ASE 211 Homework 9 Solution

1. Develop a Matlab code which will construct a least squares approximating function of any polynomial order q. The code should consist of several m-files which do the following:

(i) Input the data (x_i, y_i), $i = 1, \ldots, n$ and q.

(ii) Build the matrix A and right hand side $\mathbf{b}$ as discussed in class.

(iii) Solve the linear system $A^T A\mathbf{x} = A^T\mathbf{b}$ for the coefficient vector $\mathbf{x}$.

(iv) Plot the resulting polynomial.

Test your code using the data in problem A9.6 and for $q = 1$, 2, 3. Plot the least squares polynomials for each q.

```
function lsquare(x,y,n,q)
% program which computes and plots a least squares polynomial of degree q
%
% initialize b so that it is a column vector of zeros
b=zeros(n,1);
for i=1:n
   for j=1:q+1
      A(i,j)=x(i)^(q-j+1);
   end
   b(i)=y(i);
end
C=A'*A;
d=A'*b;
z=C\d;
h=(x(n)-x(1))/100;
for k=1:100
   xx(k)=x(1)+(k-1)*h;
   yy(k)=0;
   for j=1:q+1
      yy(k)=yy(k)+z(j)*xx(k)^(q-j+1);
   end
end
plot(xx,yy,x,y,'+')
```

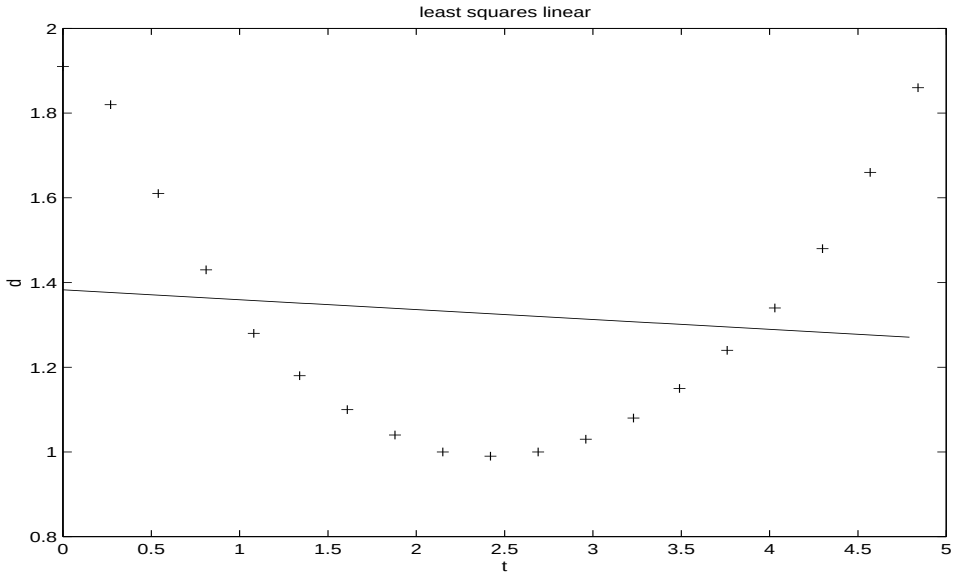

Figure 1: Plot of least squares linear

2. The data below measures vertical height above a reference height vs. spatial location (e.g., for mapping terrain). Construct least squares polynomials of degree $q = 1, \ldots, 7$ for the data. Plot your least squares polynomials versus the actual data as given in the sample figure below. Also generate and plot a cubic spline which interpolates the data, and compare to your least squares solutions. Which of the functions do you think best represents the data (justify your answer)? Hand in all plots and m-files. You do not have to hand in a diary.

Here is the data:

