ASE 211 Homework 10 Solution

1. Consider the following data:

x_i	f_i
0	0
.1	.0309
.2	.1176
.3	.2427
.4	.3804
.5	.5

Using a divided difference table, construct the Lagrange polynomial that interpolates the data. Use the Lagrange polynomial to compute approximations to $f'(.25)$ and $f''(.25)$. The data is from the function $x\sin(\pi x)$. Compare the approximations to the first and second derivatives with their actual values.

x_i	f_i	$f[x_i, x_{i+1}]$	$f[x_i, x_{i+1}, x_{i+2}]$	$f[x_i, \ldots, x_{i+3}]$	$f[x_i, \ldots, x_{i+4}]$	$f[x_i, \ldots, x_{i+5}]$
0	0	.309	2.79	-2.9	-3.5	2.917
.1	.0309	.867	1.92	-4.3	-2.0415	
.2	.1176	1.251	.63	-5.1167		
.3	.2427	1.377	-.905			
.4	.3804	1.196				
.5	.5					

The Lagrange polynomial is then:

$$\begin{aligned} P(x) &= 2.917(x-0)(x-.1)(x-.2)(x-.3)(x-.4) - 3.5(x-0)(x-.1)(x-.2)(x-.3) \\ &\quad -2.9(x-0)(x-.1)(x-.2) + 2.79(x-0)(x-.1) + .309(x-0) + 0 \end{aligned}$$

$$\begin{aligned} P'(x) &= 2.917[(x-.1)(x-.2)(x-.3)(x-.4) + (x-0)(x-.2)(x-.3)(x-.4) \\ &\quad +(x-0)(x-.1)(x-.3)(x-.4) + (x-0)(x-.1)(x-.2)(x-.4) \\ &\quad +(x-0)(x-.1)(x-.2)(x-.3)] \\ &\quad -3.5[(x-.1)(x-.2)(x-.3) + (x-0)(x-.2)(x-.3) \\ &\quad +(x-0)(x-.1)(x-.3) + (x-0)(x-.1)(x-.2)] \\ &\quad -2.9[(x-.1)(x-.2) + (x-0)(x-.2) + (x-0)(x-.1)] \\ &\quad +2.79[(x-.1) + (x-0)] + .309. \end{aligned}$$

$$P'(.25) = 1.2619$$

$$P''(.25) = 2.6885$$

The actual values are $f'(.25) = 1.2625$ and $f''(.25) = 2.6981$.

2. A rocket's distance traveled along its trajectory is measured, giving the following data:

time (sec)	distance (m)
0	0
1	25
2	65
3	140
4	275
5	444
6	621
7	899
8	1244
9	1680

Write a matlab program which constructs a divided difference table of the data, builds a Lagrange polynomial and differentiates the polynomial once. Use your program to determine the velocity of the rocket at t=7.4 seconds. The following m-file may be of use. It differentiates the polynomial

$$(x - x_1)(x - x_2) \cdots (x - x_k)$$

and evaluates it at $x = t$.

```
function [d]=pderiv(x,k,t)
d=0;
for j=1:k
  prod=1;
  for i=1:k
     if (i~=j) prod=prod*(t-x(i));
     end
  end
  d=d+prod;
end
```

Function which creates the divided difference table:

```
function f=difftable(x,y,n)
%
% construct a divided difference table using the data
% stored in x and y
%
for i=1:n
   f(i,1)=y(i);
end
for j=2:n
   for i=1:n-j+1
     f(i,j)=(f(i+1,j-1)-f(i,j-1))/(x(i+j-1)-x(i));
   end
end
```

```
function pp=difflagrange(x,f,n,t)
%
% function which differentiates a Lagrange polynomial and
% evaluates the derivative at point t
%
% f is the array from the divided difference table
%
% P(t) = f(1,n)*(t-x(1))*(t-x(2))*...*(t-x(n-1))
%    +f(1,n-1)*(t-x(1))*...*(t-x(n-2))
%    + .... + f(1,2)*(t-x(1))+f(1,1)
%
pp=0;
for k=1:n-1
    pp=pp+f(1,k+1)*pderiv(x,k,t);
end
```

Script for this problem:

```
%  Script for assignment 10
%
n=input(' enter number of data points ')
for i=1:n
   x(i)=input(' enter x value ')
   y(i)=input(' enter y value ')
end
```

```
f=difftable(x,y,n);
t=input(' enter point at which to evaluate derivative ')
pp=difflagrange(x,f,n,t)
```

Answer: pp=363.1142 (m/sec)