ASE 211 Homework 7 Solution

1. Using the code you developed for assignment 6, construct and plot a Lagrange interpolant of the function $f(x) = \frac{1}{1+25x^2}$ using

(a) 5 equally spaced points between -1 and 1,
(b) 11 equally spaced points between -1 and 1,
(c) 21 equally spaced points between -1 and 1.

Plot the Lagrange polynomial versus the function f for each case. This is a classical example of where Lagrange interpolation goes bad.

Script for problem 1:

```
%ass7script
%
n=input('enter n')
for i=1:n
  x(i)=-1 + 2*(i-1)/(n-1);
  y(i)=1./(1+25*x(i)^2);
end
lagrange(x,y,n)

function lagrange(x,y,n)
%
% This function evaluates a Lagrange polynomial at xx
%
%
dx=(x(n)-x(1))/100;
for j=1:101
  xx(j)=x(1)+(j-1)*dx;
  yy(j)=0;
  for i=1:n
    yy(j)=yy(j)+y(i)*polyl(i,xx(j),x,n);
  end
  ytrue(j)=1./(1+25*xx(j)^2);
end
plot(x,y,'*',xx,yy,xx,ytrue,'+')
%plot(x,y,'*',xx,yy)

>> ass7script
```

```
enter n 5

n =

     5

>> title('plot of Lagrange vs. true function for 5 points')
>> print fig1.ass7.ps
>> ass7script
enter n 11

n =

    11

>> title('plot of Lagrange vs. true function for 11 points')
>> print fig2.ass7.ps
>> ass7script
enter n 21

n =

    21

>> title('plot of Lagrange vs. true function for 21 points')
>> print fig3.ass7.ps
```

2. Construct, by hand, the cubic spline which interpolates the points $(0,0)$, $(1,1)$ and $(2,1)$.

$$\begin{aligned} h_1 &= 1 \\ h_2 &= 1 \\ 2(h_1+h_2)S_2 &= 6\left(\frac{y_3-y_2}{h_2}-\frac{y_2-y_1}{h_1}\right) \\ &= 6(0-1) \\ S_2 &= -3/2 \\ S_1 &= 0 \end{aligned}$$

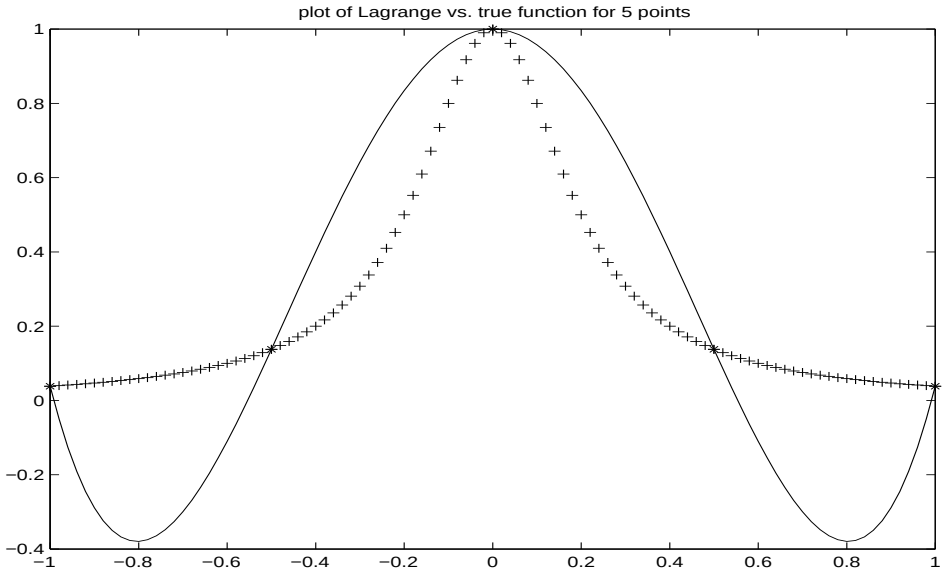

Figure 1: Plot of the Lagrange interpolant for 5 points

$$
\begin{aligned}
S_3 &= 0 \\
b_1 &= S_1/2 = 0 \\
a_1 &= \frac{S_2 - S_1}{6h_1} = -1/4 \\
c_1 &= \frac{y_2 - y_1}{h_1} - \frac{S_2 - S_1}{6}h_1 - \frac{S_1}{2}h_1 \\
&= 5/4 \\
d_1 &= y_1 = 0 \\
b_2 &= -3/4 \\
a_2 &= 1/4 \\
c_2 &= 1/2 \\
d_2 &= y_2 = 1
\end{aligned}
$$