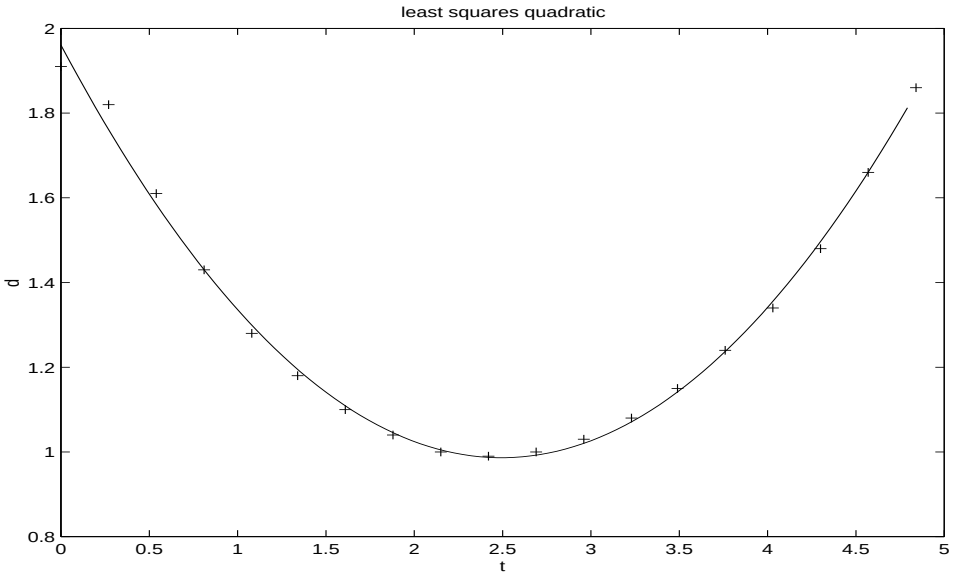

Figure 2: Plot of least squares quadratic

x	y
0	3.3
.5	2.1
1.5	-.5
1.6	1.2
2.8	6.9
3.1	8.1
4.0	7.9
5.5	4.3
6.2	1.1
6.5	-.1
7.0	-3.2
8.2	-4.5
9.6	-2.4
9.8	-2.4
11.1	1.9
12.5	2.6
13.0	4.7

From an aesthetic point of view, I would say the $q = 7$ least squares

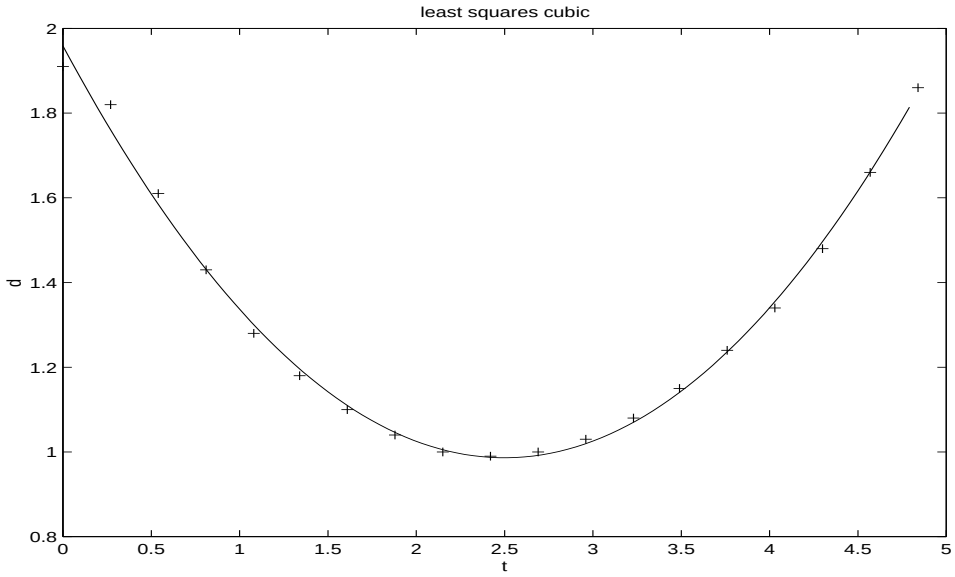

Figure 3: Plot of least squares cubic

polynomial is the best, with the spline a close second.

