ASE 211 Homework 8

1. Develop a Matlab code which will construct a cubic spline interpolant. The code should consist of several m-files which do the following:

(i) Input the data (x_i, y_i), $i = 1, \ldots, n$.

(ii) Build the matrix and right hand side.

(iii) Solve the linear system for $S_2, \ldots, S_{n-1}$, and set $S_1 = S_n = 0$.

(iv) Compute the coefficients a_i, b_i, c_i, d_i, $i = 1, \ldots, n-1$.

Test your code on the function from the last assignment,

$$f(x) = \frac{1}{1 + 25x^2}.$$

Interpolate the function at

(a) 5 equally spaced points between -1 and 1,

(b) 11 equally spaced points between -1 and 1,

(c) 21 equally spaced points between -1 and 1.

Plot your solution for each case and compare your answer to the true function.

Script for problem 1:

```
n=input('enter number of data points  ')
x(1)=-1;
y(1)=1/(1+25*x(1)^2);
x(n)=1;
y(n)=1/(1+25*x(n)^2);
dx=(x(n)-x(1))/(n-1);
for i=2:n-1
  x(i)=x(i-1)+dx;
  y(i)=1/(1+25*x(i)^2);
end
[a,b,c,d]=genspline(x,y,n);
%  plot spline
dx=(x(n)-x(1))/100;
for i=1:101
   xx(i)=x(1)+(i-1)*dx;
   yy(i)=evalspline(a,b,c,d,xx(i),x,n);
   ytrue(i)=1/(1+25*xx(i)^2);
```

```
end
plot(xx,yy,xx,ytrue,'+')
xlabel('x')
ylabel('y')

function [a,b,c,d]=genspline(x,y,n)
% generates coefficients of a cubic spline
%
%
% compute the interval lengths and store in h
for i=1:n-1
  h(i)=x(i+1)-x(i);
end
%
% compute the matrix A and right hand side f
%
A(1,1)=1;
A(n,n)=1;
f=zeros(n,1);
for i=2:n-1
  A(i,i)=2*(h(i)+h(i-1));
  f(i)=6*((y(i+1)-y(i))/h(i)-(y(i)-y(i-1))/h(i-1));
end
for i=2:n-2
  A(i,i+1)=h(i+1);
end
for i=3:n-1
  A(i,i-1)=h(i);
end
%
% solve for the vector s
%
s=A\f;
%
% compute the coefficients of the spline
%
for i=1:n-1
   a(i)=(s(i+1)-s(i))/(6*h(i));
   b(i)=s(i)/2;
```

```
   c(i)=(y(i+1)-y(i))/h(i)-(2*h(i)*s(i)+h(i)*s(i+1))/6;
   d(i)=y(i);
end

function yy=evalspline(a,b,c,d,xx,x,n)
%
% function to evaluate a spline at a point xx
%
%
% determine which interval xx is in
%
i=1;
while (xx > x(i+1) & i <= n-1)
   i=i+1;
end
%
% xx is between x(i) and x(i+1)
%
yy=a(i)*(xx-x(i))^3+b(i)*(xx-x(i))^2+c(i)*(xx-x(i))+d(i);

function plotspline(a,b,c,d,x,n)
%  function to plot the spline at 100 points between x(1) and x(n)
%
dx=(x(n)-x(1))/100;
for j=1:101
   xx(j)=x(1)+dx*(j-1);
   yy(j)=evalspline(a,b,c,d,xx(j),x,n);
end
plot(xx,yy)
```

2. Use your cubic spline code to work problem A8.2.

Input script for problem 2:

```
n=input('enter number of data points  ')
for i=1:n
   x(i)=input('enter x value ');
   y(i)=input('enter y value ');
end
[a,b,c,d]=genspline(x,y,n);
```

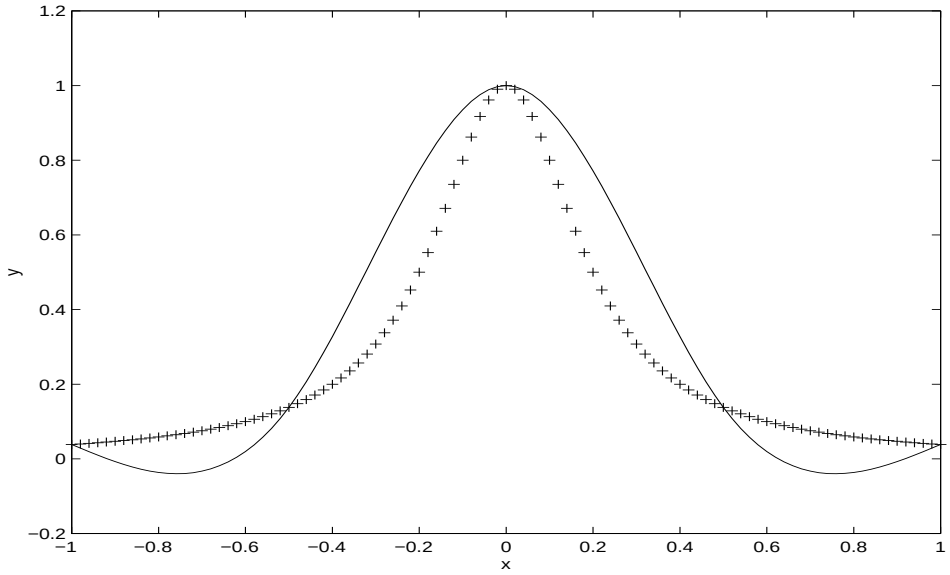

Figure 1: Comparison of spline and true function for n=5

```
%  plot spline
dx=(x(n)-x(1))/100;
for i=1:101
   xx(i)=x(1)+(i-1)*dx;
   yy(i)=evalspline(a,b,c,d,xx(i),x,n);
end
plot(xx,yy)
xlabel('x')
ylabel('y')
```

