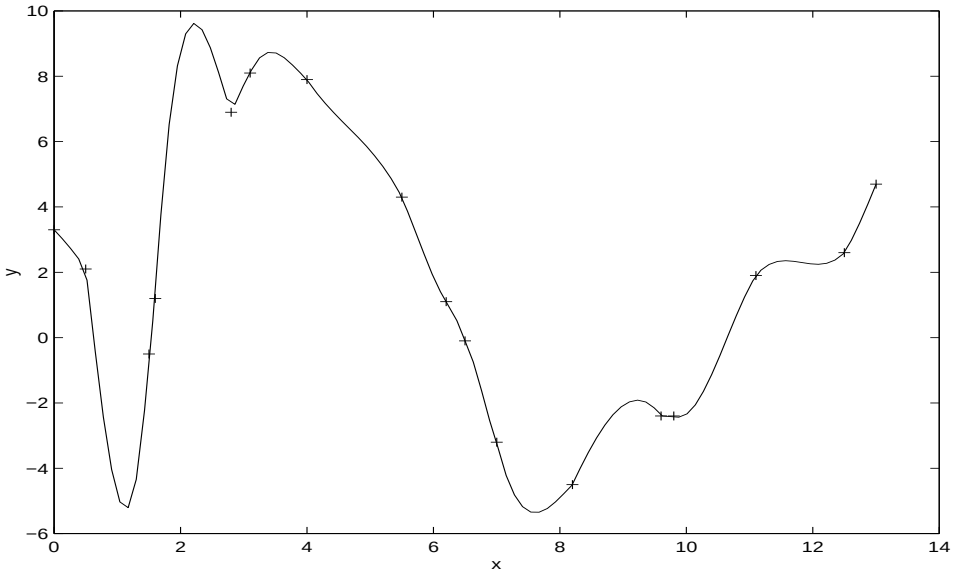

Figure 11: Cubic spline solution