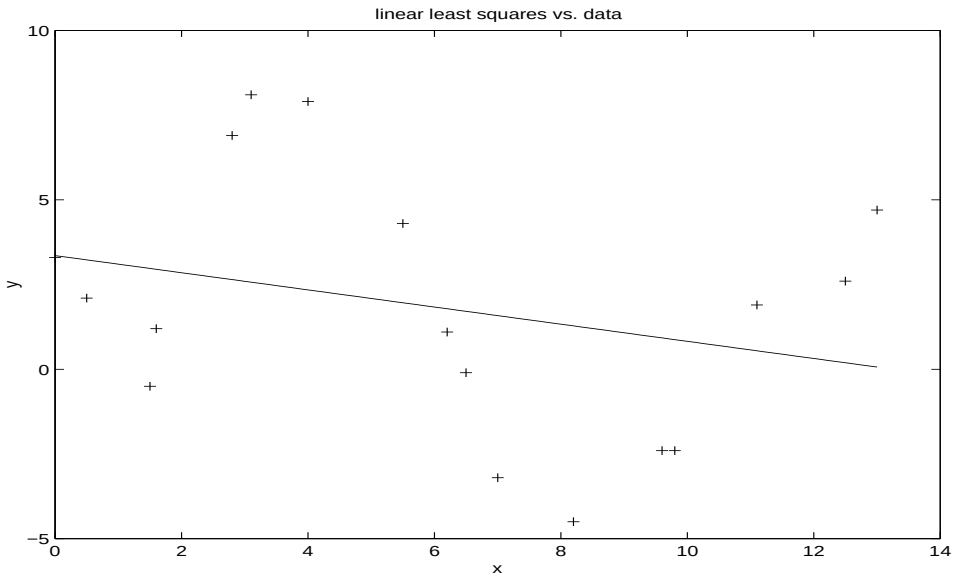

Figure 4: $q = 1$

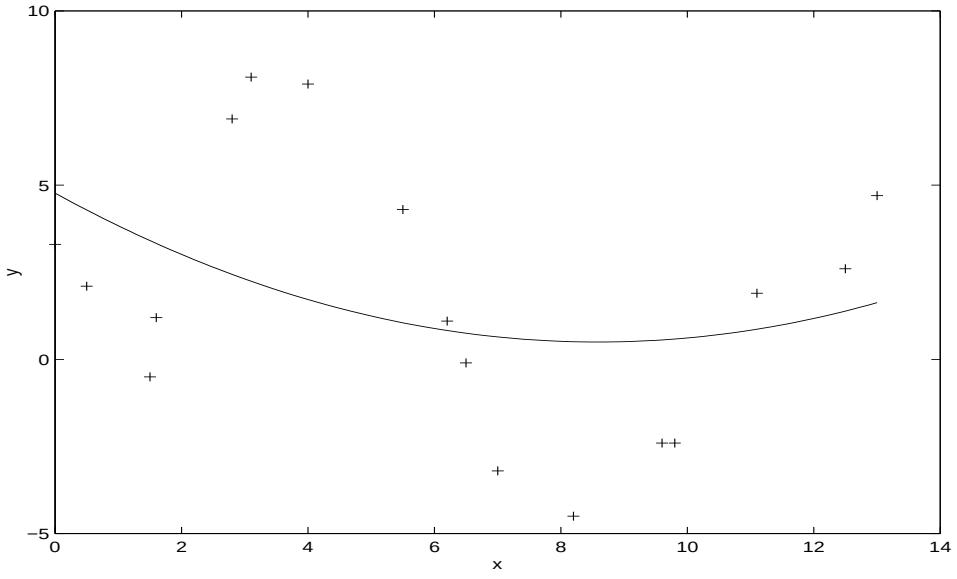

Figure 5: $q = 2$

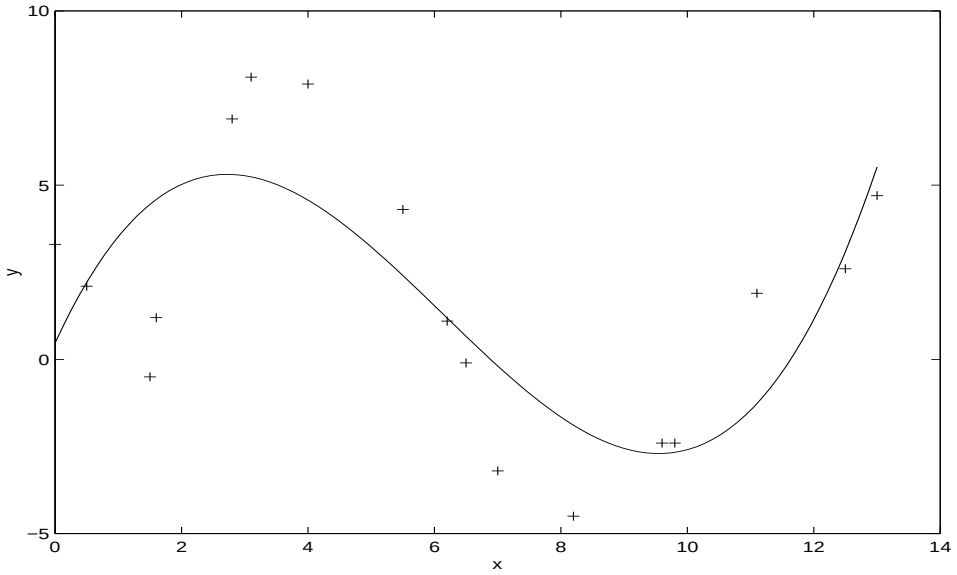