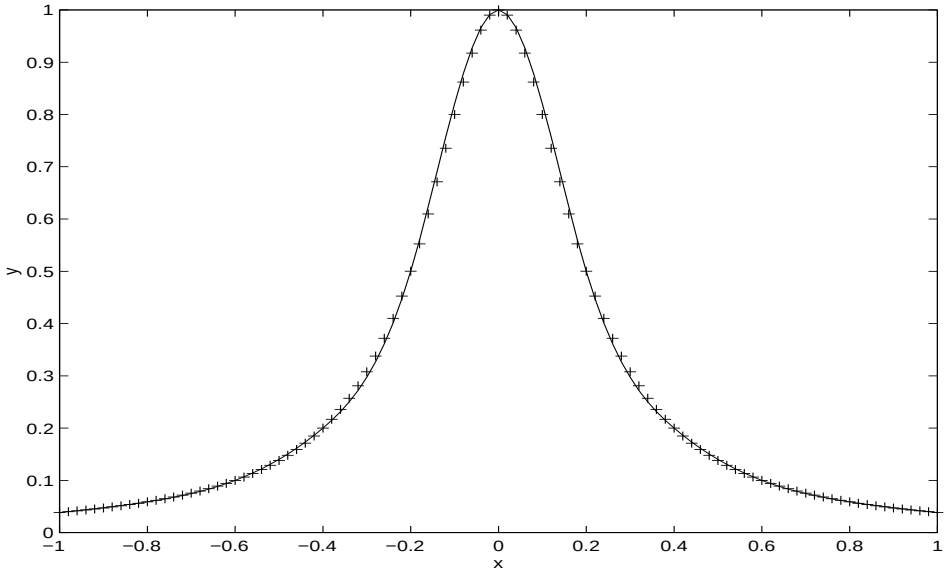

Figure 2: Comparison of spline and true function for n=11

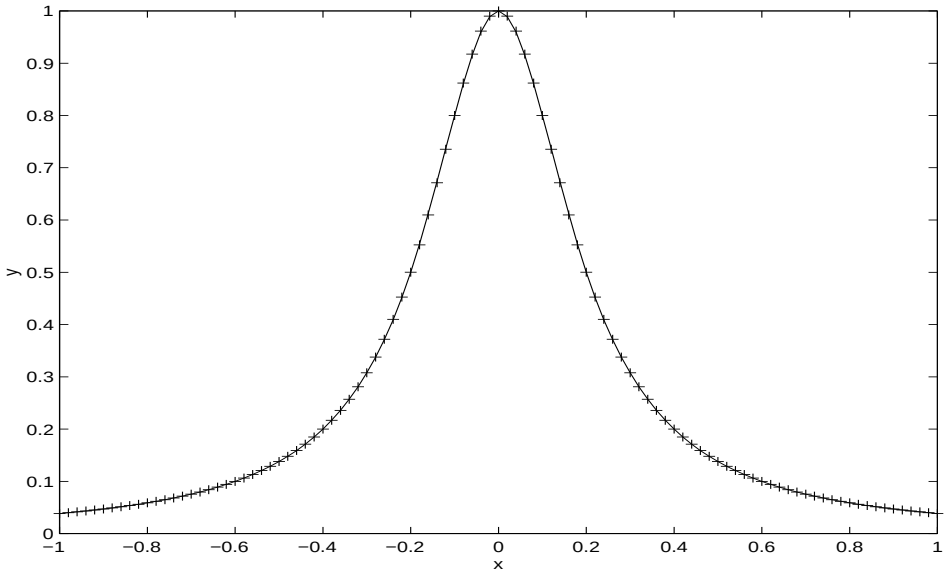

Figure 3: Comparison of spline and true function for n=21

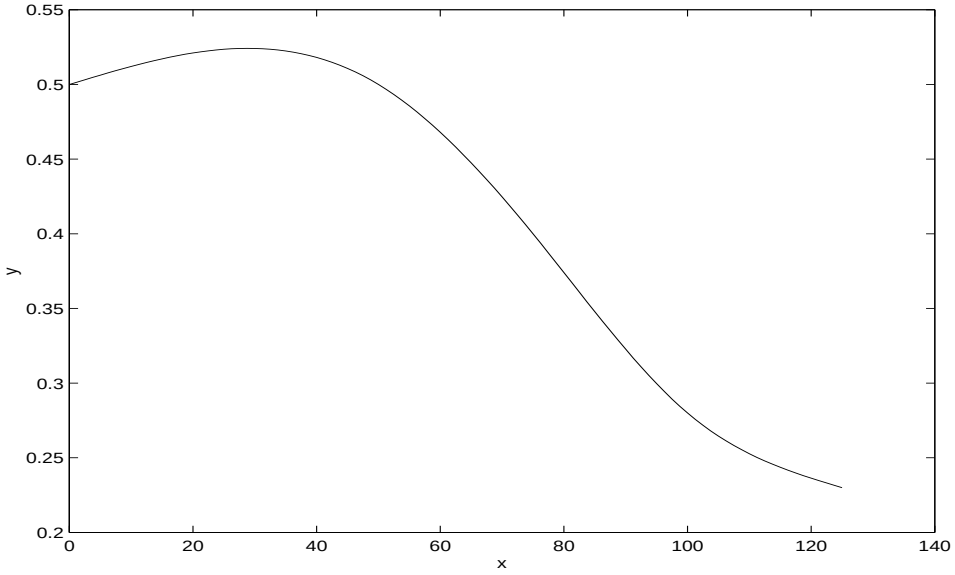

Figure 4: Comparison of spline and true function for n=11