ASE 211 Homework 6 Solution

1. Given the following data

i	x_i	y_i
1	0	2
2	1	-1
3	1.5	0
4	2.2	1.6

First, compute the linear interpolant of the data, then compute the Lagrange interpolant. For both interpolants, give the value at $x = 2$.

For the linear interpolant, the linear functions which interpolate the data are:

$$l_1(x) = \frac{(-1-2)}{(1-0)}(x-0)+2 = -3x+2, \quad 0 \le x \le 1$$
$$l_2(x) = \frac{(0-(-1))}{(1.5-1)}(x-1)-1 = 2x-3, \quad 1 \le x \le 1.5$$
$$l_3(x) = \frac{(1.6-0)}{(2.2-1.5)}(x-1.5)+0 = 2.2857x-3.4286, \quad 1.5 \le x \le 2.2$$

Thus the value of the linear interpolant at $x = 2$ is $l_3(2) = 1.1428$.

For the Lagrange interpolant, we compute

$$L_1(x) = \frac{(x-1)(x-1.5)(x-2.2)}{(0-1)(0-1.5)(0-2.2)}$$
$$L_2(x) = \frac{(x-0)(x-1.5)(x-2.2)}{(1-0)(1-1.5)(1-2.2)}$$
$$L_3(x) = \frac{(x-0)(x-1)(x-2.2)}{(1.5-0)(1.5-1)(1.5-2.2)}$$
$$L_4(x) = \frac{(x-0)(x-1)(x-1.5)}{(2.2-0)(2.2-1)(2.2-1.5)}$$

Then

$$P_4(x) = 2 * L_1(x) - L_2(x) + 1.6L_4(x).$$

Thus

$$P_4(2) = 1.26.$$

2. Write a Matlab code which allows a user to input data points, constructs the Lagrange interpolant, and plots the interpolant over the interval from x_1 to x_n (assuming the data is ordered so that $x_1 < x_2 < \ldots < x_n$.) Test your code on the data in problem A8.11 and plot the approximation to enthalpy E versus temperature T.

```
function lagrange(x,y,n)
%
% This function evaluates a Lagrange polynomial at 100 equally
% spaced points between x(1) and x(n) and plots the polynomial
%
%
dx=(x(n)-x(1))/100;
for j=1:101
  xx(j)=x(1)+(j-1)*dx;
  yy(j)=0;
  for i=1:n
    yy(j)=yy(j)+y(i)*polyl(i,xx(j),x,n);
  end
end
plot(x,y,'*',xx,yy)
```

```
function y=polyl(i,xx,x,n)
%
%  this function evaluates L_i(xx)
%
y=1;
for j=1:n
   if (j == i)
   else
     y=y*(xx-x(j))/(x(i)-x(j));
   end
end
```

Script for input:

```
% enthalpy.m
% Enthalpy vs. Temperature data
T= [60 80 100 120 140 160 180 200]
E = [0.0 17.2 45.2 92.9 178.8 349.4 764.3 2648.4]
n=size(E,2);
lagrange(T,E,n)
title('Lagrange interpolant for Enthalpy vs. Temperature')
print fig1.ass6.ps
```

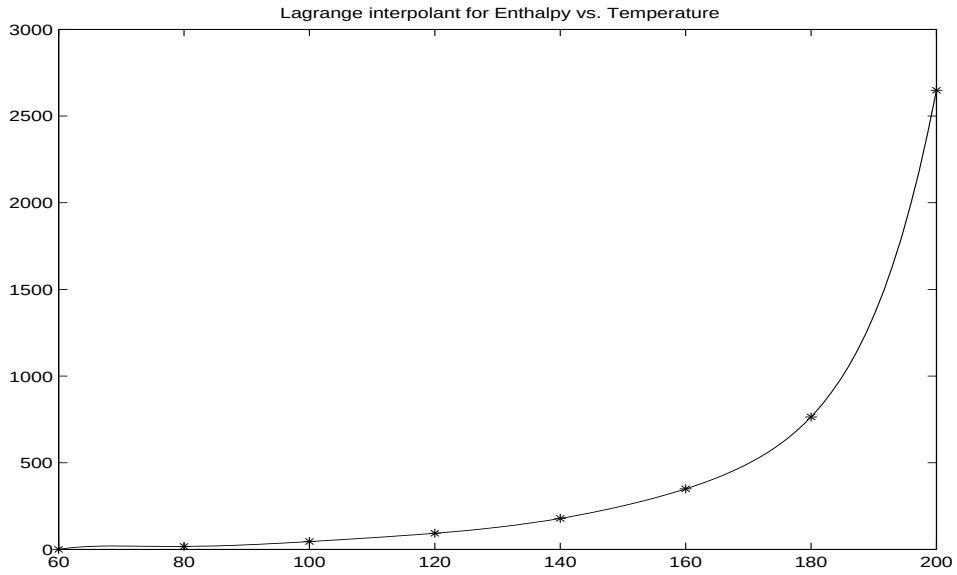

Figure 1: Plot of the Lagrange interpolant