Figure 6: $q = 3$

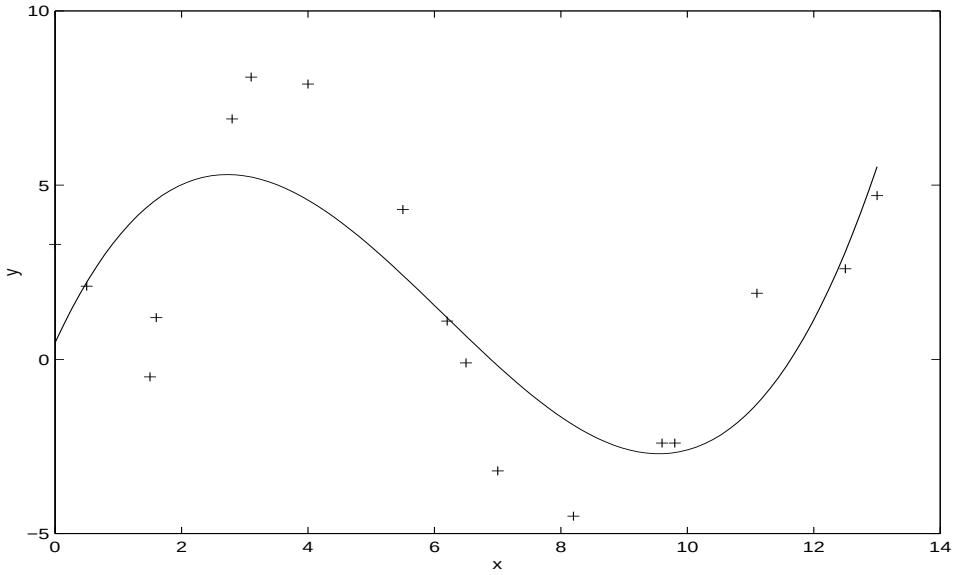

Figure 7: $q = 4$

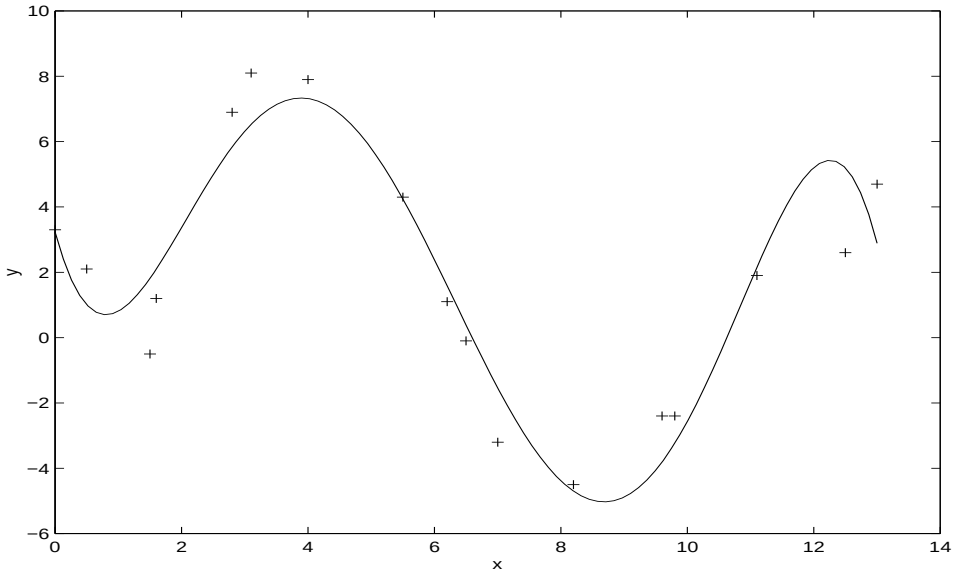

Figure 8: $q = 5$

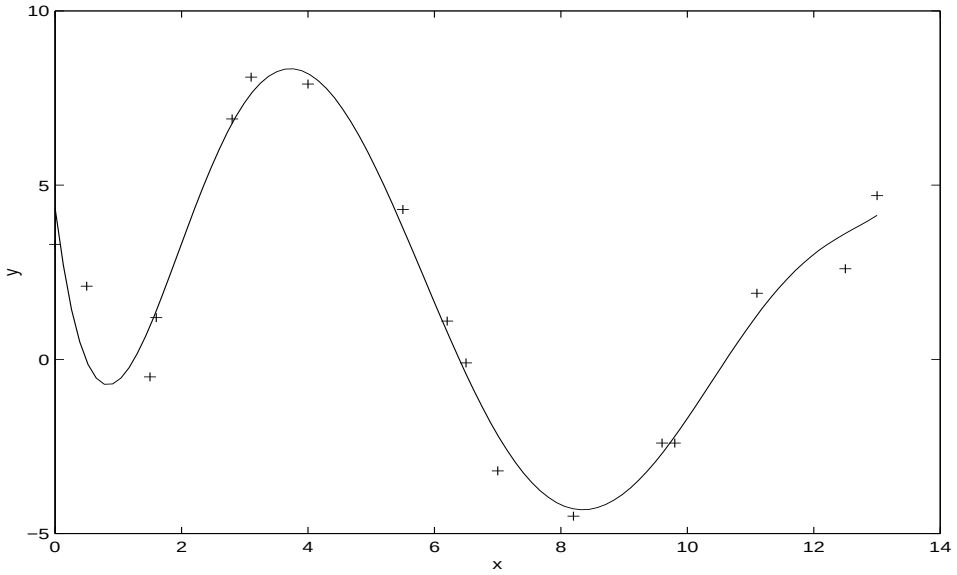