Thus,

$$
g(x) = \begin{cases} g_1(x) = -x^3/4 + 5x/4, & 0 \le x \le 1, \\ g_2(x) = (x-1)^3/4 - 3(x-1)^2/4 + (x-1)/2 + 1, & 1 \le x \le 2. \end{cases}
$$

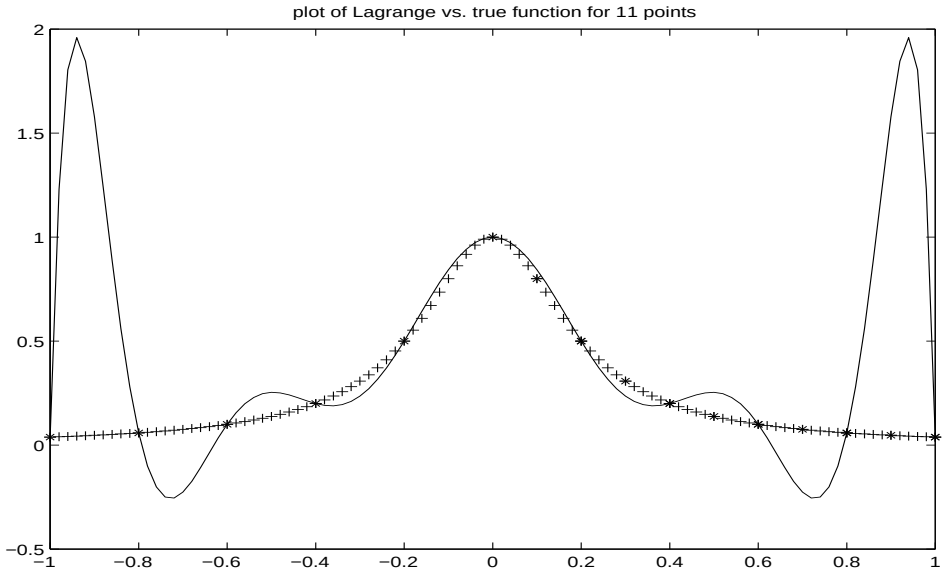

Figure 2: Plot of the Lagrange interpolant for 11 points

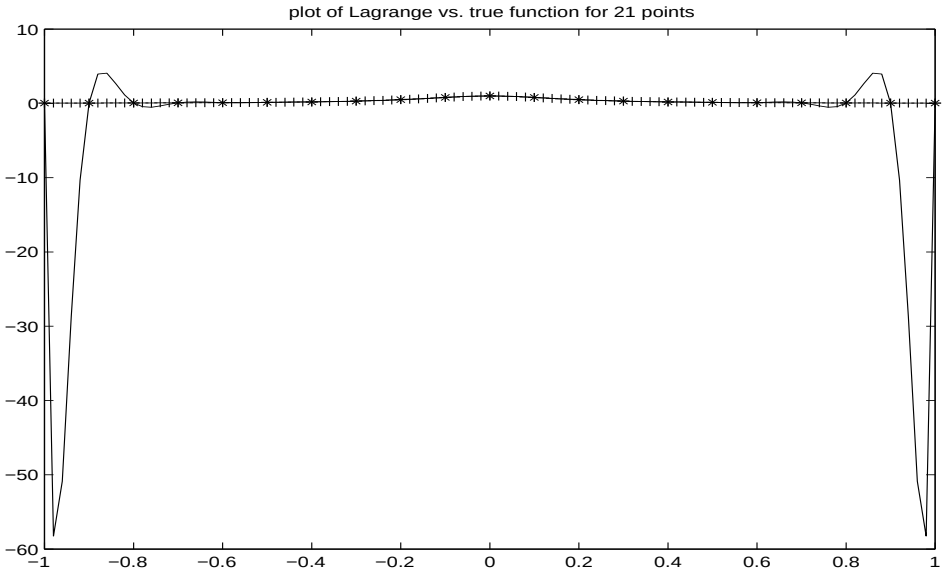

Figure 3: Plot of the Lagrange interpolant for 21 points