ASE 211 Homework 9

1. Consider the function $f(x) = x^3 \sin(x) - \cos(x^2)$. Compute approximations to $f'(\pi)$ (use radians, not degrees) using forward, backward and central differences for $h = .1$, $.01$, and $.001$. Compare your results to the true answer.

h	forward difference	backward difference	central difference	true value
.1	-38.3470	-28.9148	-33.6309	-33.7099
.01	-34.1877	-33.2306	-33.7092	–
.001	-33.7578	-33.6621	-33.7099	–

2. Consider the rocket trajectory data given in assignment 7.

t (s)	d (m)
1	100
2	225
3	365
4	514
5	780
6	844
7	1010
8	1147
9	1313
10	1510

Compute the rocket's velocity along its trajectory at $t = 2, \ldots, 9$ in two ways:

(i) using central differences

(ii) by interpolating the trajectory data by a cubic spline, and using the derivative of the spline to approximate the velocity. Recall that velocity is the time derivative of distance traveled.

For part (ii), you should use your spline code written for assignment 7. Plot the computed velocities for both (i) and (ii).

Using central differences:

$$d'(2) \approx \frac{d(3) - d(1)}{3 - 1} = \frac{365 - 100}{2} = 132.5, \text{ etc.}$$

The spline approximation gives:

$$g'(x) = 3a_i(x - x_i)^2 + 2b_i(x - x_i) + c_i, \quad x_i \leq x \leq x_{i+1}.$$

Therefore $g'(x_i) = c_i$. The plots are given in Figures 1 and 2.

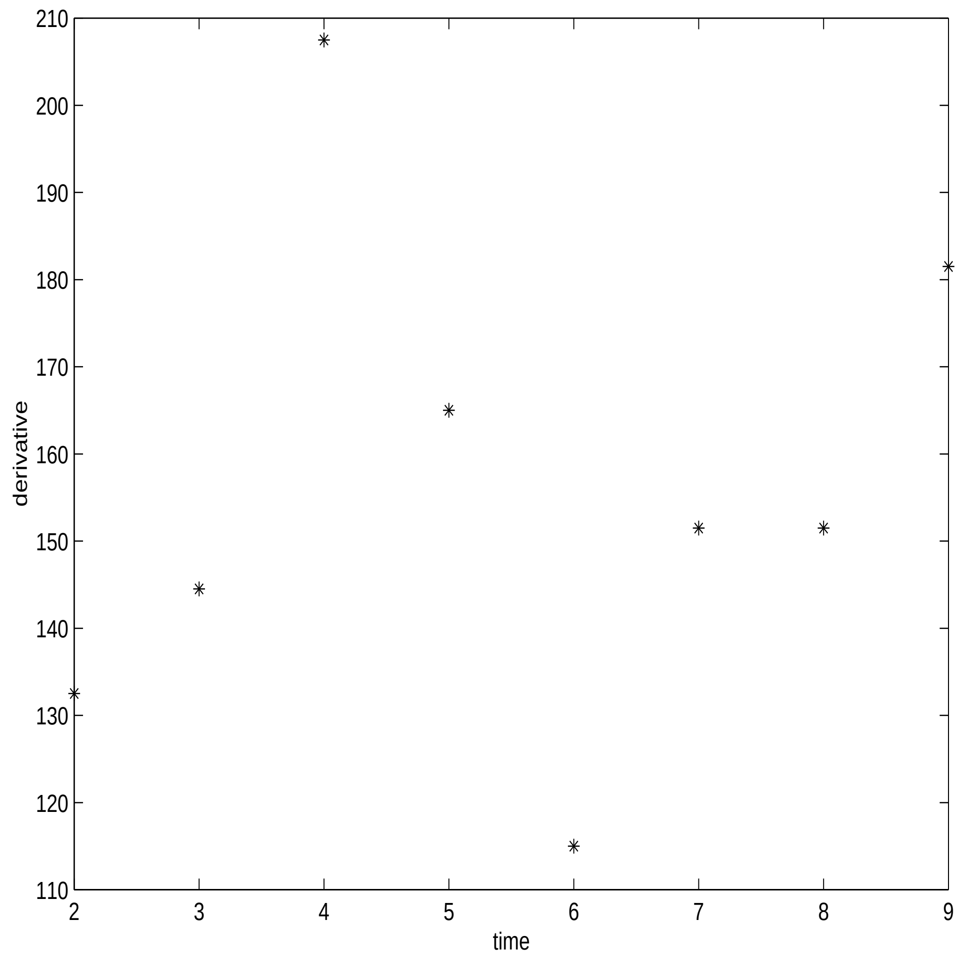

Figure 1: Derivative using central differences

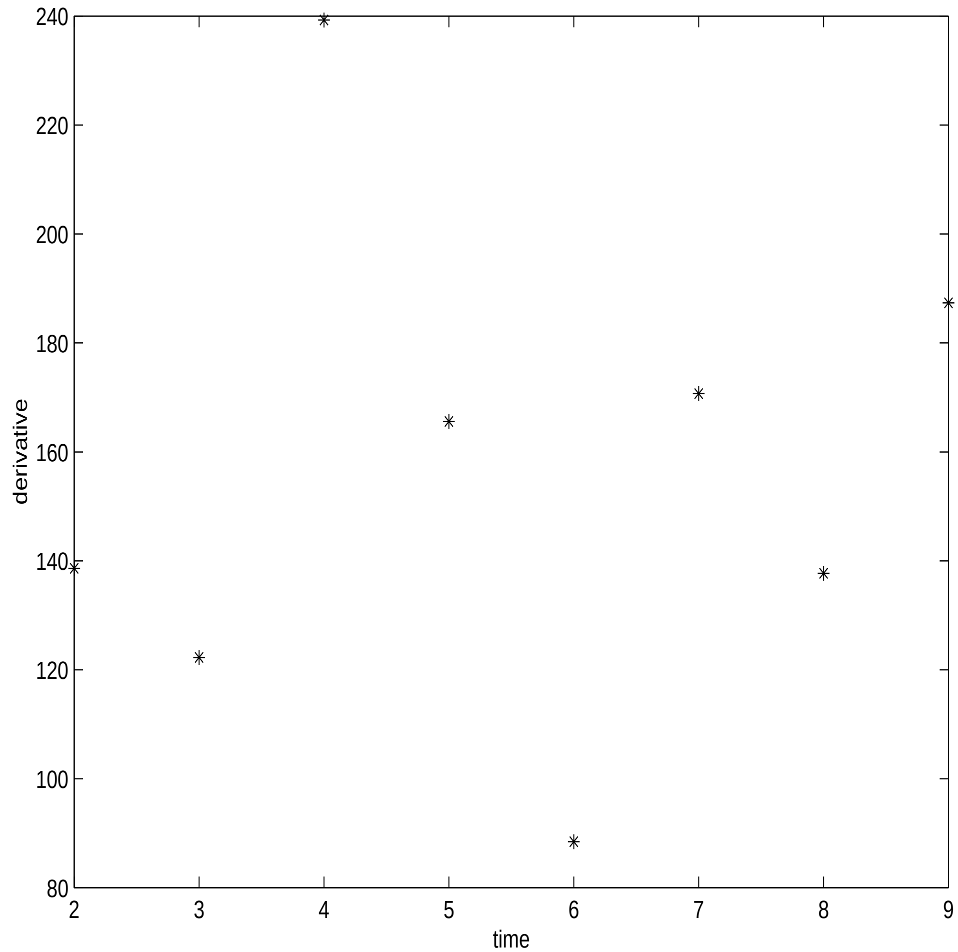

Figure 2: Derivative using cubic spline