Figure 9: $q = 6$

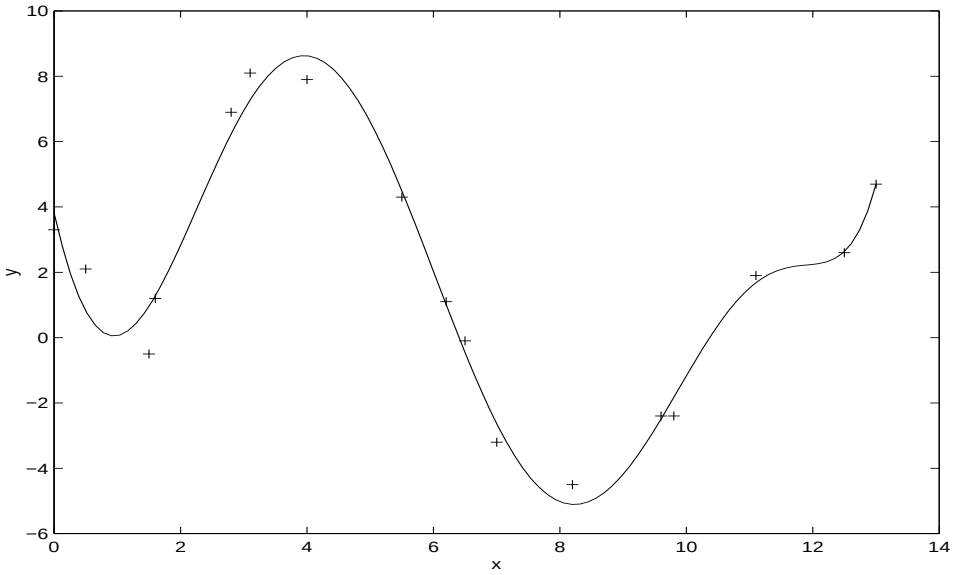

Figure 10: $q = 7$

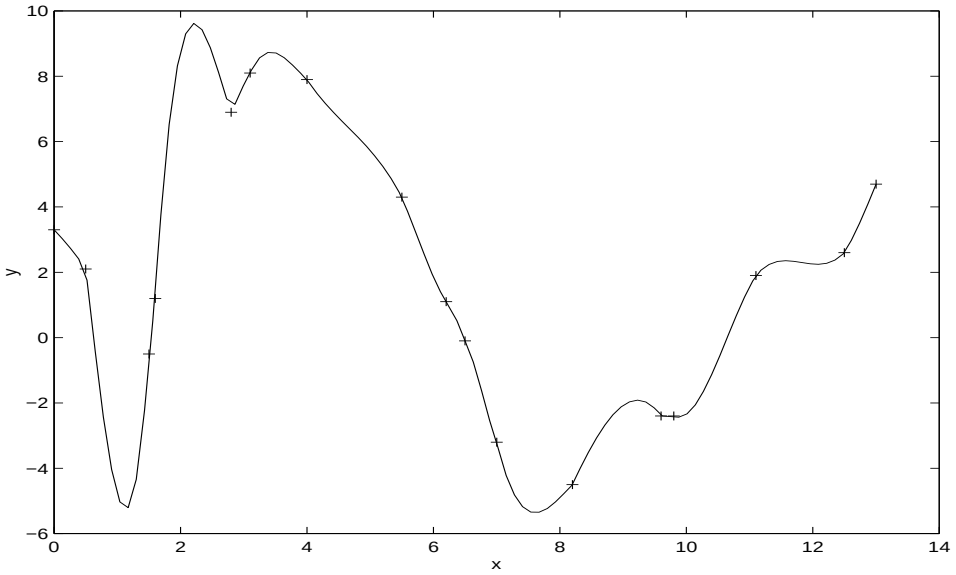

Figure 11: cubic spline