ASE 211 Homework 3 Solution

1. By hand, use Gaussian elimination to find the solution of the problem:

$$A\mathbf{x} = \begin{bmatrix} 3 & 2 & -1 \\ 6 & 1 & 0 \\ -3 & 6 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 21 \end{bmatrix} = \mathbf{b}.$$

Using Gaussian elimination, you should end up with the system:

$$\begin{aligned} 3x_1 + 2x_2 - x_3 &= 4 \\ -3x_2 + 2x_3 &= 0 \\ \frac{25}{3}x_3 &= 25 \end{aligned}$$

Thus using backward substitution:

$$\begin{aligned} x_3 &= 3 \\ x_2 &= 2 \\ x_1 &= 1. \end{aligned}$$

2. Solve the problem above using Matlab. Enter the matrix A and the column vector $\mathbf{b}$, and use the command

```
A\b
```

to solve for $\mathbf{x}$.

```
>> A=[3 2 -1;6 1 0; -3 6 4]

A =

     3     2    -1
     6     1     0
    -3     6     4

>> b=[4;8;21]

b =
```

```
     4
     8
    21

>> A\b

ans =

     1
     2
     3
```

3. Solve problem A3.6 in the book using Matlab.

```
>> k=[1 1 1 1 5]

k =

     1     1     1     1     5

>> L=[2 2 2 2 2]

L =

     2     2     2     2     2

>> for i=1:4
A(i,i)=-k(i)-k(i+1);
end
>> for i=1:3
A(i,i+1)=k(i+1);
A(i+1,i)=k(i+1);
end
>> A

A =
```

```
    -2     1     0     0
     1    -2     1     0
     0     1    -2     1
     0     0     1    -6

>> for i=1:3
b(i)=-k(i)*L(i)+k(i+1)*L(i+1);
end
>> b(4)=-k(4)*L(4)+k(5)*(L(5)-8);
>> b

b =

     0
     0
     0
   -32

>> A\b

ans =

    1.5238
    3.0476
    4.5714
    6.0952
```

The solution gives the positions of the blocks when the distance between the walls is compressed to 8 feet.

4. Suppose we change the matrix A in problem 1 as follows:

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 6 & 1 & 0 \\ -3 & 1 & -1 \end{bmatrix}.$$

By hand, attempt to solve the system in problem 1 with this matrix. What happens?

In this case, Gaussian elimination breaks down, we get

$$\begin{aligned} 3x_1 + 2x_2 - x_3 &= 4 \\ -3x_2 + 2x_3 &= 0 \\ 0 &= 25 \end{aligned}$$

The matrix is singular.