ASE 211 Homework 2 Solution

Write a Matlab m-file which will implement Newton's method. The outline of the m-file is as follows:

```
function newton(x0,xtol,maxiter)
%
%  Matlab function which uses Newton's method to find the
%  roots of a given function funcf.
%
%  m-files funcf.m and funcfp.m which specify the function and its derivative
%  must be provided.
%
%  xtol is the tolerance used for stopping
%  x0 is the starting guess for the method
%  maxiter is the maximum number of iterations allowed
%
%
k=0;
x1=x0-funcf(x0)/funcfp(x0);
%
%  do until convergence
%
while (abs(x1-x0)>xtol & k <= maxiter)
.....
.....
.....
end
k
x1
funcf(x1)
```

Use your m-file to solve the following problem. The position of a ball, thrown upward with a given initial velocity v_0 and initial position y_0, subject to air resistance proportional to its velocity, is given as a function of time x by

$$y(x) = \rho^{-1}(v_0 + v_r)(1 - e^{-\rho x}) - v_r x + y_0,$$

where ρ is the drag coefficient, g is the gravitational constant, and $v_r = g/\rho$ is the terminal velocity. Find when the ball hits the ground if $y_0 = 0$,

$v_0 = 20m/s$, $\rho = .35$ and $g = 9.8m/s^2$. Take as your initial guess $x_0 = 1$, set $xtol = .0001$ and $maxiter = 50$.

Keep a diary of your matlab session. Hand in all m-files and your diary.

```
function newton(x0,xtol,maxiter)
%
%  function which uses Newton's method to find the
%  roots of a given function funcf
%  xtol is the tolerance used for stopping
%  x0 is the starting guess for the method
%  maxiter is the maximum number of iterations allowed
%
%
k=0;
x1=x0-funcf(x0)/funcfp(x0);
%
%  do until convergence
%
while (abs(x1-x0)>xtol & k <= maxiter)
  x0=x1;
  x1=x0-funcf(x0)/funcfp(x0);
  k=k+1;
end
k
x1
funcf(x1)
```

```
function y=funcf(x)
rho=.35;
v0=20;
```

```
vr=9.8/.35;
y0=0;
y=1/rho*(v0+vr)*(1-exp(-rho*x))-vr*x+y0;
```

```
function y=funcfp(x)
rho=.35;
v0=20;
vr=9.8/.35;
y0=0;
y=1/rho*(v0+vr)*(1-exp(-rho*x))-vr*x+y0;
```

```
>> x0=1

x0 =

     1

>> xtol=.0001

xtol =

   1.0000e-04

>> maxiter=50

maxiter =

    50

>> newton(x0,xtol,maxiter)

k =
```

```
     5


x1 =

  -9.7952e-15


ans =

  -1.8251e-13

>> x0=2;
>> newton(x0,xtol,maxiter)

k =

     4


x1 =

    3.4165


ans =

  -1.0400e-09

>> x0=5;
>> newton(x0,xtol,maxiter)

k =

     3


x1 =
```

```
    3.4165


ans =

  -4.5279e-10

>> diary
```

With an initial guess of 1, Newton's method converges to the root at $x = 0$, which is not what we wanted, actually. So we tried larger values of x_0. For example, if $x_0 = 2$, then we converge to a root at 3.4165 meters, also if $x_0 = 5$, we converge to the same root. Therefore this is probably the root we're looking for. In fact, if we plot $y(x)$, it looks like:

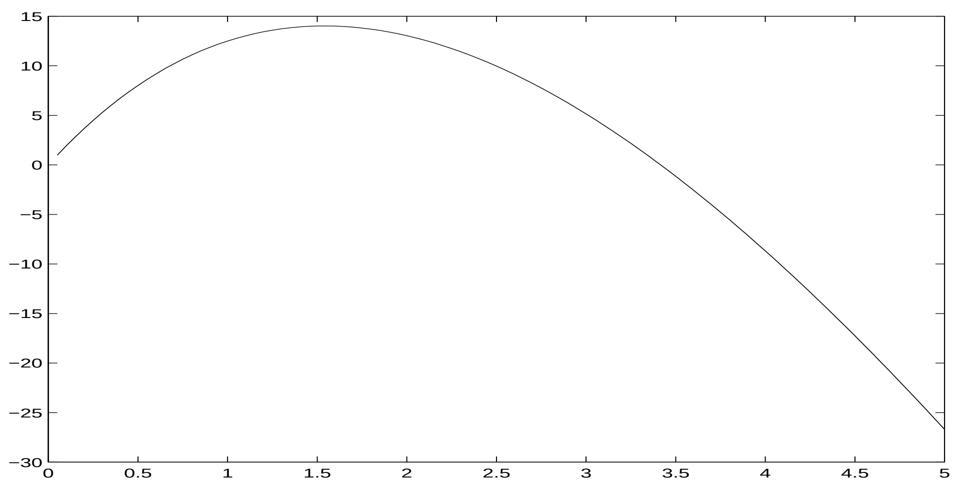

Figure 1: Plot of $y(x)$

Therefore, the ball hits the ground at $x = 3.4165$ meters.